

2.4 STEADY STATE ERROR CONSTANT

The deviation of the output of control system from desired response during steady state is known as **steady state error**. It is represented as e_{ss} . We can find steady state error using the final value theorem as follows.

$$\lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s (R(s) - C(s))$$

Where,

$E(s)$ is the Laplace transform of the error signal, $e(t)$

Let us discuss how to find steady state errors for unity feedback and non-unity feedback control systems one by one.

Steady State Errors for Unity Feedback Systems

Consider the following block diagram of closed loop control system, which is having unity negative feedback.

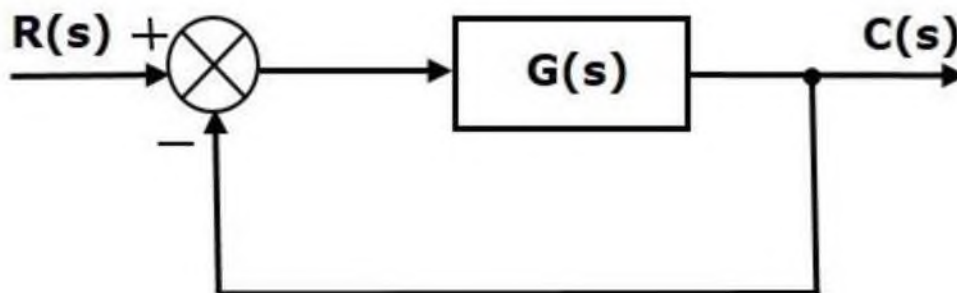


Figure 2.4.1: unit closed loop system

[Source: "Control System Engineering" by Nagoor Kani, page-2.33]

Where,

- $R(s)$ is the Laplace transform of the reference Input signal $r(t)$
- $C(s)$ is the Laplace transform of the output signal $c(t)$

We know the transfer function of the unity negative feedback closed loop control system as

$$\frac{R(s)G(s)}{1 + G(s)}$$

$$\frac{R(s)G(s)}{1 + G(s)}$$

The output of the summing point is -

$$E(s) = R(s) - C(s)$$

Substitute C(s) value in the above equation.

$$\frac{R(s)G(s)}{1 + G(s)}$$

$$E(s) = \frac{R(s) + R(s)G(s) - R(s)G(s)}{1 + G(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

Substitute E(s) value in the steady state error formula

$$= \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)}$$

The following table shows the steady state errors and the error constants for standard input signals like unit step, unit ramp & unit parabolic signals.

Input signal	Steady state error	Error constant
unit step signal	$\frac{1}{1 + k_p}$	$\lim_{s \rightarrow 0} (s)$
unit ramp signal	$\frac{1}{k_v}$	(s)
unit parabolic signal	1	$\lim_{s \rightarrow 0} (s^2)$

Where, k_p , k_v and k_a are position error constant, velocity error constant and acceleration error constant respectively.

Note - If any of the above input signals has the amplitude other than unity, then multiply corresponding steady state error with that amplitude.

Note - We can't define the steady state error for the unit impulse signal because, it exists only at origin. So, we can't compare the impulse response with the unit impulse input as t denotes infinity.

Example

Let us find the steady state error for an input signal $r(t) = (5 + 2t + t^2/2)u(t)$ of unity

negative feedback control system with $G(s)$

The given input signal is a combination of three signals step, ramp and parabolic. The following table shows the error constants and steady state error values for these three signals.

Input signal	Error constant	Steady state error
$r_1(t)=5u(t)$	$k_p = \lim_{s \rightarrow 0} (s) = \infty$	$e_{ss1} \sim \frac{1}{1+k_p} \sim 0$
$R_2(t)=2tu(t)$	$k_v = \lim_{s \rightarrow 0} (s)$	$e_{ss2} \sim \frac{2}{k_v} \sim T$
$r_3(t)=t^2/2 u(t)$	$k_a = \lim_{s \rightarrow 0} s^2 G(s)$	$e_{ss3} \sim \frac{1}{k_a} > 1$

We will get the overall steady state error, by adding the above three steady state errors.

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$$

$$e_{ss} = 0 + 0 + 1 = 1$$

Therefore, we got the steady state error e_{ss} as 1 for this example.

Steady State Errors for Non-Unity Feedback Systems

Consider the following fig.2.4.2 block diagram of closed loop control system, which is having nonunity negative feedback.

