Active filters:

An electric filter is often a frequency selective circuit that passes a specified band of frequencies and blocks or alternates signal and frequencies outside this band. Filters may be classified as

1. Analog or digital.
2. Active or passive
3. Audio (AF) or Radio Frequency (RF)

1. Analog or digital filters:

Analog filters are designed to process analog signals, while digital filters process analog signals using digital technique.

2. Active or Passive:

Depending on the type of elements used in their construction, filter may be classified as passive or Active elements used in passive filters are Resistors, capacitors, inductors. Elements used in active filters are transistor, or op-amp.

Active filters offer the following advantages over passive filters:

1. Gain and Frequency adjustment flexibility:

Since the op-amp is capable of providing gain, the i/p signal is not attenuated as it is in a passive filter. [Active filter is easier to tune or adjust].

2. No loading problem:

Because of the high input resistance and low o/p resistance of the op-amp, the active filter does not cause loading of the source or load.

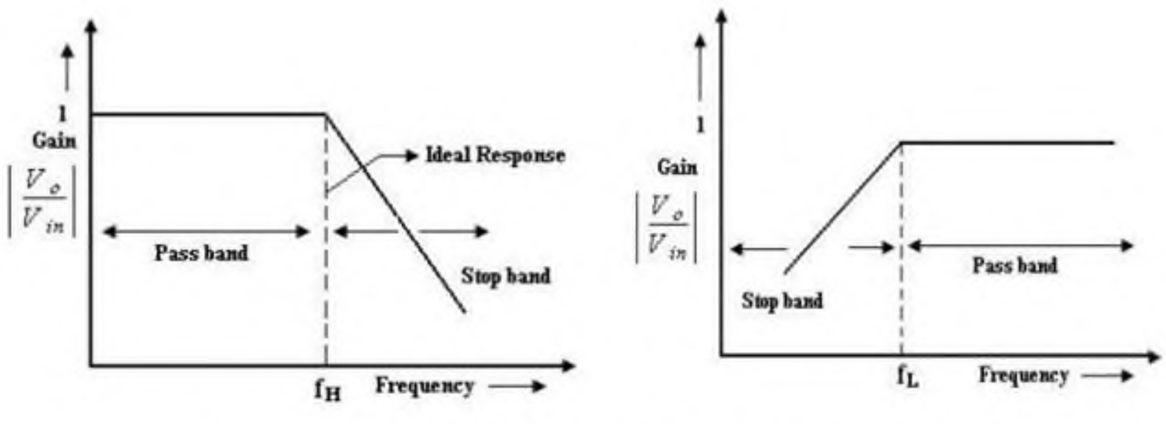
3. Cost:

Active filters are more economical than passive filter. This is because of the variety of cheaper op-amps and the absence of inductors.

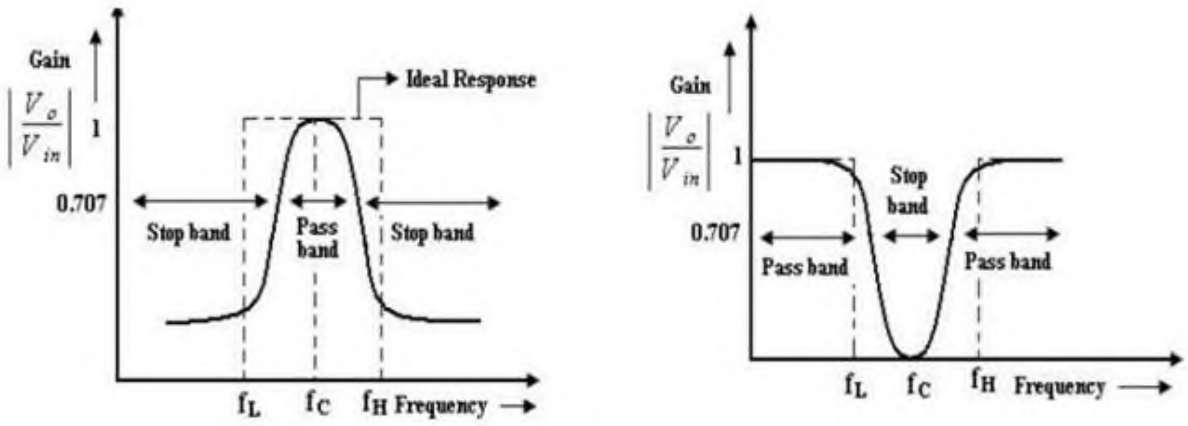
The most commonly used filters are these:

1. Low pass Filters
2. High pass Filters
3. Band pass filters
4. Band –reject filters

Frequency response of the active filters:



Frequency response of Low Pass filter and High pass Filter



Frequency response of Band Pass filter and Band reject Filter

Low pass filters:

- It has a constant gain from 0 Hz to a high cutoff frequency f_1.
- At f_H the gain in down by 3db.
- The frequency between 0 Hz and f_H are known as the pass band frequencies where as the range of frequencies those beyond fH, that are attenuated includes the stop band frequencies.

High pass filter:

High pass filter with a stop band $0 < f < f_L$ and a pass band $f > f_L$

f_L -> low cut off frequency

f -> operating frequency.

