Hybrid-Pi Model

Bipolar transistors are commonly used in circuits that amplify time-varying or sinusoidal signals. In these linear amplifier circuits, the transistor is biased in the forward-active region and small sinusoidal voltages and currents are superimposed on dc voltages and currents. In these applications, the sinusoidal parameters are of interest, so it is convenient to develop a small-signal equivalent circuit of the bipolar transistor using the small-signal admittance parameters of the pn junction.

Figure 1a shows an npn bipolar transistor in a common emitter configuration with the small-signal terminal voltages and currents. Figure 1b shows the cross section of the npn transistor. The C, B, and E terminals are the external connections to the transistor, while the C', B', and E' points are the idealized internal collector, base, and emitter regions.

We can begin constructing the equivalent circuit of the transistor by considering the various terminals individually. Figure 2a shows the equivalent circuit between the external input base terminal and the external emitter terminal. The résistance rb is the series resistance in the base between the external base terminal B and the internal base region B'. The B'-E' junction is forward biased, so C, is the junction diffusion capacitance and r, is the junction diffusion resistance. The diffusion capacitance C is,

$$C_d = \left(\frac{1}{2V_t}\right)(I_{p0}\tau_{p0} + I_{n0}\tau_{n0})$$

the diffusion resistance r,

$$r = \frac{V}{I_{DQ}}$$

The values of both parameters are functions of the junction current. These two elements are in parallel with the junction capacitance, which is C. Finally, r is the series resistance between the external emitter terminal and the internal emitter region. This resistance is usually very small and may be on the order of 1 to2 Ω.

Figure 2b shows the equivalent circuit looking into the collector terminal. The ***r,*** resistance is the series resistance between the external and internal collector connections and the capacitance **C,** is the junction capacitance of the reverse-biased collector-substrate junction

Figure 3 shows the equivalent circuit of the reverse-biased B'-C' junction. The C, parameter is the reverse-biased junction capacitance and r, is the 1 reverse-biased diffusion resistance.

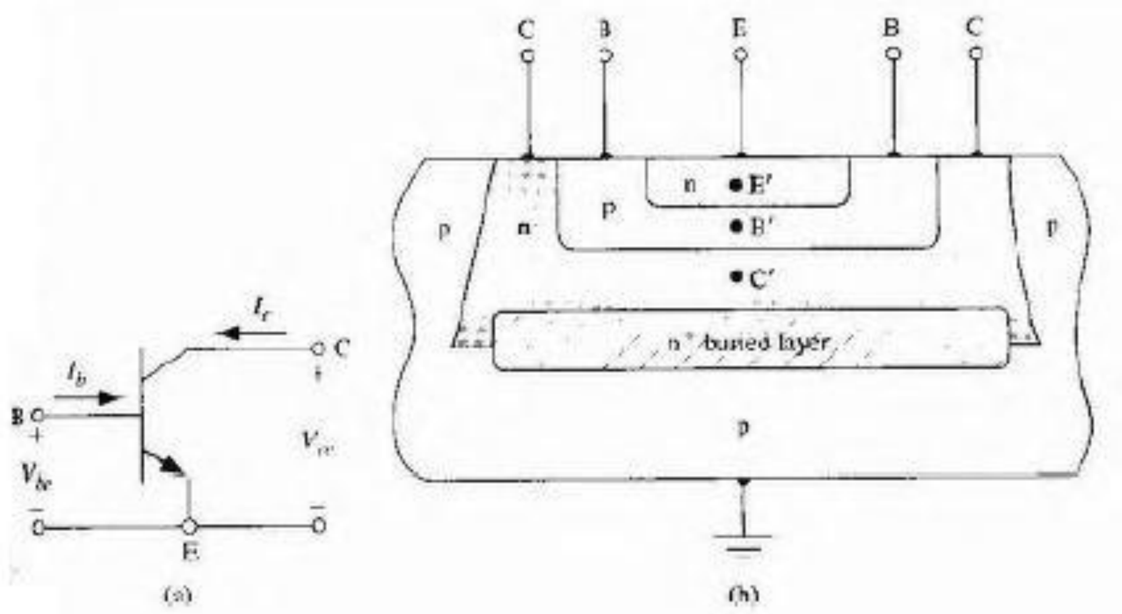
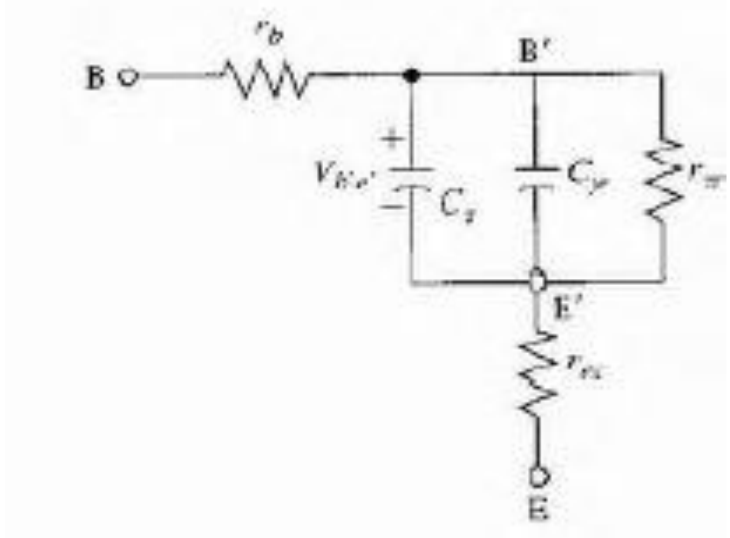


