UNIT II
ANGLE MODULATION

FREQUENCY & PHASE MODULATION –

Representation of FM and PM signals, Spectral characteristics of angle modulated signal FM demodulator – Foster Seeley Discriminator

Introduction

Besides using the amplitude of carrier to carrier information, one can also use the angle of a carrier to carrier information. This approach is called angle modulation, and includes frequency modulation (FM) and phase modulation (PM). The amplitude of the carrier is maintained constant. The major advantage of this approach is that it allows the trade-off between bandwidth and noise performance. he other type of modulation in continuous-wave modulation is the **Angle Modulation**. Angle Modulation is the process in which the frequency or the phase of the carrier varies according to the message signal. This is further divided into frequency and phase modulation.

- Frequency Modulation is the process of varying the frequency of the carrier signal linearly with the message signal.
- Phase Modulation is the process of varying the phase of the carrier signal linearly with the message signal.

Frequency Modulation

In amplitude modulation, the amplitude of the carrier varies. But in Frequency Modulation (FM), the frequency of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal. The amplitude and the phase of the carrier signal remains constant whereas the frequency of the carrier changes. This can be better understood by observing the following figures Figure 2.1.1, Figure 2.1.2, Figure 2.1.3 represents Base band Signal , Carrier Signal and FM Signal.

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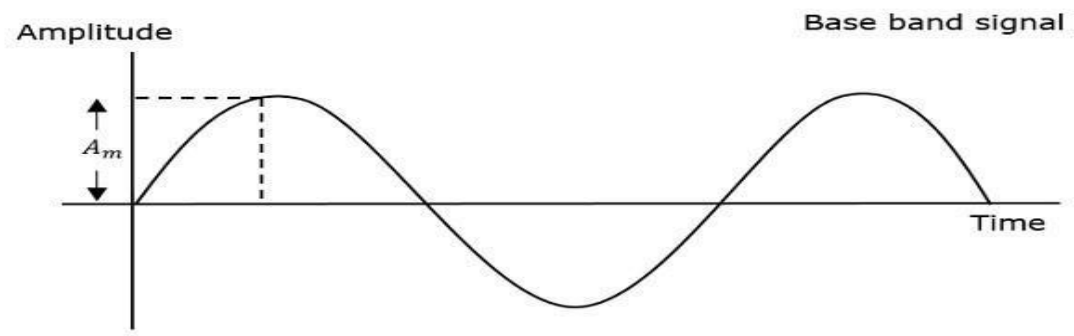

Figure 2.1.1 base Band Signal

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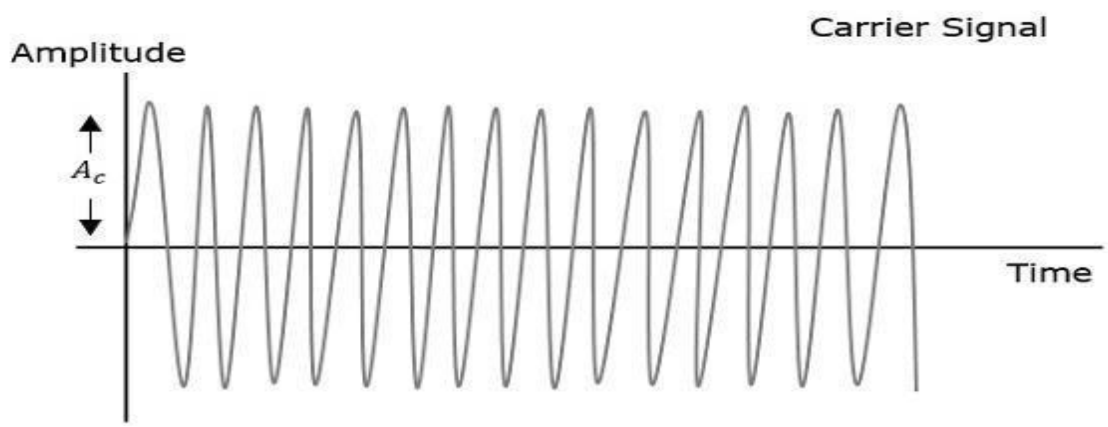