Band pass filter:

It has a pass band between 2 cut off frequencies f_H and f_L where $f_H > f_L$ and two, stop bands: $0 < f < f_L$ and $f > f_H$ between the band pass filter (equal to $f_H - f_L$).

Band –reject filter: (Band stop or Band elimination). It performs exactly opposite to the band pass.It has a band stop between 2 cut-off frequency f_L and f_H and 2 pass bands: $0 < f < f_L$ and $f > f_H$ f_C -> center frequency.

First order LPF Butterworth filter:

First order LPF that uses an RC for filtering op-amp is used in the non inverting configuration. Resistor R_1 & R_f determine the gain of the filter. According to the voltage –divider rule, the voltage at the non-inverting terminal (across capacitor) C is,

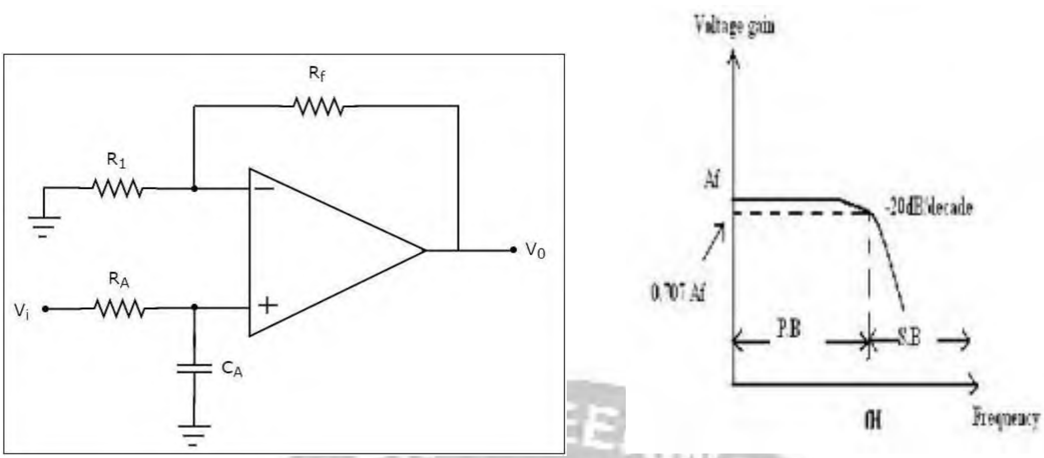


Fig a) First order low pass filter b)frequency response

Gain A= $(1+R_f/R_1)$

Voltage across capacitor $V_1 = V_i / (1+j2\pi fRC)$

Output voltage V0 for non inverting amplifier =A V_1

= $(1+R_f/R_1)$ Vi/(1+j2πfRC)

Overall gain $V_0/V_i = (1+R_f/R_1)$ Vi/(1+j2πfRC)

Transfer function H(s) =A/(jf/f_h+1) if f_h =1/2πRC

H (jω) = A/(jωRC+1) = A/(jωRC+1).

The gain magnitude and phase angle of the equation of the LPF can be obtained by converting eqn. (1) b into its equivalent polar form as follows.

1. At very lowω)|frequency, $f < f_H$

|H (jω) =A

2. At $f = f_H$

|H (jω)| =A/√2=0.707A

3. At $f > f_H$

|H (jω)|<<A ≅ 0

When the frequency increases by tenfold (one decade), the volt gain is divided by 10. The gain falls by 20 dB (=20log10) each time the frequency is reduces by 10. Hence the rate at which the gain rolls off f_H = 20 dB or 6dB/octave (twofold Rin frequency). The frequency $f = f_H$ is called the cut off frequency because the gain of the filter at this frequency is down by 3 dB (=20 log 0.707).

Second order LP Butterworth filter:

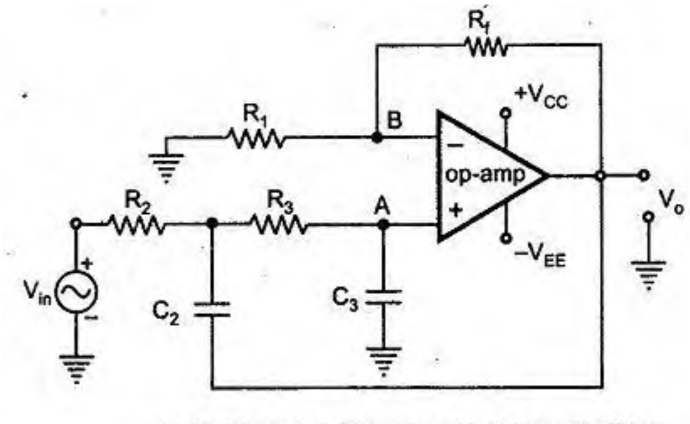
A second order LPF having a gain 40dB/decade in stop band. A First order LPF can be converted into a II order type simply by using an additional RC network.