Figure 1 (a) Common emitter npn bipolar transistor with small-signal current and voltages. (b) Cross section of an npn bipolar transistor for the hybrid-pi model.



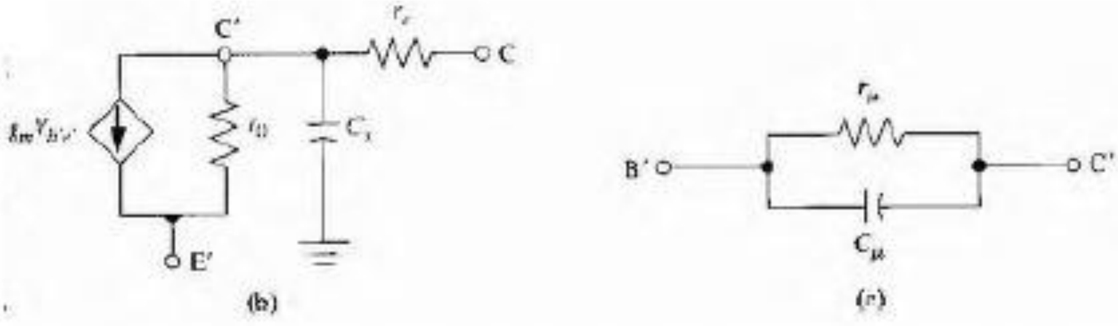


Figure 2 | Components of the hybrid-pi equivalent circuit between (a) the base and emitter, (b) the collector and emitter, and (c) the base and collector.

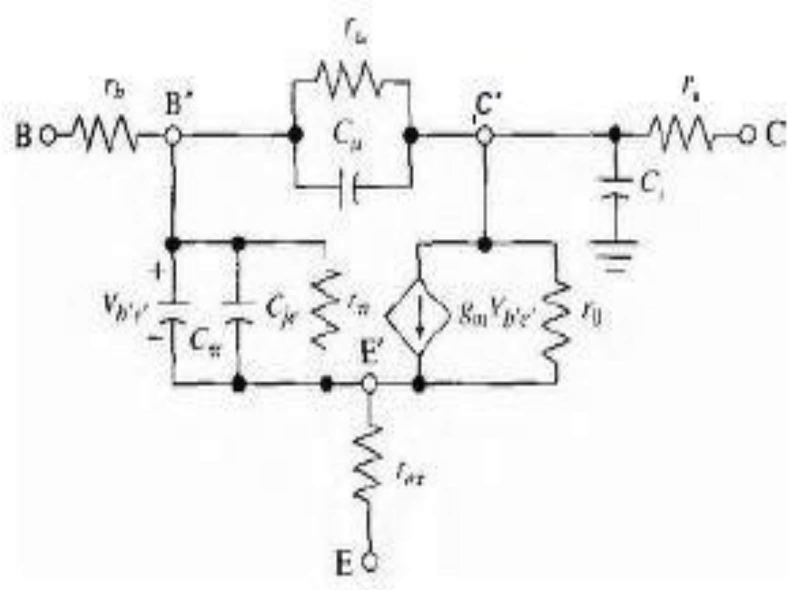


Figure 3 Hybrid-pi equivalent circuit.

