

2.4 VOLTAGE AND CURRENT FOR DISSIPATION LESS LINE

The voltage at any point at distance 's' from the receiving end of a transmission line is given by,

$$E = \frac{E_R}{2} \left[\frac{Z_R + Z_0}{Z_R} \right] \left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right] e^{-\sqrt{ZY}S} \right]$$

For dissipation less line,

$$\gamma = \sqrt{ZY}$$

$$\gamma = j\omega \sqrt{LC}$$

$$\beta = \omega \sqrt{LC}$$

$$\gamma = j\beta$$

$$E = \frac{E_R}{2Z_R} \left[(Z_R + Z_0) e^{j\beta S} + (Z_R - Z_0) e^{-j\beta S} \right]$$

$$E = \frac{E_R}{2Z_R} \left[Z_R e^{j\beta S} + Z_0 e^{j\beta S} + Z_R e^{-j\beta S} - Z_0 e^{-j\beta S} \right]$$

$$E = \frac{E_R}{2Z_R} \left[Z_R (e^{j\beta S} + e^{-j\beta S}) + Z_0 (e^{j\beta S} - e^{-j\beta S}) \right]$$

- $\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- $\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$E = \frac{E_R}{Z_R} \left[Z_R \frac{(e^{j\beta S} + e^{-j\beta S})}{2} + Z_0 \frac{(e^{j\beta S} - e^{-j\beta S})}{2} \right]$$

$$E = \frac{E_R}{Z_R} \left[Z_R \cos \beta S + jZ_0 \sin \beta S \right]$$

$$E = E_R \cos \beta S + \frac{jZ_0 E_R}{Z_R} \sin \beta S$$

- $I_R = \frac{E_R}{Z_R}$

$$E = E_R \cos \beta S + jZ_0 I_R \sin \beta S$$

This is the voltage equation for the dissipation less line.

$$I = \frac{I_R}{2} \left[\frac{Z_0 + Z_R}{Z_0} \right] \left[e^{\sqrt{ZY}S} - \frac{I_R}{2} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right] e^{-\sqrt{ZY}S} \right]$$

$$\gamma = \sqrt{ZY} = j\beta$$

$$I = \frac{I_R}{2Z_0} [(Z_R + Z_0) e^{j\beta S} - (Z_R - Z_0) e^{-j\beta S}]$$

$$I = \frac{I_R}{2Z_0} [Z_R e^{j\beta S} + Z_0 e^{j\beta S} - Z_R e^{-j\beta S} + Z_0 e^{-j\beta S}]$$

$$I = \frac{I_R}{2Z_0} [Z_R (e^{j\beta S} - e^{-j\beta S}) + Z_0 (e^{j\beta S} + e^{-j\beta S})]$$

$$I = \frac{I_R}{Z_0} \left[Z_R \frac{(e^{j\beta S} - e^{-j\beta S})}{2} + Z_0 \frac{(e^{j\beta S} + e^{-j\beta S})}{2} \right]$$

$$I = \frac{I_R}{Z_0} [Z_0 \cos \beta S + jZ_R \sin \beta S]$$

$$I = I_R \cos \beta S + \frac{jZ_R I_R}{Z_0} \sin \beta S$$

$$E_R = Z_R I_R$$

$$I = I_R \cos \beta S + \frac{jE_R}{Z_0} \sin \beta S$$

This is the current equation for the dissipation less line.

We know that,

$$Z_0 = \sqrt{\frac{L}{C}} = R_0$$

$$E = I_R \cos \beta S + \frac{jE_R}{R_0} \sin \beta S$$

These are the voltage and current equation of the dissipation less line.

The above equation represents the voltage in terms of receiving end voltage and current as well as current in terms of receiving end voltage and current.

The voltage and current distribution is the sum of cosine and sine distribution.

Wkt,

$$\beta = \frac{2\pi}{\lambda}$$

Sub in above equ,

$$E = E_R \cos \left(\frac{2\pi}{\lambda} S \right) + jR_0 I_R \sin \left(\frac{2\pi}{\lambda} S \right)$$

$$I = I_R \cos \left(\frac{2\pi}{\lambda} S \right) + \frac{jE_R}{R_0} \sin \left(\frac{2\pi}{\lambda} S \right)$$

CASE (i):