Figure 2.1.2 Carrier Signal

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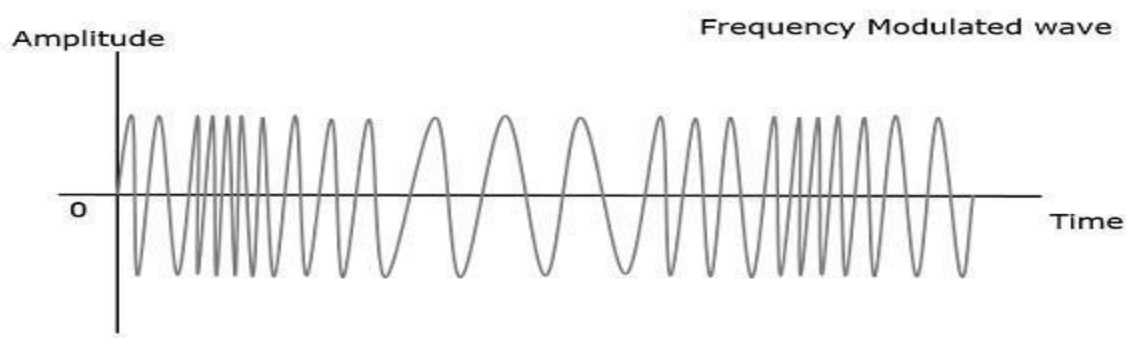

Figure 2.1.3 FM Signal

Diagram Source elprocus.com

EQUATIONS OF FM AND PM WAVES

An angle modulated signal can be written as

$$s(t) = A\cos\theta(t) \qquad (1)$$

where $\theta(t)$ is usually of the form $\theta\ t = 2\pi f_c t + \emptyset(t)$ and fc is the carrier frequency. The signal $\emptyset(t)$ is derived from the message signal m(t) . If $\emptyset\ t = k_p m(t)$ for some constant kp ,the resulting modulation is called phase modulation. The parameter kp is called the phase sensitivity.In telecommunications and signal processing, frequency modulation (FM) is the encoding of information in a carrier wave by varying the instantaneous frequency of the wave. (Compare with amplitude modulation, in which the amplitude of the carrier wave varies, while the frequency remains constant.) Frequency modulation is known as phase modulation when the carrier phase modulation is the time integral of the FM signal.

Modulation index:

As in other modulation systems, the value of the modulation index indicates by how much the modulated variable varies around its unmodulated level. It relates to variations in the carrier frequency. The modulation index of FM is defined as the ratio of the frequency deviation of the carrier to the frequency of the modulating signal

$$m_f = \text{Modulation Index of FM} = \Delta f/f_m \qquad (2)$$

The FM equation include the following

$$v = A \sin [\, \omega_c t + (\Delta f / f_m) \sin \omega_m t \,]$$

$$= A \sin [\, \omega_c t + m_f \sin \omega_m t \,] \qquad (3)$$

A = Amplitude of the FM signal. Δf = Frequency deviation

mf = Modulation Index of FM

$m_f = \Delta f/f_m$

mf is called the **modulation index** of frequency modulation.

m = 2π fm ωc = 2π fc (4)

The Bandwidth of Frequency Modulation Signal

Bandwidth is one of the main elements of FM signal. In FM signal, the sidebands will extend either side which will extend to infinity; however, the strength of them drops away. Auspiciously, it is the potential to restrict the BW of an FM signal without changing its value excessively. Recall, the bandwidth of a complex signal like FM is the difference between its highest and lowest frequency components, and is expressed in Hertz (Hz). Bandwidth deals with only frequencies. AM has only two sidebands (USB and LSB) and the bandwidth was found to be 2 fm. In FM it is not so simple. FM signal spectrum is quite complex and will have an infinite number of sidebands as shown in the Figure**.** This figure 2.1.4 gives an idea, how the spectrum expands as the modulation index increases. Sidebands are separated from the carrier by fc ± fm, fc ± 2fm, fc ± 3fm, and so on.