Figure 3 shows the complete hybrid-pi equivalent circuit. A computer simulation is usually required for this complete model because of the large number of elements. However, some simplifications can be made in order to gain an appreciation for the frequency effects of the bipolar transistor.

h-PARAMETER BJT MODEL

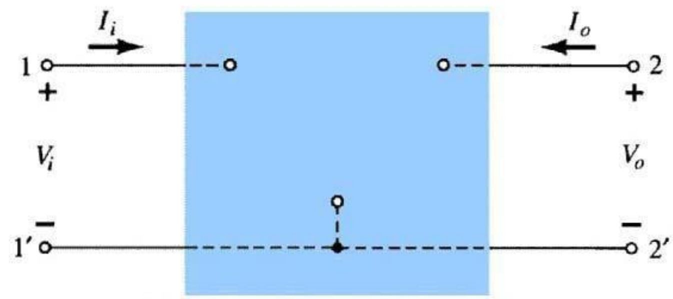
The h-parameter model is typically suited to transistor circuit modeling. It is important because:

1. its values are used on specification sheets
2. it is one model that may be used to analyze circuit behavior
3. it may be used to form the basis of a more accurate transistor model

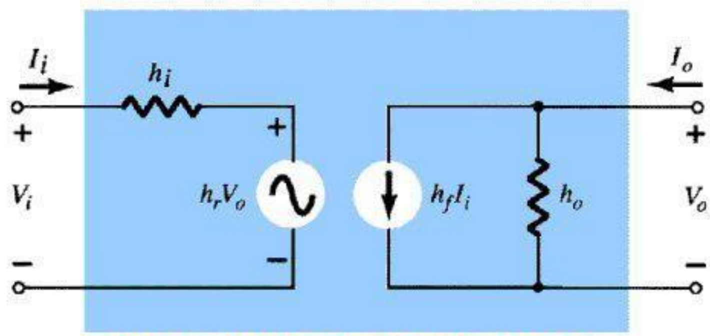
The h parameter model has values that are complex numbers that vary as a function of:

1. Frequency
2. Ambient temperature
3. Q-Point

The revised two port network for the h parameter model is shown on the right. At low and mid-band frequencies, the h parameter values are real values. Other models exist because this model is not suited for circuit analysis at high frequencies



Hybrid Equivalent Model



Hybrid Equivalent Circuit

The h-parameter model is defined by:

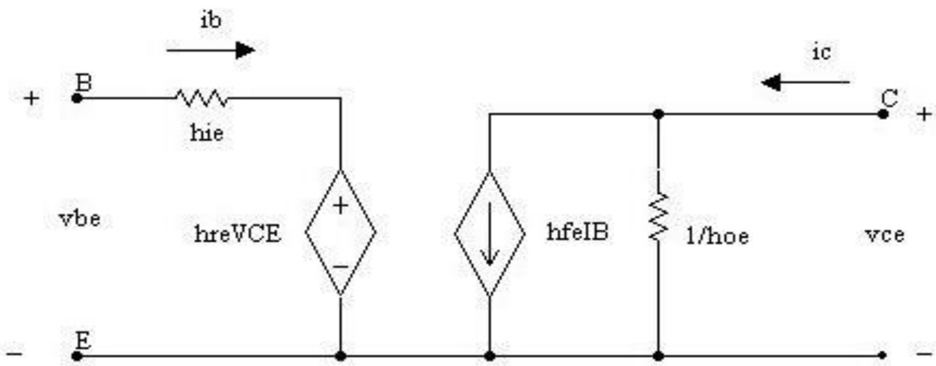
$$\begin{vmatrix} V_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ V_2 \end{vmatrix}$$

V1 = h11I1 +h12V2 (KVL)
I2 = h21I1 + h22V2 (KCL)

The h-parameter model for the common emitter circuit is on the fig. On spec sheet:

h11 = hix
h12 = hrx
h21 = hfx
h22 = hox

hrx and hfx are dimensionless ratios
hix is an impedance <Ω>
hox is an admittance <S>
where x = lead based on circuit configuration
e = emitter for common emitter
c = collector for common collector
b = base for common base

