Magnetic Flux Density:

In simple matter, the magnetic flux density $\vec{B}$ related to the magnetic field intensity $\vec{H}$ as $\vec{B} = \mu\vec{H}$ where μ called the permeability. In particular when we consider the free space $\vec{B} = \mu_0\vec{H}$ where $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m ^{2} .

The magnetic flux density through a surface is given by:

$$\psi = \int_s \vec{B}.d\vec{s} \quad \text{Wb} \quad \text{.......................................(4.18)}$$

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 4.7 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.

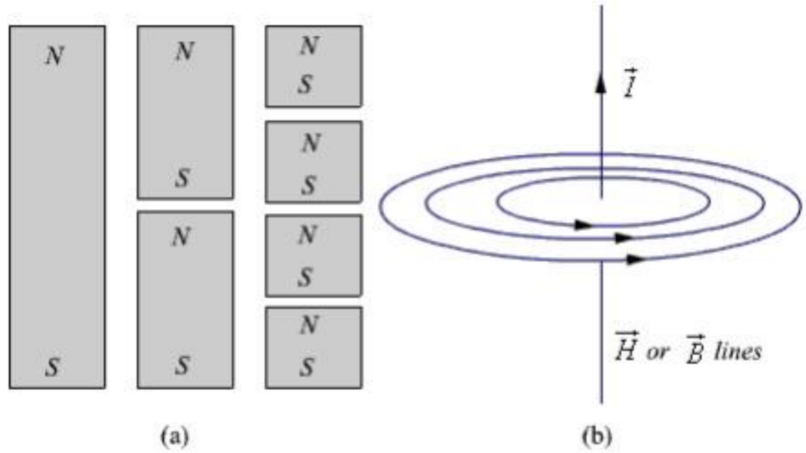

Fig. 4.7: (a) Subdivision of a magnet (b) Magnetic field/ flux lines of a straight current carrying conductor

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 4.7 (b), we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B}.d\vec{s} = 0 \quad \text{.......................................(4.19)}$$

which is the Gauss's law for the magnetic field.

By applying divergence theorem, we can write:

$$\oint_S \vec{B}.d\vec{s} = \int_V \nabla.\vec{B}dv = 0$$

Hence, $\nabla.\vec{B} = 0$ ....(4.20)

which is the Gauss's law for the magnetic field in point form.

Magnetic Scalar and Vector Potentials:

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a **scalar magnetic potential** and write:

$\vec{H} = -\nabla V_m$ ....(4.21)

From Ampere's law , we know that

$\nabla \times \vec{H} = \vec{J}$ ....(4.22)

Therefore, $\nabla \times (-\nabla V_m) = \vec{J}$ ....(4

But using vector identity, $\nabla \times (\nabla V) = 0$ we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$. Thus scalar magnetic potential is defined only in the region where $\vec{J} = 0$. Moreover, V_m in gener not a single valued function of position.

This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig 4.8.

In the region $a<\rho<b$, $\vec{J}=0$ and $\vec{H}=\frac{I}{2\pi\rho}\hat{a}_\phi$

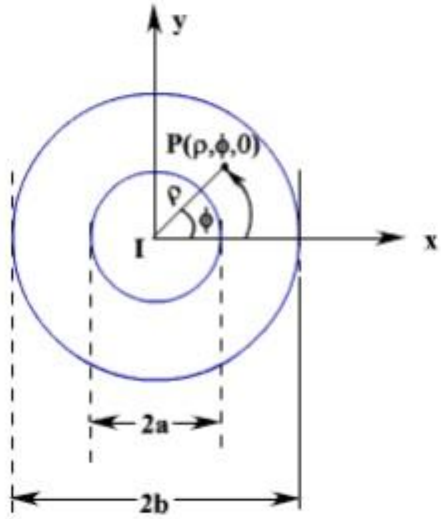

Fig. 4.8: Cross Section of a Coaxial Line

If V_m is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$$
$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\phi + c$$