When the line is open circuited at the receiving end . $Z_R = \infty$, $I_R = 0$

$$E_{OC} = E_R \cos\left(\frac{2\pi}{\lambda}\right) S$$

$$I_{OC} = \sin\left(\frac{2\pi}{\lambda}\right) S$$

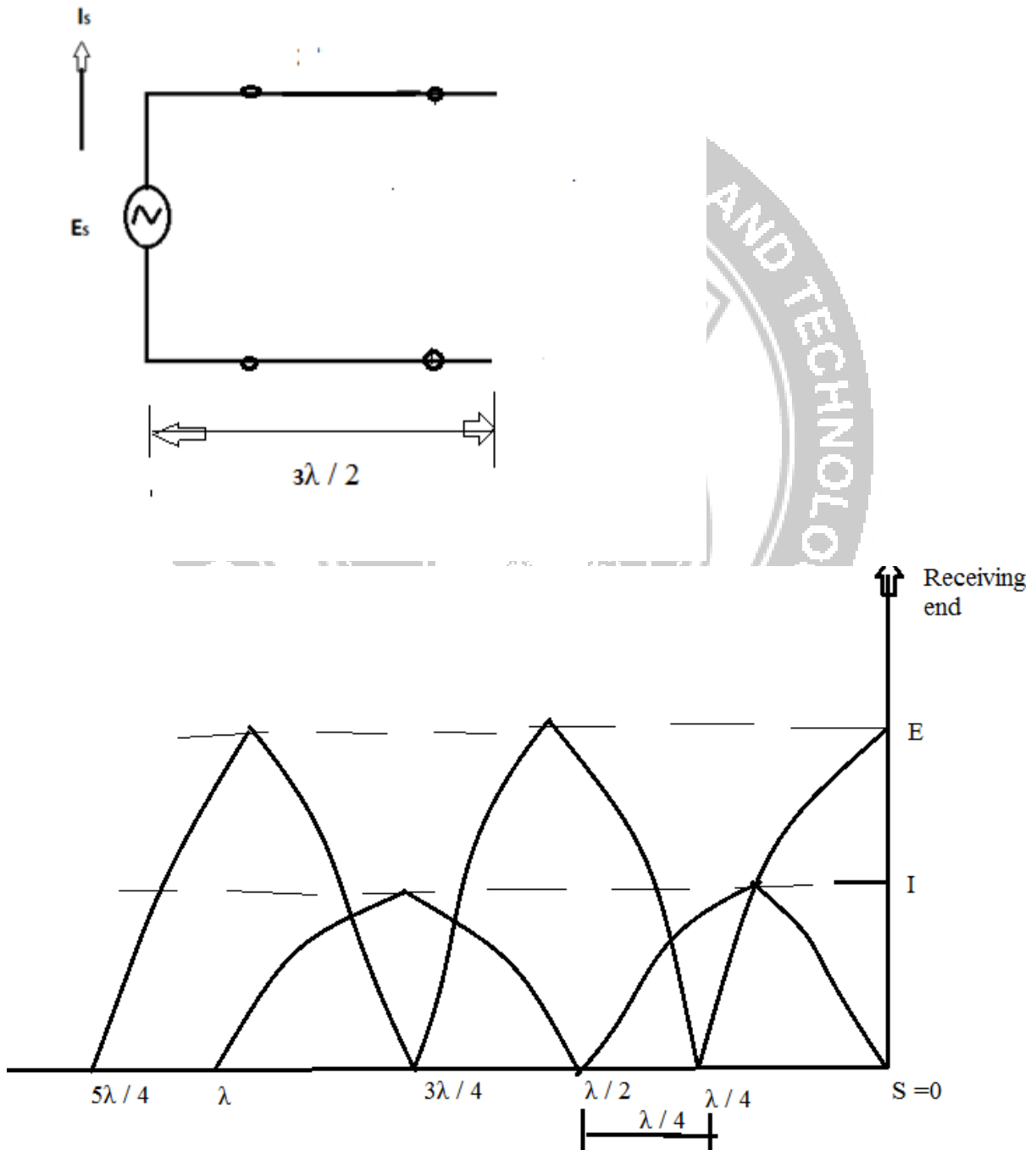


Fig: 2.4.1 Open circuit

The magnitude of voltage and current distribution for an open circuit line for $3\lambda/2$ distance is drawn in Fig 2.4.1. For every $\lambda/4$ distance the voltage changes from maximum to minimum and viceversa.

