1.4 INPUT AND TRANSFER IMPEDANCE:

INPUT IMPEDANCE:

The input impedance of a transmission line is given by,

$$Z_{in} = Z_S = Z_O \left[\frac{Z_R \cosh \gamma l + Z_O \sinh \gamma l}{Z_O \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

$$Z_S = Z_O \frac{Z_R \cosh \gamma l}{Z_R \cosh \gamma l}\left[\frac{1 + \frac{Z_O \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_O}{Z_R} + \frac{\sinh \gamma l}{\cosh \gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{1 + \frac{Z_O \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_O}{Z_R} + \tanh \gamma l}\right] \quad \text{.......(1)}$$

TRANSFER IMPEDANCE:

Let,

$$Z_T = \frac{E_S}{I_R} = \frac{Voltage\ at\ the\ receiving\ end}{Current\ at\ the\ sending\ end}$$

Transfer impedance of the transmission line, Now, the terminating current (I_R) cab be written as,

$$I_R = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$$

Divide the above equ by I_R,

$$1 = \frac{I_S}{I_R} \cosh\gamma l - \frac{E_S}{Z_O I_R} \sinh\gamma l$$

$$1 = \frac{I_S}{I_R} \cosh\gamma l - \frac{Z_T}{Z_O} \sinh\gamma l \quad \text{.......(2)}$$

We know that,

$$E_R = E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$$

$$I_S Z_O \sinh\gamma l = E_S \cosh\gamma l - E_R$$

$$I_S = \frac{E_S \cosh\gamma l - E_R}{Z_O \sinh\gamma l} \quad \text{........(3)}$$

Sub equ (3) in equ (2),

$$1 = \frac{\left(\frac{E_S \cosh\gamma l - E_R}{Z_O \sinh\gamma l}\right)}{I_R} \cosh\gamma l - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 = \left(\frac{E_S \cosh\gamma l - E_R}{I_R Z_O}\right) \frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O} \sinh\gamma l$$

$$1 = \left(\frac{E_S\cosh\gamma l}{I_R\, Z_O} - \frac{E_R}{I_R\, Z_O}\right)\frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O}\sinh\gamma l$$

$$1 = \left(\frac{Z_T\cosh\gamma l}{Z_O} - \frac{Z_R}{Z_O}\right)\frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O}\sinh\gamma l$$

$$1 = \frac{Z_T\cosh^2\gamma l}{Z_O\sinh\gamma l} - \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l} - \frac{Z_T}{Z_O}\sinh\gamma l$$

$$1 + \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T\cosh^2\gamma l}{Z_O\sinh\gamma l} - \frac{Z_T}{Z_O}\sinh\gamma l$$

$$1 + \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T}{Z_O}\left(\frac{\cosh^2\gamma l}{\sinh\gamma l} - \sinh\gamma l\right)$$

$$1 + \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T}{Z_O}\left(\frac{\cosh^2\gamma l - \sinh^2\gamma l}{\sinh\gamma l}\right)$$

$$\cosh^2\theta - \sinh^2\theta = 1$$

$$1 + \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l} = \frac{Z_T}{Z_O}\left(\frac{1}{\sinh\gamma l}\right)$$

$$Z_T = Z_O\sinh\gamma l\left[1 + \frac{Z_R}{Z_O}\frac{\cosh\gamma l}{\sinh\gamma l}\right]$$

$$Z_T = Z_O\sinh\gamma l\left[\frac{Z_O\sinh\gamma l + Z_R\cosh\gamma l}{Z_O\sinh\gamma l}\right]$$

$$Z_T = Z_O\sinh\gamma l + Z_R\cosh\gamma l$$

OPEN AND SHORT CIRCUIT IMPEDANCE:

FINITE LINE TERMINATED IN Z_O:

In Fig 1.4.1 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

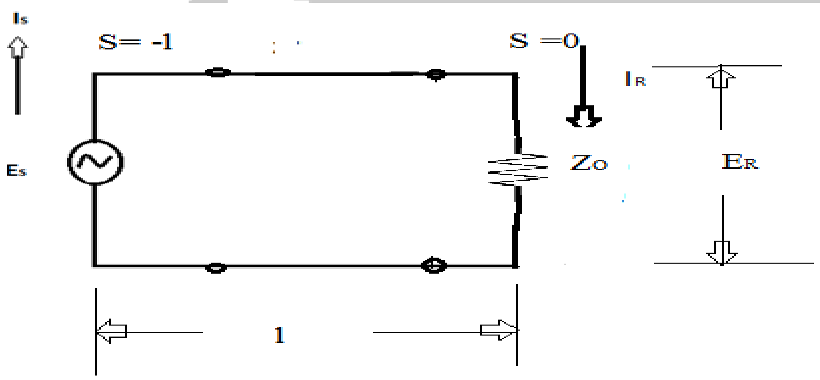

Fig: 1.4.1 Finite line terminated in Z_O

At distance S= l , E = E_R and I = I_R

E_R = $E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

I_R = $I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$Z_R = \frac{E_R}{I_R}$

$$Z_R = \frac{E_S \cosh\gamma l - I_S Z_O \sinh\gamma l}{I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l}$$