- An improved filter response can be obtained by using a second order active filter.
- A second order active filter consists of two RC pairs & has roll off rate of -40db/decade.
- The op-amp is connected as non-inverting amplifier hence

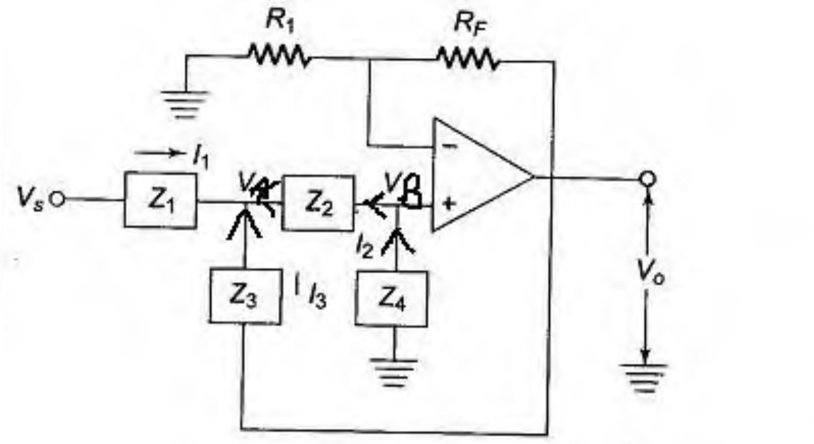
$$V_o = \left(1 + \frac{R_f}{R_1}\right) V_B = A_o V_B$$

$$where,\ A_o = \left(1 + \frac{R_f}{R_1}\right)$$

$$and\ V_B \rightarrow voltage\ at\ node\ B$$



Second order low pass butterworth filter



General Prototype Second Order Filter Circuit

KCL at node A,

$$(V_i - V_A)Z_1 + (V_0 - V_A)Z_3 + (V_B - V_A)Z_2 = 0$$

$$V_iZ_1 + V_oZ_3 + V_BZ_2 - V_A(Z_1 + Z_2 + Z_3) = 0$$

$$V_iZ_1 = V_A(Z_1 + Z_2 + Z_3) - V_BZ_2 - V_oZ_3$$

$$A_o = \frac{V_o}{□}$$

$$□ = \frac{V_o}{□}$$

$$V_iZ_1 = V_A(Z_1 + Z_2 + Z_3) - V_BZ_2 - \frac{V_o}{□}Z_3 \text{-----(1)}$$

$KCL\ at\ node\ B$,

$$(V_B - V_A)Z_2 + V_BZ_4 = 0$$

$$\square_{\square}\square_2 = \square_{\square}(\square_4 + \square_2)$$

$$V_A Z_2 = \frac{V_o}{\square_{\square}}(Z_4 + Z_2) \text{------}(2)$$

$$V_{\square} = \frac{V_o}{\square_{\square}} \frac{(Z_2 + Z_4)}{\square_2}$$

$$Sub\ V_A\ (2)\ in\ (1)$$

$$\square\square_{1} = \frac{V_o}{A_o}\frac{(Z_2 + Z_4)}{\square_2}(Z_1 + \square_2 + Z_3) - V_{\square} Z_2 - \frac{V_o}{\square_{\square}} Z_3$$

$$V_i Z_1 = V_o\left(\frac{(Z_2 + Z_4)(Z_1 + Z_2 + Z_3) - Z_3(A_o Z_2) - Z^2{}_2}{A_o Z_2}\right)$$

$$\frac{\square_{\square}}{V_i} = \frac{\square_{\square}\square_1\square_2}{Z_1 Z_2 + Z^2{}_2 + Z_2 Z_3 + Z_1 Z_4 + Z_2 Z_4 + Z_3 Z_4 - A_o Z_2 Z_3 - Z^2{}_2}$$

$$\frac{\square_{\square}}{V_i} = \frac{\square_{\square}\square_1\square_2}{Z_1 Z_2 + Z_4(Z_1 + Z_2 + Z_3) + Z_2 Z_3(1 - A_0)} \quad --(3)$$

$To\ make\ a\ low\ pass\ filter, choose\ Z_1 = Z_2 = \frac{1}{\square}$ And $Z_3 = Z_4 =$

$\square\ \square\ \ \square\ \ \square$.

$From\ (3), we\ get\ the\ transfer\ function\ H(s) of\ a\ low\ pass\ filter\ as$

$$H(S) = \frac{\frac{\square_{\square}}{\square^2}}{\left(\frac{1}{\square^2} + SC\left(\frac{1}{R} + \frac{1}{R} + SC\right) + \frac{SC}{R}(1 - A_o)\right)}$$

$\square\ \ \square\ \ , \square\ \square$

$$H(S) = \frac{Ao}{S^2C^2R^2 + SCR(3 - A_o) + 1} \quad \text{-----}(4)$$

$\square$ (4),

$$H(s) = A_o,\ for\ S = 0$$

$$H(s) = \infty,\ for\ S = \infty$$

$The\ transfer\ function\ of\ the\ low\ pass\ second\ order\ system\ can\ be\ written\ as$

$$H(s) = \frac{A_o \omega_n^2}{S^2 + \alpha\omega_n S + \omega_{\square}^2} \quad \text{-------}(5)$$

$$Where, \qquad A_o \rightarrow the\ gain$$

$$\omega_n \rightarrow upper\ cutoff\ frequency\ in\ rad/\sec$$

$$\alpha \rightarrow sampling\ coefficient$$

□ □ (4)&(5)

$$\omega_{□} = \frac{1}{□}, \alpha = (3 - A_o)$$

$$The\ value\ of\ the\ damping\ coefficient\ \alpha\ for\ low\ pass\ active\ RC\ filter\ can\ be\ determined\ by\ the\ value\ of\ A_o\ chosen$$

