2.5 MEASUREMENT OF UNKNOWN LOAD IMPEDANCE:

The unknown value of a load impedance ZR connected to a transmission line may be determined by standing wave measurements on the open wire of slotted line. Bride circuit is used for the measurement of unknown impedance.

At the point of voltage minimum at a distance s' from the load it can be shown that

$$Z_S = R_{min} = \frac{R_O}{S} \quad \ldots\ldots\ldots(1)$$

S = Standing wave ratio

At any point on the line, the input impedance is given by,

$$Z_S = R_O \left[\frac{Z_R + jR_O \tan\beta s'}{R_O + jZ_R \tan\beta s'}\right] \quad \ldots\ldots.(2)$$

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in equ (2),

$$Z_S = R_O \left[\frac{Z_R + jR_O \tan\left(\frac{2\pi s'}{\lambda}\right)}{R_O + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right)}\right] \quad \ldots\ldots.(3)$$

Equating equ (1) and (3),

$$R_O \left[\frac{Z_R + jR_O \tan\left(\frac{2\pi s'}{\lambda}\right)}{R_O + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right)}\right] = \frac{R_O}{S}$$

Solving for Z_R gives,

$$R_O + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right) = s\left[Z_R + jR_O \tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$$R_O + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right) = sZ_R + SjR_O \tan\left(\frac{2\pi s'}{\lambda}\right)$$

$$-sZ_R + jZ_R \tan\left(\frac{2\pi s'}{\lambda}\right) = -R_O + SjR_O \tan\left(\frac{2\pi s'}{\lambda}\right)$$

$$-Z_R\left[S - j\tan\left(\frac{2\pi s'}{\lambda}\right)\right] = -R_O\left[1 - jS\tan\left(\frac{2\pi s'}{\lambda}\right)\right]$$

$Z_R = R_O \left[\frac{\left[1 - j\, S \tan\left(\frac{2\pi s'}{\lambda}\right)\right]}{\left[S - j \tan\left(\frac{2\pi s'}{\lambda}\right)\right]}\right]$

Where $\beta = \frac{2\pi}{\lambda}$,

$Z_R = R_O \left[\frac{1 - j\, S \tan \beta S}{S - j \tan \beta S}\right]$

POWER AND IMPEDANCE MEASUREMENT ON LINE:

Expression for voltage and current for the dissipationless line is given by,

$E = I_R \left(\frac{Z_R + Z_O}{2}\right) [1 + |K| \angle \emptyset - 2\beta s]$ ……..(1)

$I = I_R \left(\frac{Z_R + Z_O}{2R_O}\right) [1 + |K| \angle \emptyset - 2\beta s]$ ………(2)