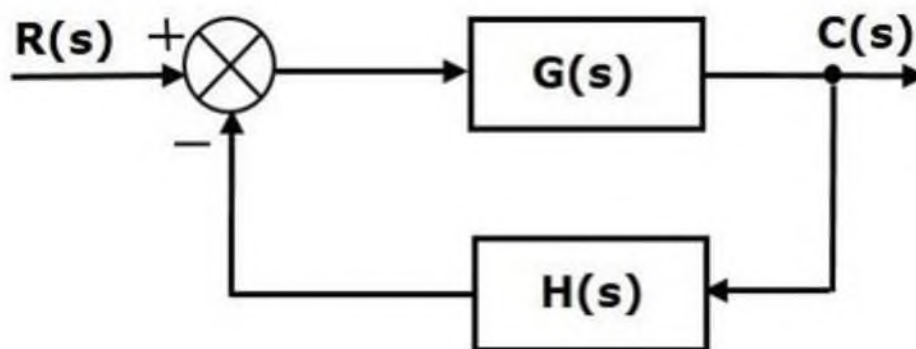


Figure 2.4.2: closed loop system

[Source: "Control System Engineering" by Nagoor Kani, page-2.33]

We can find the steady state errors only for the unity feedback systems. So, we have to convert the non-unity feedback system into unity feedback system. For this, include one unity positive feedback path and one unity negative feedback path in the above block diagram. The new block diagram looks like as shown below fig 2.4.3.

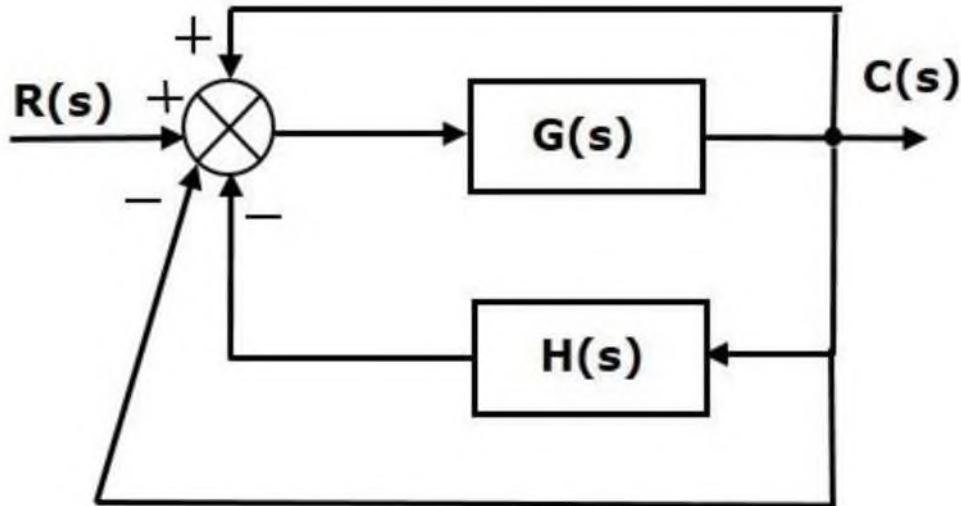


Figure 2.4.3: simplified block diagram

[Source: "Control System Engineering" by Nagoor Kani, page-2.33]

Simplify the above block diagram by keeping the unity negative feedback as it is. The following is the simplified block diagram in fig 2.4.4.

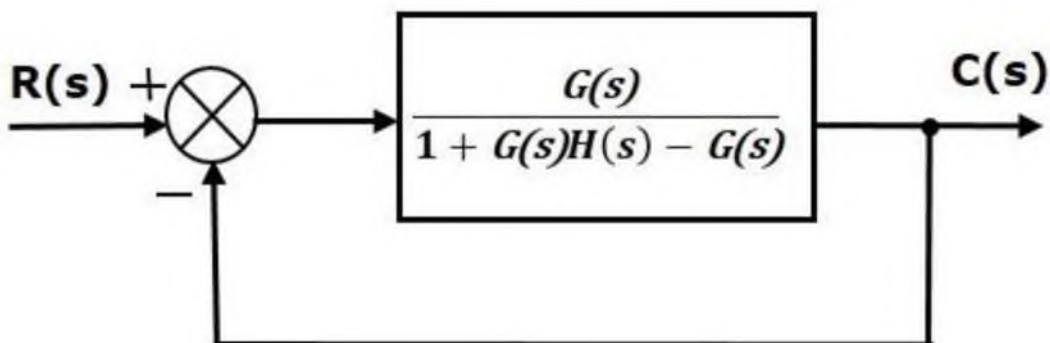


Figure 2.4.4: simplified unity feedback system

[Source: "Control System Engineering" by Nagoor Kani, page-2.34]

This block diagram resembles the block diagram of the unity negative feedback closed loop control system. Here, the single block is having the transfer function $G(s)/1+G(s)H(s)-G(s)$ instead of $G(s)$. You can now calculate the steady state errors by using steady state error formula given for the unity negative feedback systems.