$$Sub\ S = j\omega\ in\ (5)$$

$$H(j\omega) = \frac{A_o \omega_n^2}{(j\omega)^2 + \alpha\omega_n j\omega + \omega_{□}^2}$$

$$H(j\omega) = \frac{A_o}{\left(\frac{j\omega}{\omega_n}\right)^2 + j\alpha\frac{\omega}{\omega_n} + 1}$$

$$The\ normalised\ expression\ for\ lowpass\ filter\ is$$

$$H(j\omega) = \frac{A_o}{S_{□}^2 + \alpha S_{□} + 1}$$

$$Where,\ normalised\ frequency\ S_n = j\left(\frac{\omega}{\omega_n}\right)$$

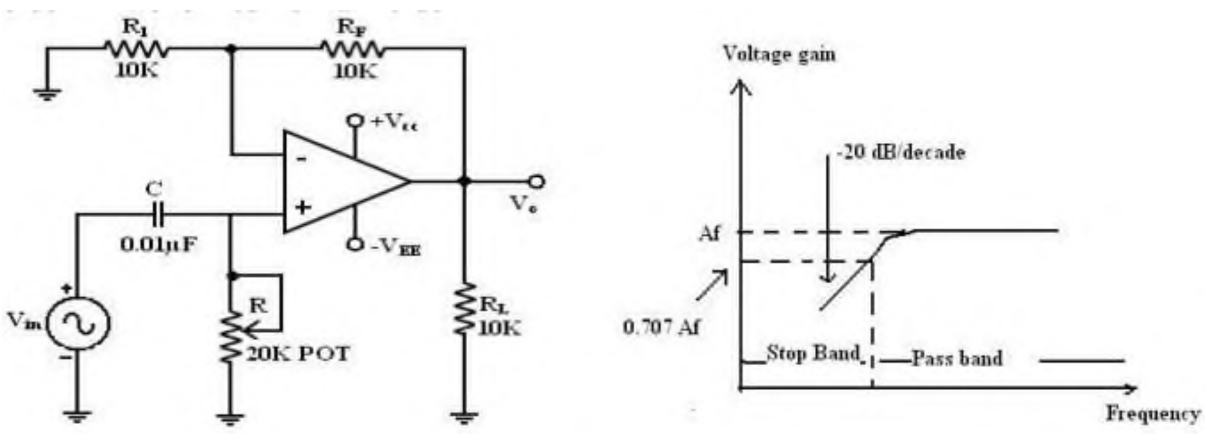
$$The\ expression\ of\ magnitude\ in\ db\ of\ the\ transfer\ function\ is$$

$$20\ log|H(j\omega)| = 20\ l0g\left(\frac{Ao}{1 + j\alpha\left(\frac{\omega}{\omega n}\right) + \left(\frac{j\omega}{\omega n}\right)^2}\right)$$

$$= 20\ log\left(\frac{Ao}{\left(1 - \frac{\omega^2}{\omega_{□}^2}\right)2 + \left(\alpha\frac{\omega}{\omega_n}\right)^2}\right)$$

First order HP Butterworth filter:

High pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. (i.e) I order HPF is formed from a I order LPF by interchanging components R & C. Similarly, II order HPF is formed from a II order LPF by interchanging R & C.



I order HPF and its frequency response

Here I order HPF with a low cut off frequency of f_L. This is the frequency at which the magnitude of the gain is 0.707 times its passband value.
Here all the frequencies higher than f_L are passband frequencies.
The output voltage V_0 of the first order active high pass filter is

$$V_o = \left(1+\frac{R_f}{R_i}\right)\frac{j2\pi fRC}{1+j2\pi fRC}V_i$$

The gain of the filter:

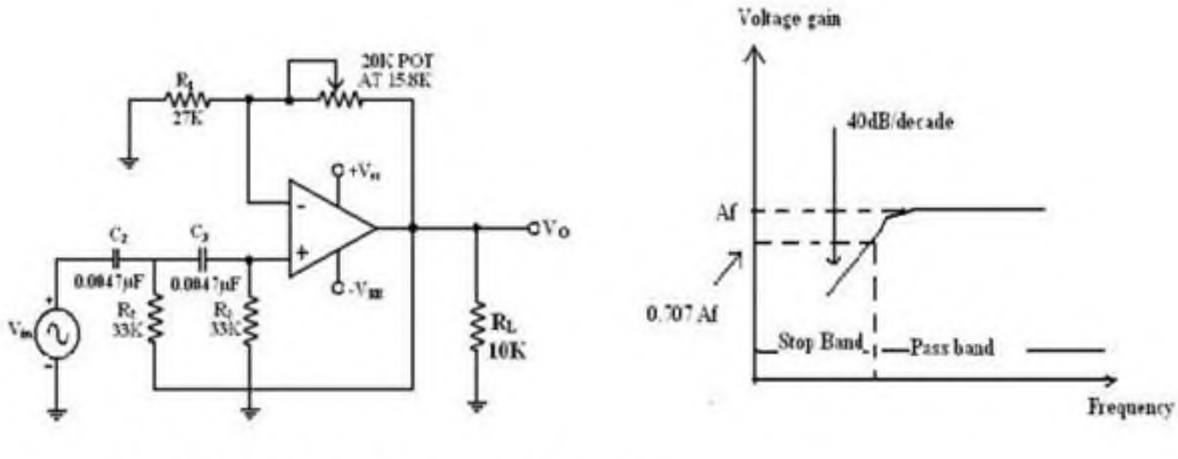
$$\frac{V_o}{V_i} = A\left(\frac{j\left(\frac{f}{f_L}\right)}{1+j\left(\frac{f}{f_L}\right)}\right)$$