If we set V_m = 0 at $\phi = 0$ then c=0 and $V_m = -\frac{I}{2\pi}\phi$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0$$

We observe that as we make a complete lap around the current carrying conductor , we reach ϕ_0 again but V_m this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

We observe that value of V_m keeps changing as we complete additional laps to pass through the same point. We introduced V_m analogous to electostatic potential V. But for static electric fields, $\nabla\times\vec{E}=0$ and $\oint\vec{E}\cdot d\vec{l}=0$, whereas for steady magnetic field $\nabla\times\vec{H}=0$ wherever $\vec{J}=0$ but $\oint\vec{H}\cdot d\vec{l}=I$ even if $\vec{J}=0$ along the path of integration.

We now introduce the **vector magnetic potential** which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since $\nabla.\vec{B}=0$ and we have the vector identity that for any vector $\vec{A}$, $\nabla.\left(\nabla\times\vec{A}\right)=0$, we can write $\vec{B}=\nabla\times\vec{A}$.

Here, the vector field $\vec{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\vec{A}$ of a given current distribution, $\vec{B}$ can be found from $\vec{A}$ through a curl operation.

We have introduced the vector function $\vec{A}$ and related its curl to $\vec{B}$. A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla.\vec{A}$ is made as follows.

$\nabla\times\nabla\times\vec{A}=\mu\nabla\times\vec{H}=\mu\vec{J}$ ....(4.24)

By using vector identity, $\nabla\times\nabla\times\vec{A}=\nabla(\nabla.\vec{A})-\nabla^2\vec{A}$ ....(4.25)

$\nabla(\nabla.\vec{A}) - \nabla^2\vec{A} = \mu\vec{J}$ ....(4.26)

Great deal of simplification can be achieved if we choose $\nabla.\vec{A} = 0$. Putting $\nabla.\vec{A} = 0$, we get $\nabla^2\vec{A} = -\mu\vec{J}$ which is vector poisson equation. In Cartesian coordinates, the above equation can be written in terms of the components as

$\nabla^2 A_x = -\mu J_x$ ....(4.27a)

$\nabla^2 A_y = -\mu J_y$ ....(4.27b)

$\nabla^2 A_z = -\mu J_z$ ....(4.27c)

The form of all the above equation is same as that of

$\nabla^2 V = -\frac{\rho}{\varepsilon}$ ....(4.28)

for which the solution is

$V = \frac{1}{4\pi\varepsilon}\int_{v'}\frac{\rho}{R}dv', \qquad R = \left|\vec{r} - \vec{r}'\right|$ ....(4.29)

In case of time varying fields we shall see that $\nabla.\vec{A} = \mu\varepsilon\frac{\partial V}{\partial t}$, which is known as Lorentz condition, *V* being the electric potential. Here we are dealing with static magnetic field, so $\nabla.\vec{A} = 0$.

By comparison, we can write the solution for Ax as

$A_x = \frac{\mu}{4\pi}\int_{v'}\frac{J_x}{R}dv'$ ....(4.30)

Computing similar solutions for other two components of the vector potential, the vector potential can be written as

$$\vec{A} = \frac{\mu}{4\pi}\int_{v}\frac{\vec{J}}{R}dv' \quad \text{.......................................(4.31)}$$

This equation enables us to find the vector potential at a given point because of a volume current density $\vec{J}$. Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi}\int_{L}\frac{I}{R}d\vec{l}' \quad \text{..(4.32)}$$

$$\vec{A} = \frac{\mu}{4\pi}\int_{S}\frac{\vec{K}}{R}ds' \quad \text{respectively. ....(4.33)}$$

The magnetic flux ψ through a given area S is given by

$$\psi = \int_{S}\vec{B}.d\vec{s} \quad \text{...(4.34)}$$

Substituting $\vec{B} = \nabla\times\vec{A}$

$$\psi = \int_{S}\nabla\times\vec{A}d\vec{s} = \oint_{C}\vec{A}d\vec{l} \quad \text{..(4.35)}$$

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.