CASE (ii):

If the line is short circuited $Z_R = 0$, $E_R = 0$

$$E_{SC} = jR_o I_R \sin\left(\frac{2\pi}{\lambda}\right) S$$

$$I_{SC} = I_R \cos\left(\frac{2\pi}{\lambda}\right) S$$

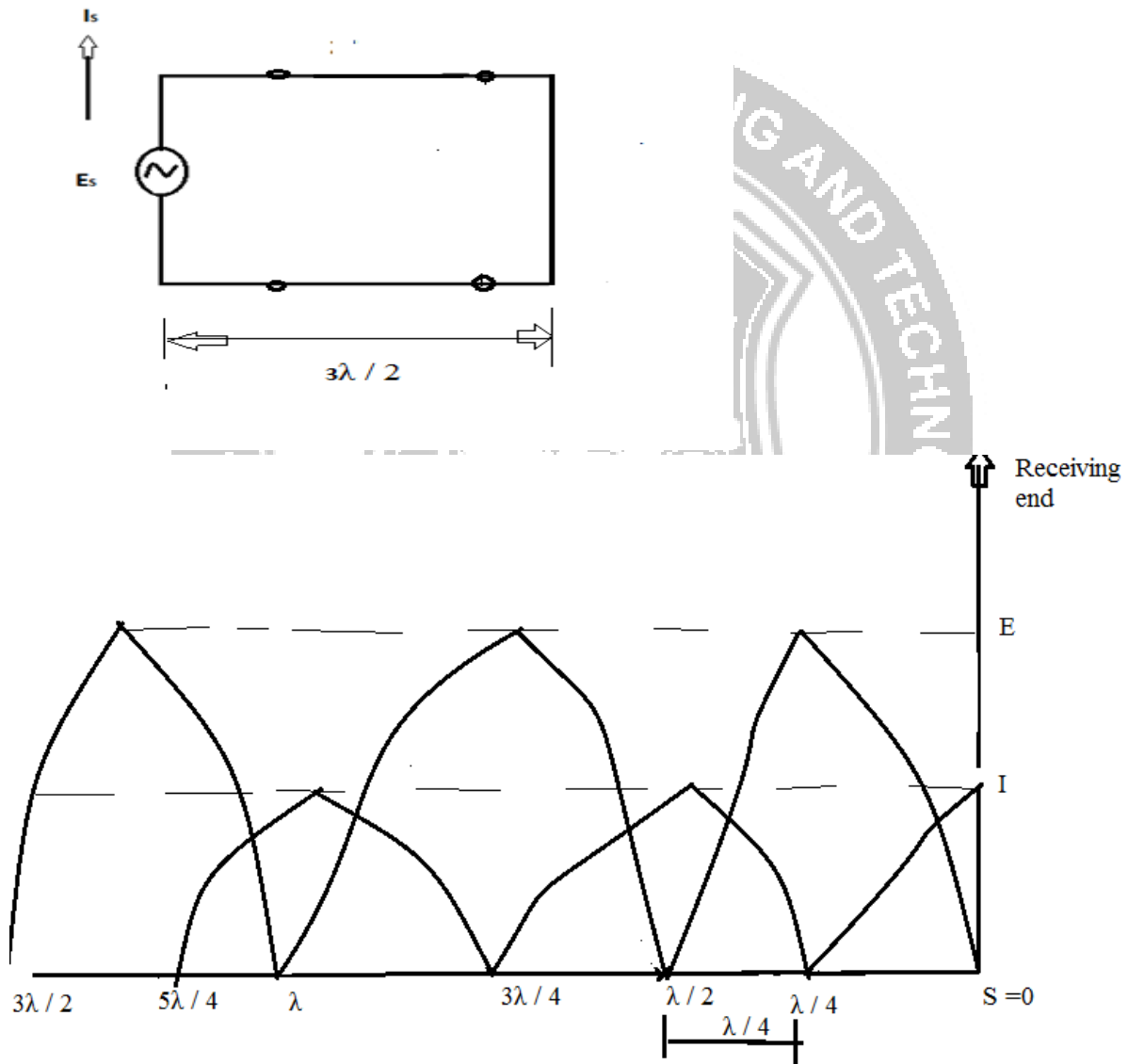


Fig: 2.4.2 Short circuit

The similarity of performance of short-circuited lines to that of series-resonant or antiresonant circuits may be readily noted by comparison of the curves of Fig 2.4.2.

CASE (iii):

In Fig 2.4.3, when the line is terminated with an impedance $Z_R = Z_o$

Wkt,

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

At high frequencies $Z_0 = R_0$

$$K = \frac{R_0 - R_0}{R_0 + R_0}$$

$$K = 0$$

This means that, the reflected wave is absent.