Frequency response of the filter $|H(jf)| = \left|\frac{V_o}{V_i}\right| = \frac{A\left(\frac{f}{f_L}\right)}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}} = \frac{A}{\sqrt{1-\left(\frac{f_L}{f}\right)^2}}$ is

- At high frequencies $f>f_L$ gain = A.
- At $f=f_L$ gain = 0.707 A.
- At $f<f_L$ the gain decreases at a rate of -20 db /decade. The frequency below cutoff frequency is stop band.

Second – order High Pass Butterworth Filter:

I order Filter, II order HPF can be formed from a II order LPF by interchanging the frequency



II order HPF and its frequency response

Band pass filters

- Filters that pass band of frequencies and attenuates others. Its high cutoff frequency and low cutoff frequency are related as $f_H > f_L$ and maximum gain at resonant frequency
- $f_r = \sqrt{f_H f_L}$
- Figure of merit $Q = f_r / (f_H - f_L) = f_r / B$ where B= bandwidth.
- 2 types of filters are Narrow band pass and wide band pass filters

Wide band pass filter:

It is connection of a low pass filter and a high pass filter in cascade.
The f_H of low pass filter and f_L of high pass filter are related as $f_H > f_L$

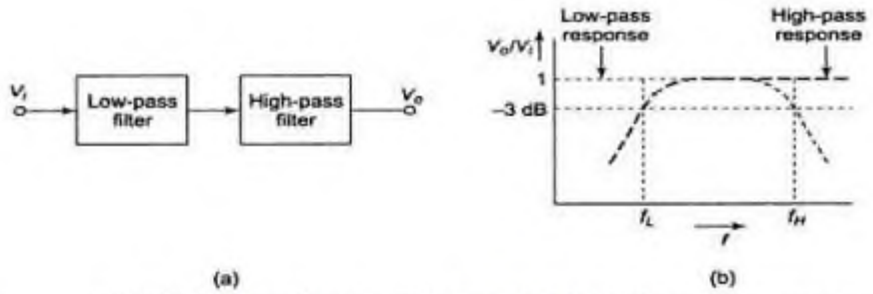
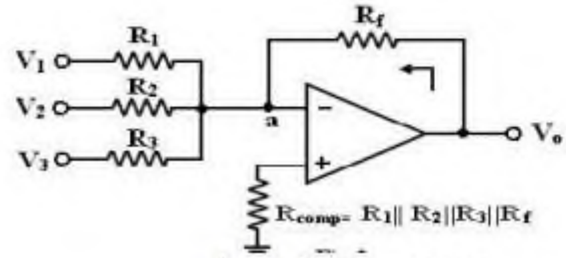


Fig. (a) Wide band pass filter and (b) its frequency response

Adder:

Op-amp may be used to design a circuit whose output is the sum of several input signals.Such a circuit is called a summing amplifier or a summer or adder.An inverting summer or a non-inverting summer may be discussed now.

Inverting Summing Amplifier:



inverting summer

A typical summing amplifier with three input voltages V_1, V_2 and V_3 three input resistors R_1, R_2, R_3 and a feedback resistor R_f is shown in figure 2.The following analysis is carried out assuming that the op-amp is an ideal one, AOL= ∞.Since the input bias current is assumed to be zero, there is no voltage drop across the resistor R_{comp} and hence the non-inverting input terminal is at ground potential.

$$I= V_1/R1+V_2/R2\ldots..+Vn/Rn;$$

$$Vo= - R_f$$

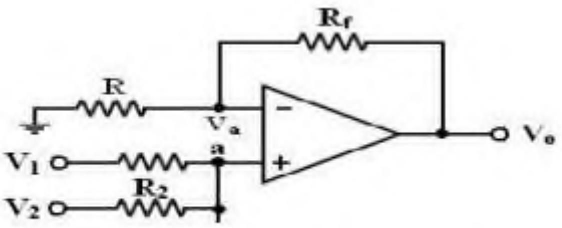
$$I=Rf/R(V_1+V_2+\ldots.V_n).$$

To find R_{comp}, make all inputs $V_1 = V_2 = V_3 = 0$.

So the effective input resistance $R_i = R_1 \parallel R_2 \parallel R_3$.

Therefore, $Rcomp = R_i \parallel R_f = R_1 \parallel R_2 \parallel R_3 \parallel R,f$.

Non-Inverting Summing Amplifier:



Non inverting summer

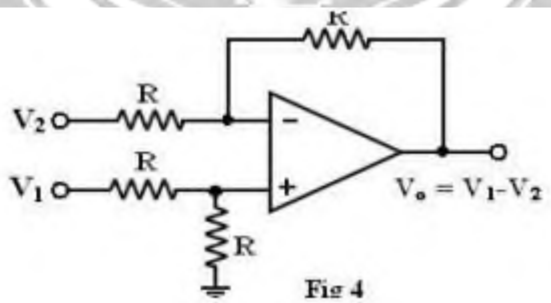
A summer that gives a non-inverted sum is the non-inverting summing amplifier of figure Let the voltage at the (-) input terminal be Va. which is a non-inverting weighted sum of inputs.