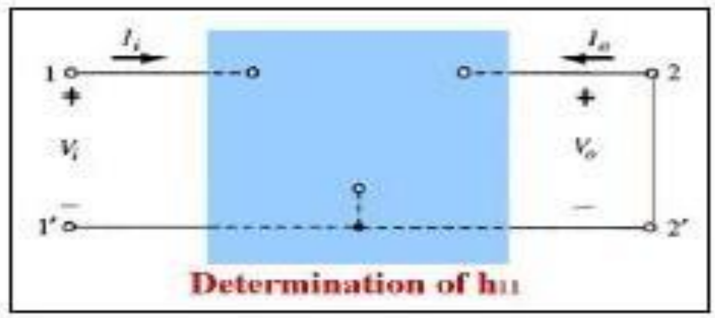


Short Circuit Input Impedance

$h_{11} = Z_{IN}$ with output shorted <Ω>

$$h_{11} = \left.\frac{V_i}{I_i}\right|_{V_O = 0}$$

1. Short terminals 2 2'
2. Apply test source V_i to terminal 1 1'
3. Measure I_i
4. Calculate h_{11}



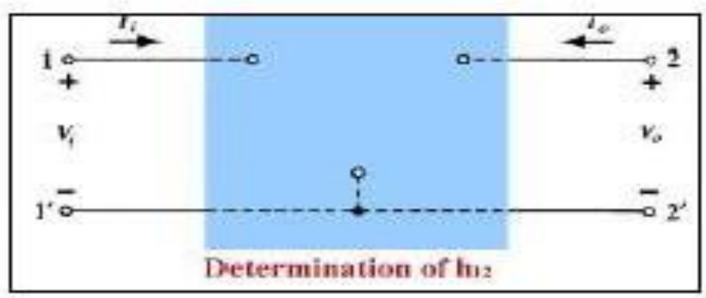
Determination of h_{11}

Open Circuit Reverse Transfer Ratio

h_{12} <dimensionless>

$$h_{12} = \left.\frac{V_i}{V_o}\right|_{I_i = 0}$$

1. Open terminals 1 1'
2. Apply test source V_2 to terminal 2 2'
3. Measure V_i
4. Measure V_o
5. Calculate h_{12}



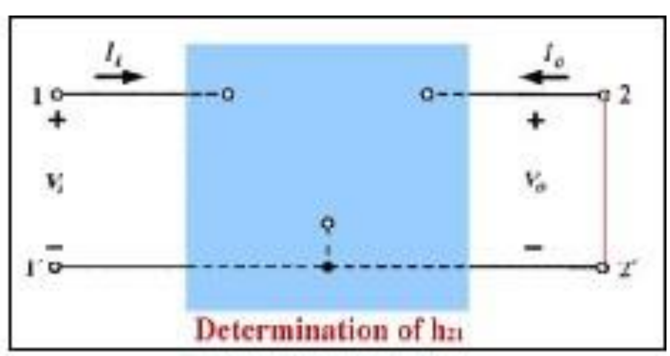
Determination of h_{12}

Short Circuit Forward Transfer Ratio

h_{21} = <dimensionless>

$$h_{21} = \left.\frac{I_o}{I_i}\right|_{V_O = 0}$$

1. Short terminals 2 2'
2. Apply test source V_i to terminal 1 1'
3. Measure I_i
4. Measure I_o
5. Calculate h_{21}



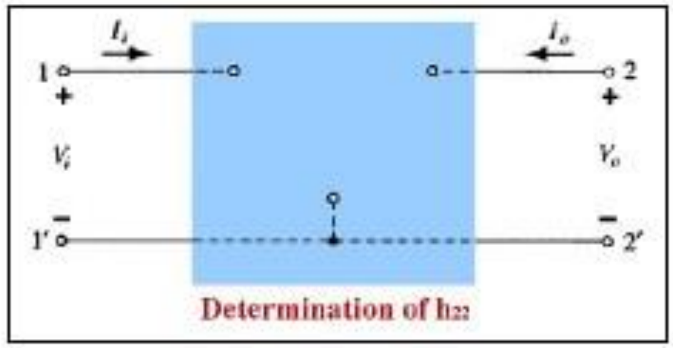
Determination of h_{21}

Open Circuit Output Admittance

h_{22} <Siemans>

$$h_{22} = \left.\frac{I_o}{V_o}\right|_{I_i = 0}$$

1. Open terminals 1 1'
2. Apply test source V_2 to terminal 2 2'
3. Measure I_o
4. Measure V_o
5. Calculate h_2



Determination of h_{22}

<u>Ebers-Moll Model</u>

The Ebers-Moll model, or equivalent circuit, is one of the classic models of the bipolar transistor. This particular model is based on the interacting diode junctions and applicable in any of the transistor operating modes. Figure shows the current directions and voltage polarities used in the Ebers Moll model. The currents are defined as all entering the terminals so that

$I_E + I_B + Ic = 0$

The direction of the emitter current is opposite to what we have considered up to point, but as long as we are consistent in the analysis, the defined direction does not matter.