The voltage and current in the line is given by,

$$E = E_R e^{j\beta s}$$

$$I = I_R e^{j\beta s}$$

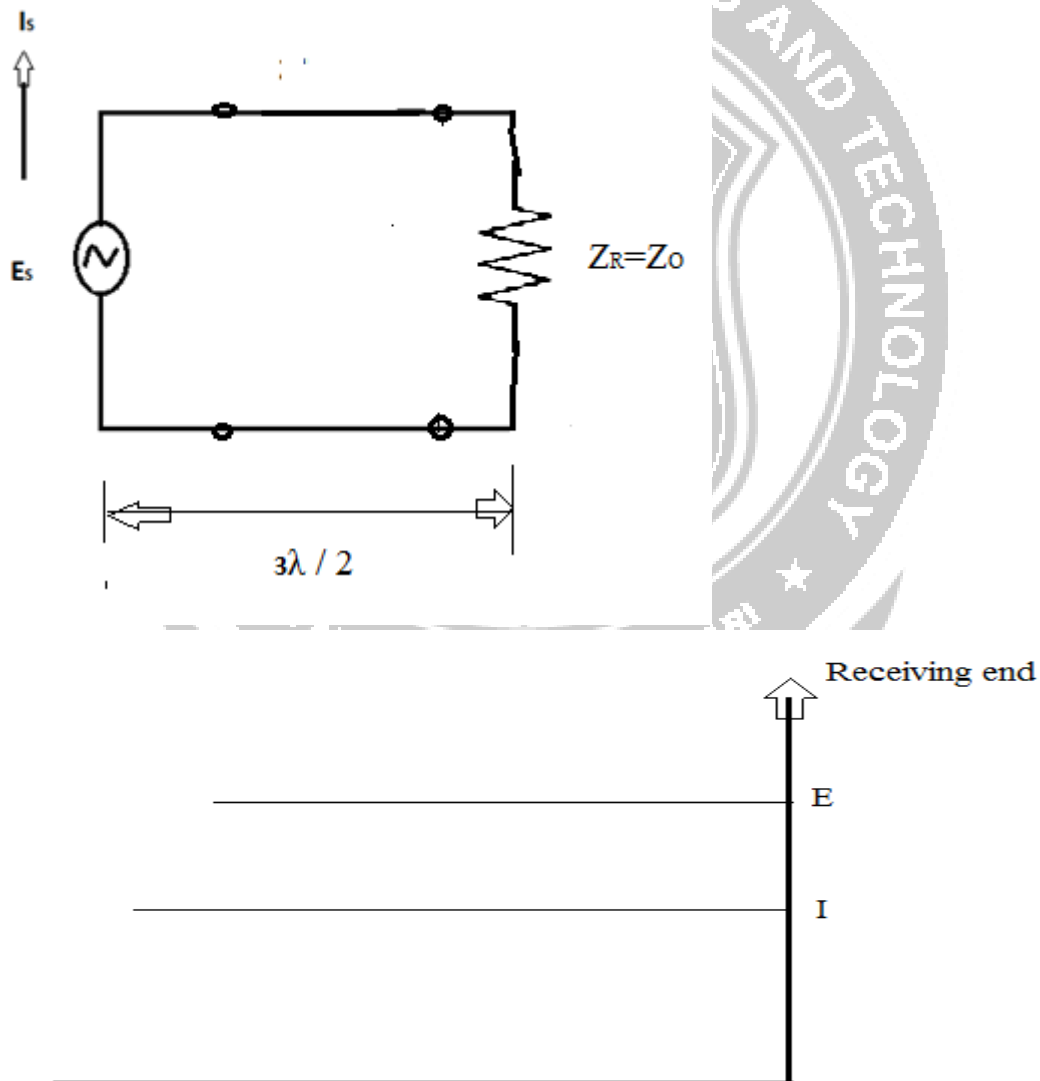
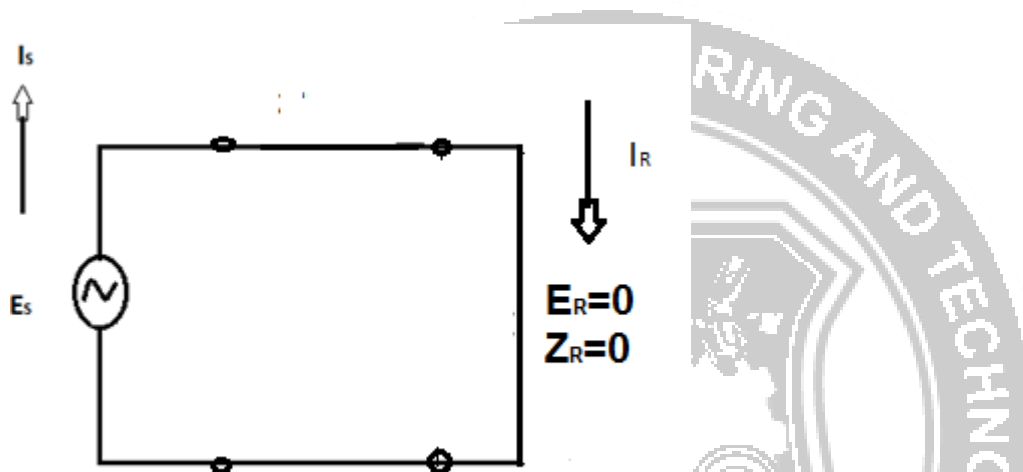