Let $R_1 = R_2 = R_3 = R = R_f/2$, then $V_o = V_1+V_2+V_3$

Subtractor using Operational Amplifier

If all resistors are equal in value, then the output voltage can be derived by using superposition principle.

Subtractor:



Subtractor

A basic differential amplifier can be used as a subtractor as shown in the above figure. If all resistors are equal in value, then the output voltage can be derived by using superposition principle.

To find the output V_{01} due to V_1 alone, make $V_2 = 0$.

Then the circuit of figure as shown in the above becomes a non-inverting amplifier having input voltage $V_1/2$ at the non-inverting input terminal and the output becomes

$V_{01} = V_1/2(1+R/R) = V_1$ when all resistances are R in the circuit.

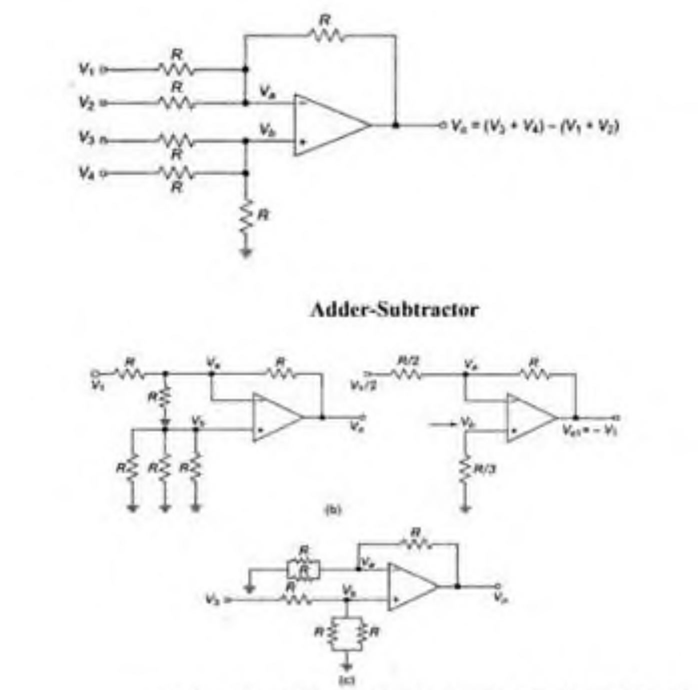
Similarly the output V_{02} due to V_2 alone (with V_1 grounded) can be written simply for an inverting amplifier as

$V_{02} = -V_2$

Thus the output voltage Vo due to both the inputs can be written as

$V_0 = V_{01} - V_{02} = V_1 - V_2$

Adder/Subtractor:



Adder-Subtractor

(b)

(c)

equivalent circuit for $V_2=V_3=V_4=0$ and (c) for $V_1=V_2=V_4=0$

It is possible to perform addition and subtraction simultaneously with a single op-amp using the circuit shown in figure 2.16.The output voltage Vo can be obtained by using superposition theorem. To find output voltage V_{01} due to V_1 alone, make all other input voltages V_2, V_3 and V_4 equal to zero.

The simplified circuit is shown in figure 2.17. This is the circuit of an inverting amplifier and its output voltage is, $V_{01}= -R/(R/2) * V_1/2= - V_1$ by Thevenin‘s equivalent circuit at inverting input terminal).Similarly, the output voltage V_{02} due to V_2 alone is,

$$V_{02} = -V_2$$

Now, the output voltage V_{03} due to the input voltage signal V_3 alone applied at the (+) input terminal can be found by setting V_1, V_2 and V_4 equal to zero.

$$V_{03} = V_3$$

The circuit now becomes a non-inverting amplifier as shown in fig.(c).So, the output voltage V_{03} due to V_3 alone is

$$V_{03} = V_3$$

Similarly, it can be shown that the output voltage V_{04} due to V_4 alone is

$$V_{04} = V_4$$

Thus, the output voltage Vo due to all four input voltages is given by

$$V_o = V_{01} = V_{02} = V_{03} = V_{04}$$

$$Vo = -V_1 - V_2 + V_3 + V_4$$

$$V_o = (V_3 + V_4) - (V_1 + V_2)$$

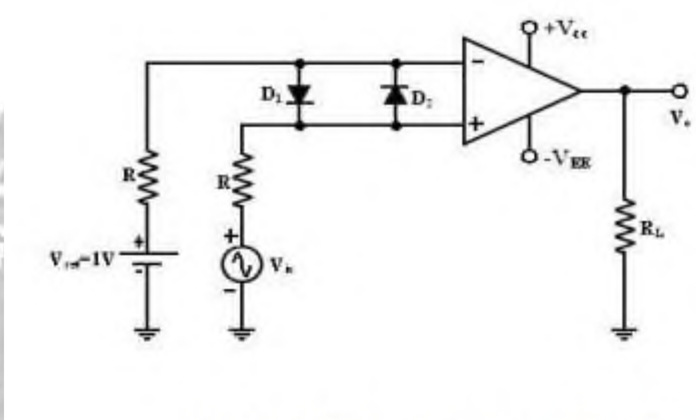
So, the circuit is an adder-subtractor.