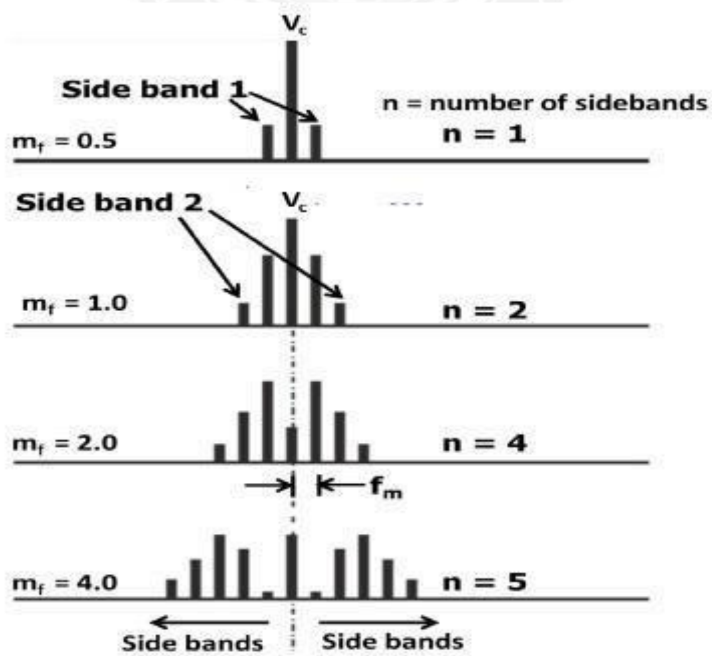

Figure 2.1.4 The Band width of FM Signal

Diagram Source elprocus.com

Only the first few sidebands will contain the major share of the power (98% of the total power) and therefore only these few bands are considered to be significant sidebands.
As a rule of thumb, often termed as Carson's Rule, 98% of the signal power in FM is contained within a bandwidth equal to the deviation frequency, plus the modulation frequency-doubled.

Carson's rule: Bandwidth of FM BWFM

$= 2 [\Delta f + fm].$

$= 2 fm [mf + 1]$

$$BT = 2(\delta + fm_{(max)}) \quad (5)$$

With a tone-modulated FM wave, if the modulation frequency is held constant and the modulation index is increased, the (non-negligible) bandwidth of the FM signal increases but the spacing between spectra remains the same; some spectral components decrease in strength as others increase. If the frequency deviation is held constant and the modulation frequency increased, the spacing between spectra increases.

Frequency modulation can be classified as narrowband if the change in the carrier frequency is about the same as the signal frequency, or as wideband if the change in the carrier frequency is much higher (modulation index >1) than the signal frequency. [6] For example, narrowband FM is used for two way radio systems such as Family Radio Service, in which the carrier is allowed to deviate only 2.5 kHz above and below the center frequency with speech signals of no more than 3.5 kHz bandwidth. Wideband FM is used for FM broadcasting, in which music and speech are transmitted with up to 75 kHz deviation from the center frequency and carry audio with up to a 20-kHz bandwidth.

The frequency of the modulated wave remains constant as the carrier wave frequency when the message signal is at zero. The frequency increases when the message signal reaches its maximum amplitude.

Which means, with the increase in amplitude of the modulating or message signal, the carrier frequency increases. Likewise, with the decrease in the amplitude of the modulating signal, the frequency also decreases.

Mathematical Representation

Let the carrier frequency be fc

The frequency at maximum amplitude of the message signal = $f_c + \Delta f$

The frequency at minimum amplitude of the message signal = $f_c - \Delta f$

The difference between FM modulated frequency and normal frequency is termed as Frequency Deviation and is denoted by Δf.

The deviation of the frequency of the carrier signal from high to low or low to high can be termed as the Carrier Swing.

$$\text{Carrier Swing} = 2 \times \text{frequency deviation}$$

$$= 2 \times \Delta f$$

Equation for FM WAVE

The equation for FM wave is

$$s(t) = A_c \cos[\omega_c t + 2\pi k_f m(t)] \qquad (6)$$

Where,

A_c = the amplitude of the carrier

ω_c = angular frequency of the carrier = $2\pi f_c$

$m(t)$ = message signal

FM can be divided into Narrowband FM and Wideband FM.

Narrowband FM

The features of Narrowband FM are as follows −

- This frequency modulation has a small bandwidth.
- The modulation index is small.
- Its spectrum consists of carrier, USB, and LSB.

This is used in mobile communications such as police wireless, ambulances, taxicabs, etc.

Wideband FM

The features of Wideband FM are as follows −

- This frequency modulation has infinite bandwidth.
- The modulation index is large, i.e., higher than 1.
- Its spectrum consists of a carrier and infinite number of sidebands, which are located around it.
- This is used in entertainment broadcasting applications such as FM radio, TV, etc.