Fig: 2.4.3 Transmission line is terminated with an impedance $Z_R = Z_0$

OPEN AND SHORT CIRCUITED IMPEDANCE:**To find short circuit impedance:**

In Fig 2.4.4 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

**Fig: 2.4.4 Short circuit**

At short circuit condition $Z_R=0$, we know,

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan \beta s}{R_0 + jZ_R \tan \beta s} \right]$$

Sub $Z_R = 0$,

$$Z_{in} = R_0 \left[\frac{jR_0 \tan \beta s}{R_0} \right]$$

$$Z_{in} = jR_0 \tan \beta s$$

We know that,

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in Z_{in} ,

$$Z_{sc} = jR_0 \tan \left(\frac{2\pi}{\lambda} s \right)$$

$$R_s + jX_s = jR_0 \tan \left(\frac{2\pi}{\lambda} s \right)$$

Equating real and imag parts,

$$R_s = 0$$

$$X_s = R_0 \tan\left(\frac{2\pi}{\lambda}\right) s$$

$$\frac{X_s}{R_0} = \tan\left(\frac{2\pi}{\lambda}\right) s$$

Open Circuit impedance:

In Fig 2.4.5 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

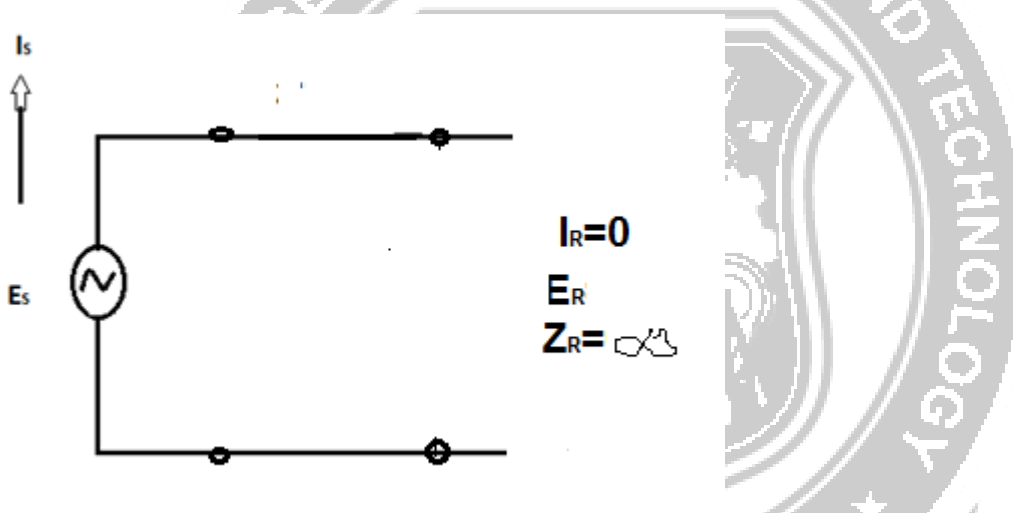


Fig: 2.4.5 Open circuit

$$Z_{in} = R_0 \left[\frac{Z_R + jR_0 \tan \beta s}{R_0 + jZ_R \tan \beta s} \right]$$

$$Z_{in} = R_0 \frac{Z_R}{Z_R} \left[\frac{1 + j \frac{R_0}{Z_R} \tan \beta s}{\frac{R_0}{Z_R} + j \tan \beta s} \right]$$

$Z_R = \infty$ sub in above equ,

$$Z_{oc} = R_0 \left[\frac{1}{j \tan \beta s} \right]$$

$$Z_{oc} = \frac{-jR_0}{\tan \beta s}$$

$$Z_{oc} = -jR_0 \cot \beta s$$

$$Z_{oc} = -jR_0 \cot \beta s$$

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in Z_{oc} ,

$$z_{oc} = -jR_0 \cot\left(\frac{2\pi}{\lambda}\right) s$$

$$R_s + jX_s = -jR_0 \cot\left(\frac{2\pi}{\lambda}\right) s$$

Equating real and imag parts,

$$R_s = 0$$

$$X_s = -R_0 \cot\left(\frac{2\pi}{\lambda}\right) s$$

$$\frac{X_s}{R_0} = -\cot\left(\frac{2\pi}{\lambda}\right) s$$