Comparator

A comparator compares a signal voltage on one input of an op-amp with a known voltage called a reference voltage on the other input. Comparators are used in circuits such as,

- Digital Interfacing
- Schmitt Trigger
- Discriminator
- Voltage level detector and oscillators

Non-inverting Comparator:



non-inverting comparator circuit

A fixed reference voltage V_{ref} of 1 V is applied to the negative terminal and time varying signal voltage Vin is applied to the positive terminal.

When Vin is less than V_{ref} the output becomes V_0 at $-V_{sat}$

$$[V_{in} < V_{ref} \Rightarrow V_0 (-V_{sat})].$$

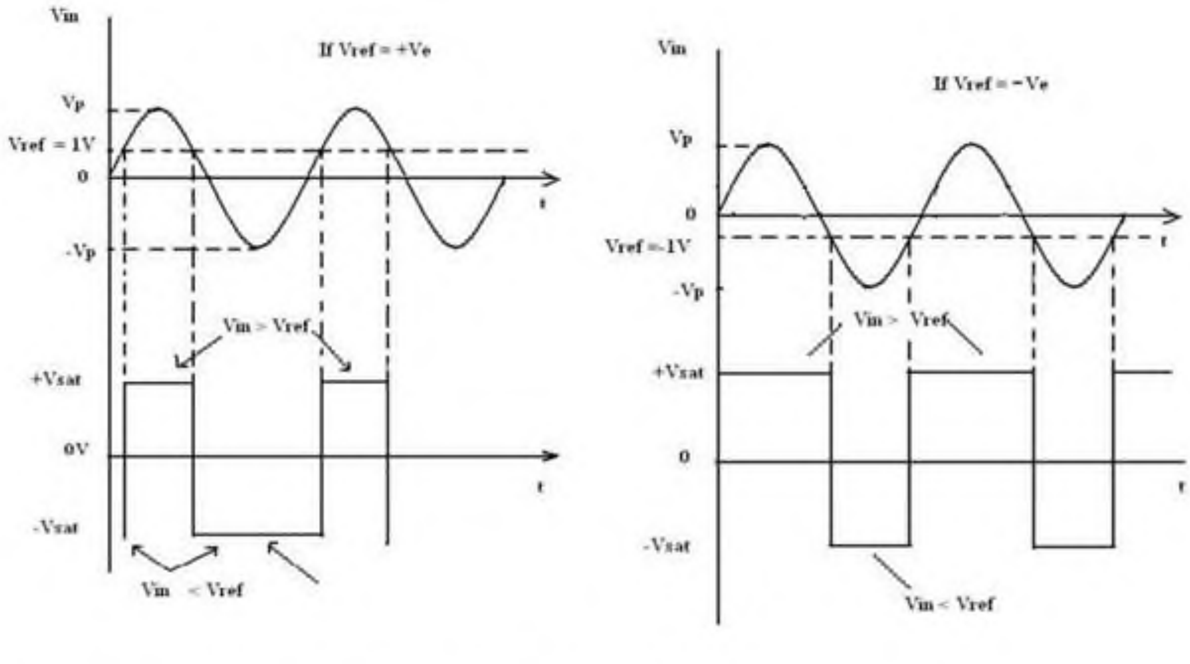
When Vin is greater than V_{ref}, the (+) input becomes positive, the V_0 goes to $+V_{sat}$.

$$[V_{in} > V_{ref} \Rightarrow V_0 (+V_{sat})].$$

Thus the V_0 changes from one saturation level to another.

The diodes D_1 and D_2 protect the op-amp from damage due to the excessive input voltage V_{in}. Because of these diodes, the difference input voltage Vid of the op-amp diodes are called clamp diodes.

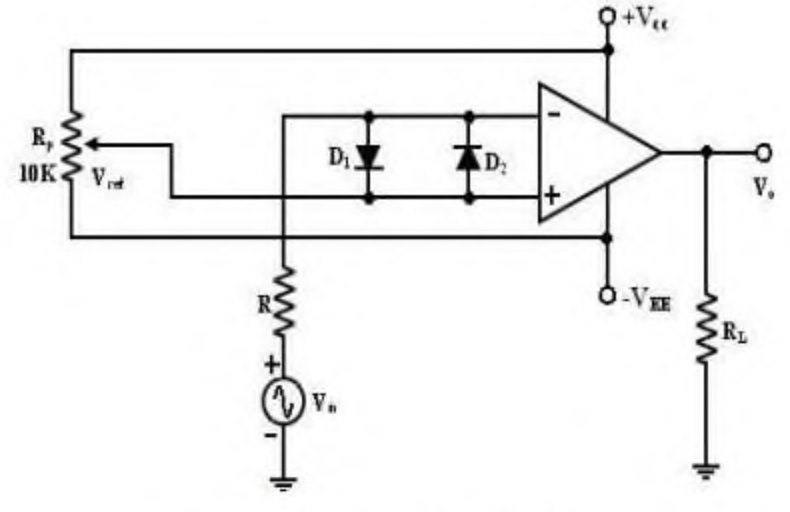
The resistance R in series with V_{in} is used to limit the current through D_1 and D_2. To reduce offset problems, a resistance $R_{comp} = R$ is connected between the (-ve) input and V_{ref}.