Phase Modulation

- In frequency modulation, the frequency of the carrier varies. But in Phase Modulation (PM), the phase of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.
- The amplitude and the frequency of the carrier signal remains constant whereas the phase of the carrier changes. This can be better understood by observing the following Figure 2.1.5 Base Band Signal, Figure 2.1.6 , Carrier Signal and Figure 2.1.67 Phase Modulated Sigal

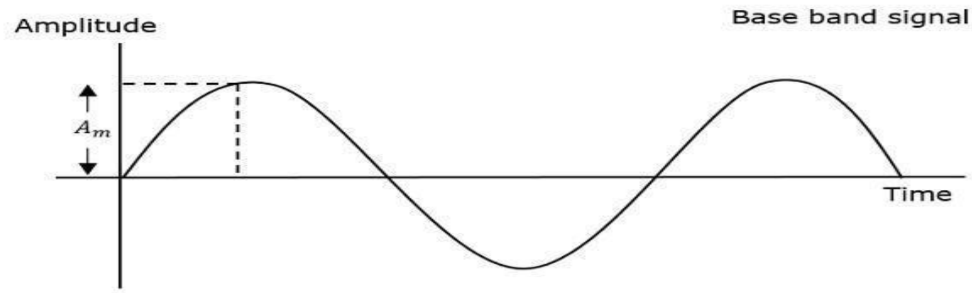

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Figure 2.1.5 Base Band Signal

Diagram Source Electronics Tutorials

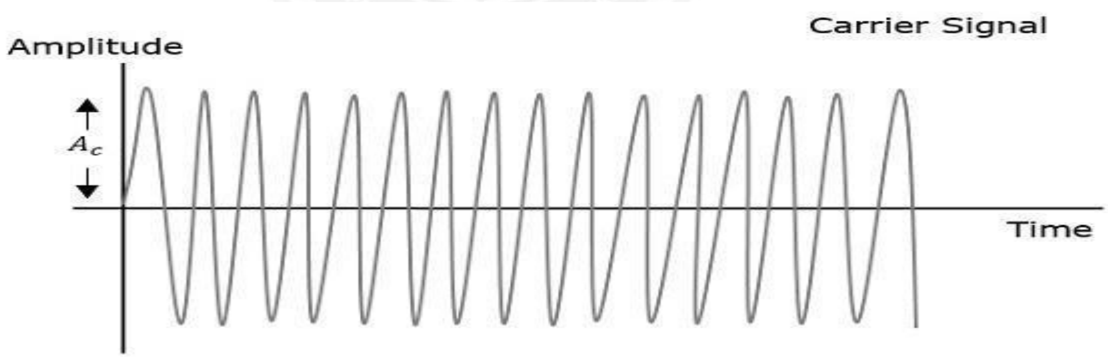

Figure 2.1.6 Carrier Signal

Diagram Source Source Brain Kart

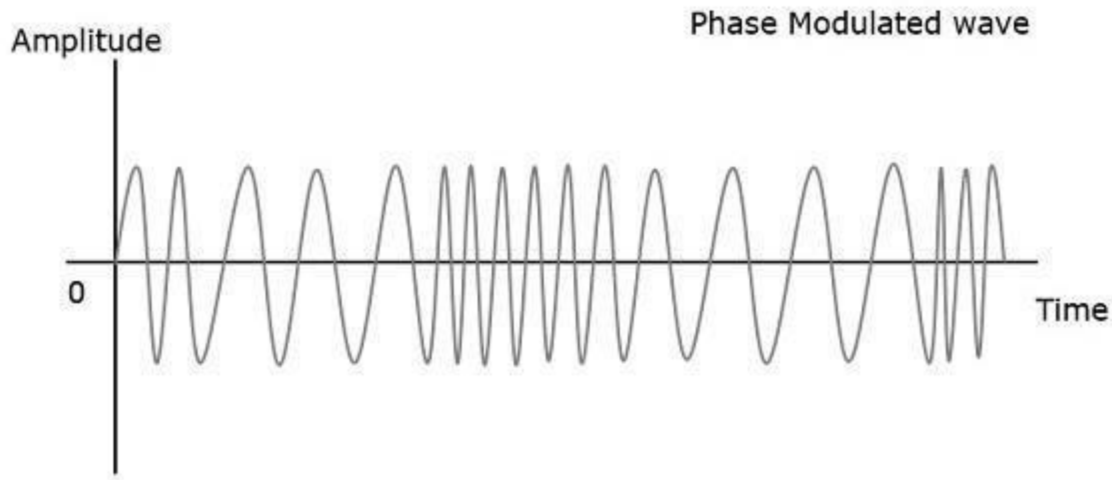

Figure 2.1.7 Phase Modulated Signal

Diagram Source Brain Kart

The phase of the modulated wave has got infinite points where the phase shift in a wave can take place. The instantaneous amplitude of the modulating signal, changes the phase of the carrier. When the amplitude is positive, the phase changes in one direction and if the amplitude is negative, the phase changes in the opposite direction.

