5.4 *Milne's Predictor and Corrector Method*

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

1. *Using Milne's Method Given* $\frac{dy}{dx} = x^3 + y$*with* $y(0) = 2$, $y(0.2) = 2.073$, $y(0.4) = 2.452$, $y(0.6) = 3.023$*find* $y(0.8)$

solution:

$$Given: y' = f(x, y) = x^3 + y$$

$x_0 = 0$	$y_0 = 2$
$x_1 = 0.2$	$y_1 = 2.073$
$x_2 = 0.4$	$y_2 = 2.452$
$x_3 = 0.6$	$y_3 = 3.023$
$x_3 = 0.8$	$y_4 =?$

$$h = x_1 - x_0 = 0.2 - 0 = 0.2$$

By Milne's Predictor Method

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$y_4, p = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$We\ have\ y' = x^3 + y$$

$y'_1 = x_1^3 + y_1$	$y'_1 = (0.2)^3 + 2.073 = 0.004 + 2.073 = 2.081$
$y'_2 = x_2^3 + y_2$	$y'_2 = (0.4)^3 + 2.452 = 0.064 + 2.452 = 2.516$
$y'_3 = x_3^3 + y_3$	$y'_3 = (0.6)^3 + 3.023 = 0.216 + 3.023 = 3.239$

$$y_4, p = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$y_4, p = 2 + \frac{4(0.2)}{3}[2(2.081) - 2.516 + 2(3.239)]$$

$$y_4, p = 2 + \frac{0.8}{3}[8.124]$$

$$= 2 + 2.1664 = 4.1664$$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

$$y_4, c = y_2 + \frac{h}{3}\left[y'_{2} + 4y'_{3} + y'_{4}\right]$$

$$y'_4 = x_4^3 + y_4 = (0.8)^3 + 4.1664$$

$$= 0.512 + 4.1664$$

$$= 4.6784$$

$$y_4, c = 2.452 + \frac{0.2}{3}[2.513 + 4(3.239) + 4.6784]$$

$$= 2.452 + \frac{0.2}{3}[20.1504]$$

$$= 3.79536$$

2. *Using Milne's Method Given* $y' = x - y^2$ *andwith* $y(0) = 0$,
$y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ *find* $y(0.8)$

solution:

Given: $y' = f(x, y) = x - y^2$

$x_0 = 0$	$y_0 = 0$
$x_1 = 0.2$	$y_1 = 0.02$
$x_2 = 0.4$	$y_2 = 0.0795$
$x_3 = 0.6$	$y_3 = 0.1762$
$x_3 = 0.8$	$y_4 = ?$

$$h = x_1 - x_0 = 0.2 - 0 = 0.2$$

By Milne's Predictor Method

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$y_4, p = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

We have $y' = x - y^2$

$y_1' = x_1 - y_1^2$	$y_1' = 0.2 - (0.02)^2 = 0.2 - 0.0004 = 0.1996$
$y_2' = x_2 - y_2^2$	$y_2' = 0.4 - (0.0795)^2 = 0.4 - 0.0063 = 0.3937$
$y_3' = x_3 - y_3^2$	$y_3' = 0.6 - (0.0795)^2 = 0.4 - 0.0063 = 0.3937$

$$y_4, p = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$y_4, p = 2 + \frac{4(0.2)}{3}[2(2.081) - 2.516 + 2(3.239)]$$

$$y_4, p = 2 + \frac{0.8}{3}[8.124]$$

$$= 2 + 2.1664 = 4.1664$$

By Milne's Corrector Method

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

$$y_4, c = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$y_4' = x_4^3 + y_4 = (0.8)^3 + 4.1664$$

$$= 0.512 + 4.1664 = 4.6784$$

$$y_4, c = 2.452 + \frac{0.2}{3}[2.513 + 4(3.239) + 4.6784]$$

$$= 2.452 + \frac{0.2}{3}[20.1504]$$

$$= 3.79536$$