<u>FINITE LINE OPEN CIRCUITED AT DISTANCE END:</u>

In Fig 1.4.2 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

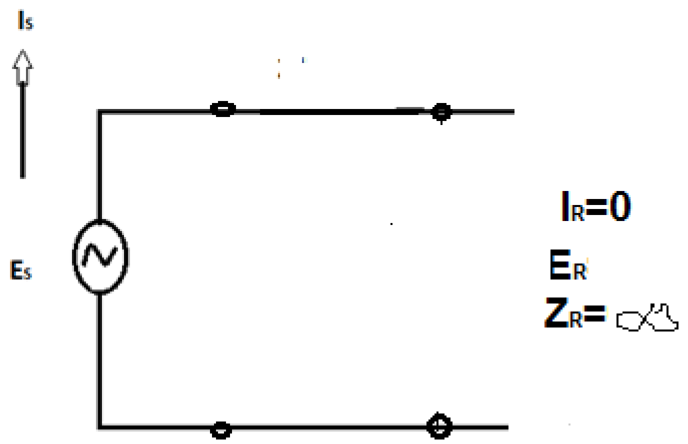

Fig: 1.4.2 Finite line open circuited at distance end

E_R = $E_S \cosh\gamma l - I_S Z_O \sinh\gamma l$

I_R = $I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

I_R = 0

$0 = I_S \cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$

$I_S \cosh\gamma l = \frac{E_S}{Z_O} \sinh\gamma l$

$$\frac{E_S}{I_S} = Z_O \frac{\cosh\gamma l}{\sinh\gamma l}$$

$$Z_{OC} = Z_O \frac{\cosh\gamma l}{\sinh\gamma l}$$

$$Z_{OC} = Z_O \coth\gamma l$$

If $l = \infty$

$$Z_{OC} = Z_O \coth\gamma(\infty)$$

$$Z_{OC} = Z_O$$

FINITE LINE SHORT CIRCUITED AT DISTANCE END:

In Fig 1.4.3 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

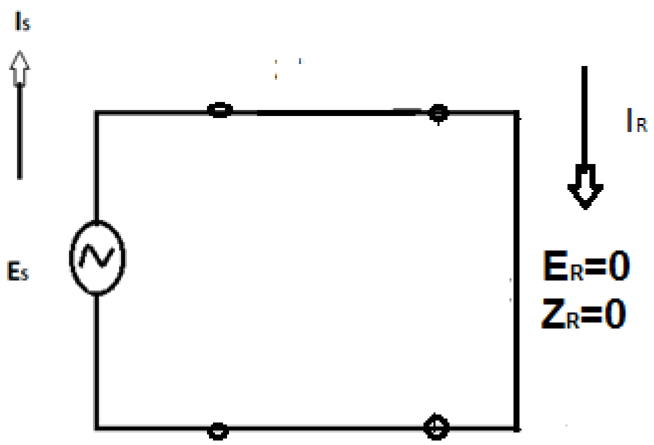

Fig: 1.4.3 Finite line short circuited at distance end

$$E_R = E_S\cosh\gamma l - I_S Z_O \sinh\gamma l$$

$$I_R = I_S\cosh\gamma l - \frac{E_S}{Z_O} \sinh\gamma l$$

$$E_R = 0$$

$$0 = E_S\cosh\gamma l - I_S Z_O \sinh\gamma l$$

$$E_S\cosh\gamma l = I_S Z_O \sinh\gamma l$$

$$\frac{E_S}{I_S} = Z_O \frac{\sinh\gamma l}{\cosh\gamma l}$$

$Z_{SC} = Z_O \tanh\gamma l$

If $l = \infty$

$Z_{SC} = Z_O \tanh\gamma(\infty)$

$Z_{SC} = Z_O$

Multiply the Z_{OC} & Z_{SC} expression.

$Z_{OC}\ Z_{SC} = Z_O \coth\gamma l Z_O \tanh\gamma l$

$Z_{OC}\ Z_{SC} = {Z_O}^2 \frac{1}{\coth\gamma l} \tanh\gamma l$

$Z_{OC}\ Z_{SC} = {Z_O}^2$

$Z_O = \sqrt{Z_{OC}\ Z_{SC}}$

INPUT IMPEDANCE INTERMS OF Z_O AND REFLECTION COFFIECIENT:

We know that,

Input impedance of the transmission line is,

$$Z_S = Z_O \left[\frac{Z_R \cosh\gamma l + Z_O \sinh\gamma l}{Z_O \cosh\gamma l + Z_R \sinh\gamma l}\right]$$

$$Z_S = Z_O \left[\frac{Z_R\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_O\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}{Z_O\left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_R\left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}\right]$$

$$Z_S = \frac{2Z_O}{2}\left[\frac{Z_R\left(e^{\gamma l} + e^{-\gamma l}\right) + Z_O\left(e^{\gamma l} - e^{-\gamma l}\right)}{Z_O\left(e^{\gamma l} + e^{-\gamma l}\right) + Z_R\left(e^{\gamma l} - e^{-\gamma l}\right)}\right]$$

$$Z_S = Z_O \left[\frac{Z_R e^{\gamma l} + Z_R e^{-\gamma l} + Z_O e^{\gamma l} - Z_O e^{-\gamma l}}{Z_O e^{\gamma l} + Z_O e^{-\gamma l} + Z_R e^{\gamma l} - Z_R e^{-\gamma l}}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l}\,[Z_R + Z_O] + e^{-\gamma l}\,[Z_R - Z_O]}{e^{\gamma l}\,[Z_R + Z_O] - e^{-\gamma l}\,[Z_R - Z_O]}\right]$$

$$Z_S = Z_O \frac{[Z_R + Z_O]}{[Z_R + Z_O]}\left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right]$$

$$Z_S = Z_O \left[\frac{e^{\gamma l} + e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}{e^{\gamma l} - e^{-\gamma l}\left[\frac{Z_R - Z_O}{Z_R + Z_O}\right]}\right]$$

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

$$Z_S = Z_O \left[\frac{e^{\gamma l} + e^{-\gamma l} K}{e^{\gamma l} - e^{-\gamma l} K}\right]$$

The above equation is interms of Z_O and reflection coefficient.