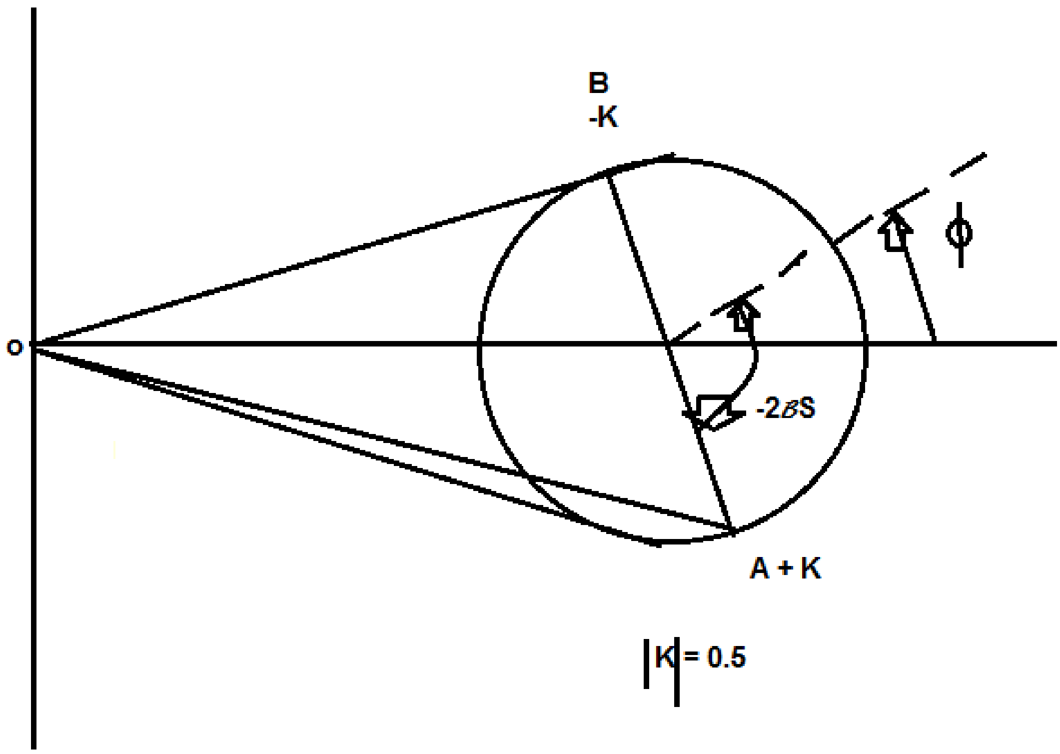

Fig: 2.5.1 Diagram illustrating Equation (1) & (2)

In Fig 2.5.1, phasors A and B being propositional to E and I, respectively,

Consider two phases A and B representing voltage and cureent.

When the incident and reflected waves are inphase, we get voltage maximum.

So it can be given by,

$\emptyset - 2\beta s = 0$, sub in (1) and (2),

$E_{max} = I_R \left(\frac{Z_R+Z_O}{2}\right) [1 + |\ K|\ ]$ …….(3)

$I_{max} = I_R \left(\frac{Z_R+Z_O}{2R_O}\right) [1 + |\ K|\ ]$ …….(4)

$\frac{E_{max}}{I_{max}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1+|\ K\ ]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1+|\ K|\ ]}$

$\frac{E_{max}}{I_{max}} = R_O$ ……(5)

From the theory of standing waves it can be obtained that the minimum values of voltage and current occurs at both incident and reflected waves are out-off phase.

$\emptyset - 2\beta s = \pi$

$E_{min} = I_R \left(\frac{Z_R+Z_O}{2}\right) [1 - |\ K|\ \ ]$ ……(6)

$I_{min} = I_R \left(\frac{Z_R+Z_O}{2R_O}\right) [1 - |\ K|\ ]$ ……...(7)

$\frac{E_{min}}{I_{min}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1-|\ K|\ \ ]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1-|\ K|\ ]}$

$\frac{E_{min}}{I_{min}} = R_O$ ……(8)

It is clear that the voltage maximum and current minimum at the same point in the transmission line.

Divide (3) and (7),

$\frac{E_{max}}{I_{min}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1+|\ K|\ \ ]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1-|\ K|\ ]}$

$\frac{E_{max}}{I_{min}} = R_O\left(\frac{1+|\ K|}{1-|\ K|}\right)$

$\frac{E_{max}}{I_{min}} = R_O$s

$R_{max} = R_O$ s

s = standing wave ratio

s= $\frac{1+|\ \ K|}{1-|\ \ K|}$

divide (6) and (4)

$$\frac{E_{min}}{I_{max}} = \frac{I_R\left(\frac{Z_R+Z_O}{2}\right)[1-|K|]}{I_R\left(\frac{Z_R+Z_O}{2R_O}\right)[1+|K|]}$$

$$\frac{E_{min}}{I_{max}} = R_O\left(\frac{1-|K|}{1+|K|}\right)$$

$$\frac{E_{min}}{I_{max}} = \frac{R_O}{S}$$

$$R_{min} = \frac{R_O}{S}$$

Here, the resistance R_{max} is known as impedance in the voltage loop and R_{min} is represented as impedance in current loop.

The effective power flowing into a resistance R_{max} is the power passing through voltage loop at voltage E_{max}.

$$P = \frac{E^2max}{R_{max}}$$

Similarly, the power can be also calculated as the power passing through current loop at voltage E_{min}.

$$P = \frac{E^2min}{R_{min}}$$

$$P^2 = \frac{E^2min\,E^2max}{R_{max}R_{min}}$$

$$P^2 = \frac{E^2min\,E^2max}{R_O\,s.\frac{R_O}{S}}$$

$$P^2 = \frac{E^2min\,E^2max}{R_O^2}$$

$$P = \frac{|E_{max}|\,|E_{min}|}{R_O}$$

$$P = |I_{max}|\,|I_{min}|\,.\,R_O$$

REFLECTION LOSSES ON THE UNMATCHED LINE:

If the line is not terminated to its load, the energy delivered by the line to the load is less than if the impedance are properly adjusted. This effect is due to the reflection coefficient at the junction and results in a reflected wave and a standing wave system. The voltage at a maximum voltage point is due to the in-phase sum of the incident and reflected waves is given by:

$$|E_{max}| = |E_i| + |E_r| = \left|\frac{I_R(Z_R+Z_O)}{2}\right| (1 + |K|) \qquad \text{........(1)}$$

The minimum voltage is due to the difference of the incident and reflected wave and is given as.

$$|E_{min}| = |E_i| - |E_r| = \left|\frac{I_R(Z_R+Z_O)}{2}\right| (1 - |K|) \qquad \text{........(2)}$$

Hence the standing wave ratio is:

$$\frac{E_{max}}{E_{min}} = \frac{|E_i| + |E_r|}{|E_i| - |E_r|} \qquad \text{.........(3)}$$

The total power along the line and delivered to the load is given by:

$$P = \frac{|E_{max}|\,|E_{min}|}{R_O} \qquad \text{........(4)}$$

$$P = \frac{(|E_i| + |E_r|)(|E_i| - |E_r|)}{R_O}$$

$$P = \frac{|E_i|^2 - |E_r|^2}{R_O} = P_i - P_r \qquad \text{.......(5)}$$

The above equation is the difference of two power flows, one being the power P_i is transmitted in the incident wave , the other being power P_r travelling back in the reflected wave.

The ratio of the power P delivered to the load to the power transmitted by the incident wave is,

$$\frac{P}{P_i} = \frac{P_i - P_r}{P_i} = \frac{|E_i|^2 - |E_r|^2}{|E_i|^2} = 1 - \frac{|E_r|^2}{|E_i|^2} = 1 - |K|^2$$

Where $K = \frac{S-1}{S+1}$

$$\frac{P}{P_i} = 1 - \left(\frac{S-1}{S+1}\right)^2 = \frac{(S+1)^2 - S^2 + 2S - 1}{(S+1)^2}$$

$$\frac{P}{P_i} = \frac{S^2 + 2S + 1 - S^2 + 2S - 1}{(S+1)^2}$$

$$\frac{P}{P_i} = \frac{4S}{(S+1)^2} \qquad \text{..........(6)}$$

The ratio of power absorbed by the load to the transmitter is plotted as a function of S as shown in the following Fig 2.5.2.

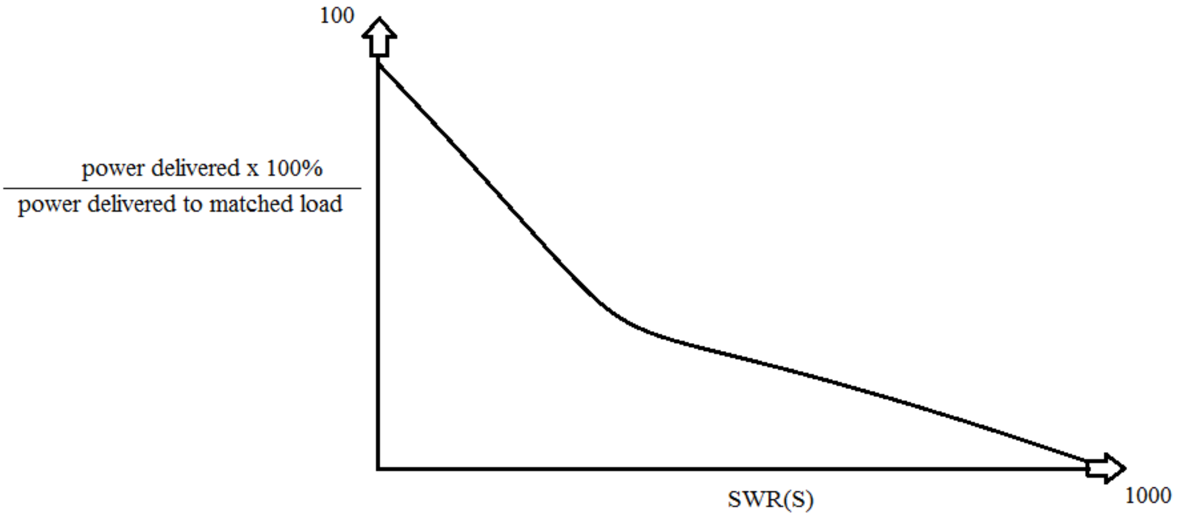

Fig: 2.5.2 Reflection losses as a function of standing wave ratio