Input and Output Waveforms of non-inverting comparator

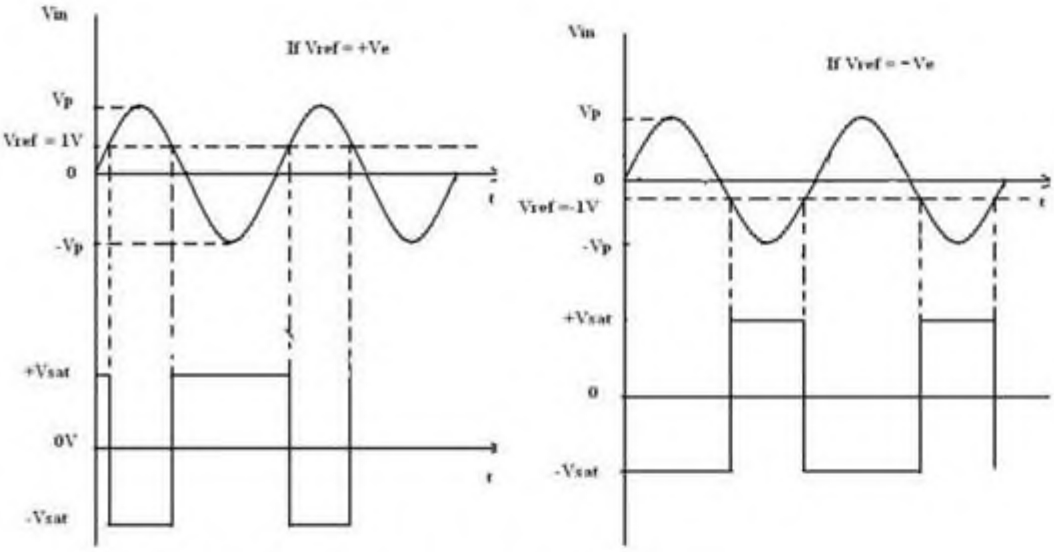
Inverting Comparator:

This fig shows an inverting comparator in which the reference voltage V_{ref} is applied to the (+) input terminal and V_{in} is applied to the (-) input terminal.



Inverting comparator circuit

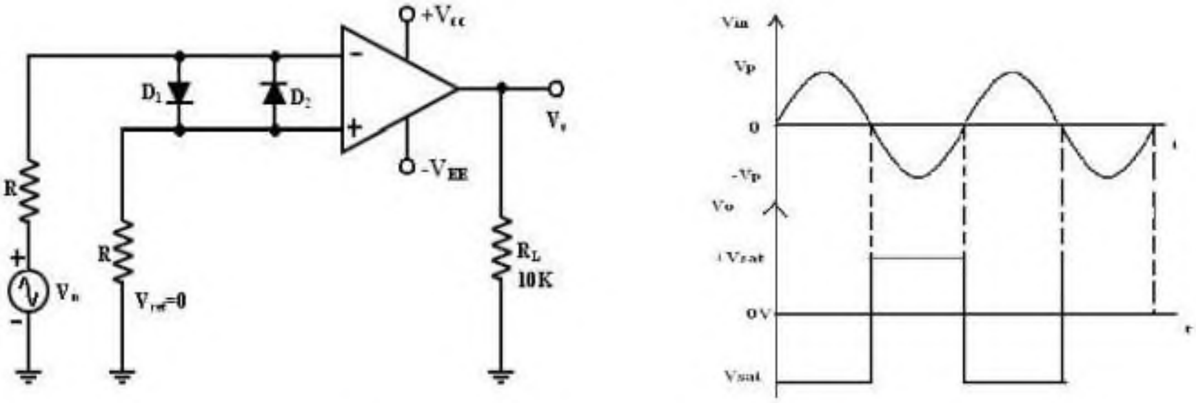
In this circuit V_{ref} is obtained by using a 10K potentiometer that forms a voltage divider with DC supply volt $+V_{cc}$ and -1 and the wiper connected to the input. As the wiper is moved towards $+V_{cc}$, V_{ref} becomes more positive. Thus a Vref of a desired amplitude and polarity can be got by simply adjusting the 10k potentiometer.



Input and Output Waveforms of non-inverting comparator

Applications:

Zero Crossing Detector: [Sine wave to Square wave converter]



Zero crossing detector circuit and input-output waveforms

One of the applications of comparator is the zero crossing detector or ―sine wave to Square wave Converter. The basic comparator can be used as a zero crossing detector by setting Vref is set to Zero.This Fig shows when in what direction an input signal V_{in} crosses zero volts. (i.e.) the o/p V_0 is driven into negative saturation when the input the signal V_{in} passes through zero in positive direction. Similarly, when Vin passes through Zero in negative direction the output V_0 switches and saturates positively.

Drawbacks of Zero- crossing detector:

In some applications, the input Vin may be a slowly changing waveform, (i.e) a low frequency signal. It will take Vin more time to cross 0V, therefore V_0 may not switch quickly from one saturation voltage to the other.

Because of the noise at the op-amp‘s input terminals the output V0 may fluctuate between 2 saturations voltages +Vsat and –Vsat. Both of these problems can be cured with the use of regenerative or positive feedback that cause the output V0 to change faster and eliminate any false output transitions due to noise signals at the input Inverting comparator with positive feedback This is known as Schmitt Trigger.