- In frequency modulation, the frequency of the carrier varies. Whereas, in Phase Modulation (PM), the phase of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.
- So, in phase modulation, the amplitude and the frequency of the carrier signal remains constant. This can be better understood by observing the following figures.

The phase of the modulated wave has got infinite points, where the phase shift in a wave can take place. The instantaneous amplitude of the modulating signal changes the phase of the carrier signal. When the amplitude is positive, the phase changes in one direction and if the amplitude is negative, the phase changes in the opposite direction.

Mathematical Representation

The equation for instantaneous phase $\phi i \phi i$ in phase modulation is

$$\phi i = kpm(t) \quad (7)$$

Where,

- kp is the phase sensitivity
- m(t) is the message signal

The standard equation of angle modulated wave is

$$s(t)=Accos(2\pi fct+\phi i) \qquad (8)$$

Substitute, ϕi value in the above equation.

$$s(t)=Accos(2\pi fct+kpm(t)$$

This is the **equation of PM wave**.

If the modulating signal, m(t)=Amcos(2πfmt), then the equation of PM wave will be

$$s(t)=Accos(2\pi fct+\beta cos(2\pi fmt) \qquad (10)$$

Where,

- β = **modulation index** = Δϕ=kpAm
- Δϕ is phase deviation

Phase modulation is used in mobile communication systems, while frequency modulation is used mainly for FM broadcasting.

Relation between PM and FM

The change in phase, changes the frequency of the modulated wave. The frequency of the wave also changes the phase of the wave. Though they are related, their relationship is not linear. Phase modulation is an indirect method of producing FM. The amount of frequency shift, produced by a phase modulator increases with the modulating frequency. An audio equalizer is employed to compensate this.

The other type of modulation in continuous-wave modulation is Angle Modulation. Angle Modulation is the process in which the frequency or the phase of the carrier signal varies according to the message signal.

The standard equation of the angle modulated wave is

$$s(t)=Accos\ \theta i(t)$$

Where,

A_c is the amplitude of the modulated wave, which is the same as the amplitude of the carrier signal

$\theta_i(t)\theta_i(t)$ is the angle of the modulated wave

Angle modulation is further divided into frequency modulation and phase modulation.

- **Frequency Modulation** is the process of varying the frequency of the carrier signal linearly with the message signal.
- **Phase Modulation** is the process of varying the phase of the carrier signal linearly with the message signal.
- In amplitude modulation, the amplitude of the carrier signal varies. Whereas, in Frequency Modulation (FM), the frequency of the carrier signal varies in accordance with the instantaneous amplitude of the modulating signal.
- Hence, in frequency modulation, the amplitude and the phase of the carrier signal remains constant. This can be better understood by observing the following figures.
- The frequency of the modulated wave increases, when the amplitude of the modulating or message signal increases. Similarly, the frequency of the modulated wave decreases, when the amplitude of the modulating signal decreases. Note that, the frequency of the modulated wave remains constant and it is equal to the frequency of the carrier signal, when the amplitude of the modulating signal is zero.

Mathematical Representation

The equation for instantaneous frequency $f_i f_i$ in FM modulation is

$f_i=f_c+k_f m(t)$ Where, f_c is the carrier frequency

k_t is the frequency sensitivity

$m(t)$ is the message signal

We know the relationship between angular frequency ω_i and angle $\theta_i(t)$ as

$$\omega_i = d\theta_i(t)/dt$$

$$\Rightarrow 2\pi f_i = d\theta_i(t)/dt \quad (11)$$

$$\Rightarrow \theta_i(t) = 2\pi\int f_i dt \quad (12)$$

Substitute, fi value in the above equation.

$$\theta i(t)=2\pi\int(fc+kfm(t))dt$$

$$\theta i(t)=2\pi fct+2\pi kf\int m(t)dt \qquad (13)$$

Substitute, θi(t) value in the standard equation of angle modulated wave.

$$s(t)=Accos(2\pi fct+2\pi kf\int m(t)dt) \qquad (14)$$

This is the **equation of FM wave**.

If the modulating signal is $m(t)=Amcos(2\pi fmt)$, then the equation of FM wave will be

$$s(t)=Accos(2\pi fct+\beta sin(2\pi fmt)$$

Where,

β = **modulation index** $=\Delta f/fm$

$$=kfAm / fm \qquad (15)$$

The difference between FM modulated frequency (instantaneous frequency) and normal carrier frequency is termed as Frequency Deviation. It is denoted by Δf, which is equal to the product of kf and Am. FM can be divided into Narrowband FM and Wideband FM based on the values of modulation index β.