The collector current can be written in general as

$Ic = \alpha_F I_F - I_R$

where α_F is the common base current rain in the forward-active mode. In this **mode.**

$$I_C = \alpha_F I_F + I_{CS}$$

where the current *lcs* is the reverse-bias B-C junction current. The current is given by

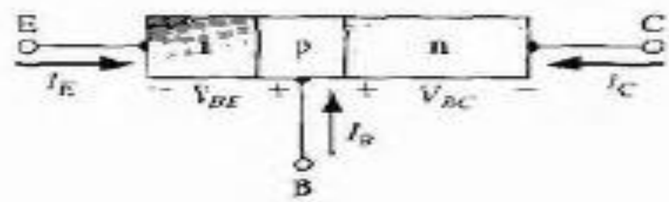
$$I_F = I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

If the B-C junction becomes forward biased, such as in saturation, then we can write the current *IR* as

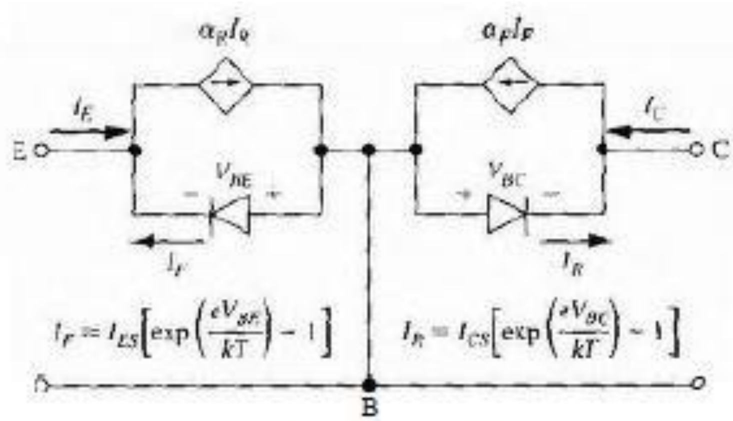
$$I_R = I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right) - 1\right]$$

Using above equations collector current written as

$$I_C = \alpha_F I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right] - I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right) - 1\right]$$



Current direction and voltage polarity definitions for the Ebers–Moll model.



Basic Ebers–Moll equivalent circuit.

We can also write the emitter current as

$$I_E = \alpha_R I_R - I_F$$

$$I_E = \alpha_R I_{CS}\left[\exp\left(\frac{eV_{BC}}{kT}\right) - 1\right] - I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

The current sources in the equivalent circuit represent current components that depend on voltages across other junctions. The Ebers-Moll model has four parameters: α_F, α_R, I_{ES} and lcs.
However, only three parameters are independent. The reciprocity relationship states that

$$\alpha_F I_{ES} = \alpha_R I_{CS}$$

Normally in electronic circuit applications, the collector-emitter voltage at saturation is of interest. We can define the C-E saturation voltage as

$$V_{CE}(\text{sat}) = V_{BE} - V_{BC}$$

Combining the previous some eqn we get

$$-(I_B + I_C) = \alpha_R I_{CS}\left[\exp\left(f\right) - 1\right] - I_{ES}\left[\exp\left(\frac{eV_{BE}}{kT}\right) - 1\right]$$

If we solve for the value of $[\exp(eV_{BC}/kT) - 1]$ and sub in previous one and simplifying we get

$$V_{BE} = V_t \ln\left[\frac{I_C(1 - \alpha_R) + I_B + I_{ES}(1 - \alpha_F\alpha_R)}{I_{ES}(1 - \alpha_F\alpha_R)}\right]$$

where Vs, is the thermal voltage. IIIly we solve $[\exp(eV_{BE}/kT) - 1]$ and substitute it, we get

$$V_{BC} = V_t \ln\left[\frac{\alpha_F I_B - (1 - \alpha_F)I_C + I_{CS}(1 - \alpha_F\alpha_R)}{I_{CS}(1 - \alpha_F\alpha_R)}\right]$$

We may neglect the I_{ES} and I_{CS} and we get

$$V_{CE}(\text{sat}) = V_{BE} - V_{CB} = V_t \ln\left[\frac{I_C(1 - \alpha_R) + I_B}{\alpha_F I_B - (1 - \alpha_F)I_C} \cdot \frac{I_{CS}}{I_{ES}}\right]$$

The ratio of Ics to I_{ES} can be written in terms of α_F and α_R and we finally get

$$V_{CE}(\text{sat}) = V_t \ln\left[\frac{I_C(1 - \alpha_R) + I_B}{\alpha_F I_B - (1 - \alpha_F)I_C} \cdot \frac{\alpha_F}{\alpha_R}\right]$$