### 2.4 INFLUENCE LINE FOR SHEARING FORCE, BENDING MOMENT AND SUPPORT REACTION COMPONENTS OF FIXED BEAMS

## Fixed beam

Consider a fixed beam ABB of span 'L' as shown in figure.


Figure : Fixed beam
Fig. 2.4.1 Fixed beam
If we consider AB as simply supported beam, the influence lines for reaction at the support can be determined by applying a cut at ' A ' and by applying unit displacement at ' A ' as shown in figure.


Fig. 2.4.2 Displacement At 'A'

## Maximum shear force diagram

Due to a given system of rolling loads the maximum shear force for every section of the girder can be worked out by placing the loads in appropriate positions. When these are plotted for all the sections of the girder, the diagram that we obtain is the maximum shear force diagram. This diagram yields the 'design shear' for each cross section.

## Bending moment diagram

Bending moment diagram represents variation of bending moment. Bending moment diagrams are drawn for only bending moments. If span longer than UDL for a maximum BM, the load on left side is equal to the load on right side in case of bending moment diagram.

## Several point loads

The maximum bending moment for a series of moving loads is obtained when the average load on the left of the section is equal to the average load on the right of the section.

The above statement exists in a system of moving point loads. In such cases, each load is passed over the section and average load on each side is calculated. The load, when the crosses the section makes the heavier side lighter and lighter side heavier and gives the maximum bending moment at the section.

## Types of connections possible with the model used with Begg's deformeter.

(i) Hinged connection
(ii) Fixed connection
(iii) Floating connection

## Principle on which indirect model analysis is based.

The indirect model analysis is based on the Muller Breslau principle.
Muller Breslau principle has lead to a simple method of using models of structures to get the influence lines for force quantities like bending moments, support moments, reactions, internal shears, thrusts, etc.

To get the influence line for any force quantity,
(i) remove the resistant due to the force,
(ii) apply a unit displacement in the direction of the
(iii) plot the resulting displacement diagram. This diagram is the influence line for the force.

## Example:

Sketch the influence line diagram for reaction at support of the fixed beam


Fig. 2.4.3

## Solution :



Fig. 2.4.4

Reaction at RA $=\mathrm{M}_{\mathrm{A}} 1 / 3+\mathrm{M}_{\mathrm{B}} 1 / 6$
Reaction at at $\mathrm{RB}=\mathrm{M}_{\mathrm{A}} \mathrm{l} / 6+\mathrm{M}_{\mathrm{B}} \mathrm{l} / 3$


Fig. 2.4.5 ILD for Reaction at AB
$\theta \mathrm{A}=\mathrm{RA}$

$$
\begin{align*}
1 / 1 & =\mathrm{M}_{\mathrm{A}} 1 / 3+\mathrm{M}_{\mathrm{B}} 1 / 6 \\
& =0 \\
1 / 1 & =-\mathrm{M}_{\mathrm{A}} 1 / 3+\mathrm{M}_{\mathrm{B}} 1 / 6 \\
1 / 1 & =-\left(\mathrm{M}_{\mathrm{A}} 1 / 3+\mathrm{M}_{\mathrm{B}} 1 / 6\right) \tag{1}
\end{align*}
$$

$\qquad$


Fig. 2.4.6
$\theta B=-R B$

$$
\begin{align*}
1 / 1 & =-\left(M_{\mathrm{A}} 1 / 6+\mathrm{M}_{\mathrm{B}} 1 / 3\right) \\
& =0 \\
1 / 1 & =\mathrm{M}_{\mathrm{A}} 1 / 6+\mathrm{M}_{\mathrm{B}} 1 / 3 \tag{2}
\end{align*}
$$

equating $1 \& 2$

$$
\begin{align*}
-\left(M_{A} 1 / 3+M_{B} 1 / 6\right) & =M_{A} 1 / 6+M_{B} 1 / 3 \\
-M_{A} 1 / 3-M_{A} 1 / 6 & =M_{B} 1 / 3+M_{B} 1 / 6 \\
-3 M_{A} 1 / 6 & =3 M_{B} 1 / 6 \\
-M_{A} & =M_{B} \\
M_{A} & =-M_{B} \tag{3}
\end{align*}
$$

Sub 3 in 2

$$
\begin{aligned}
1 / 1 & =-M_{B} 1 / 6+M_{B} 1 / 3 \\
1 / 1 & =-M_{B} 1+2 M_{B} 1 / 6
\end{aligned}
$$

$$
\begin{aligned}
1 / 1 & =-M_{B} 1 / 6 \\
M_{B} & =6 / l^{2} \\
M_{A} & =-M_{B} \\
& =-6 / 1^{2}
\end{aligned}
$$

Moment at a distance x from end B of deflection from given by

$$
\begin{aligned}
\mathrm{M}_{\mathrm{x}} & =\mathrm{M}_{\mathrm{A}} 1 / 6 \cdot x-\mathrm{M}_{\mathrm{A}} \mathrm{x}^{3} / 61+\mathrm{M}_{\mathrm{B}} 1 / 6(1-\mathrm{x})-\mathrm{M}_{\mathrm{B}}(1-\mathrm{x})^{3} / 61 \\
& =\left[-6 / l^{2} \cdot l \mathrm{x} / 6\right]-\left[-6 / l^{2} x^{3} / 61\right]+\left[6 / l^{2} .1 / 6(1-x)\right]-\left[6 / l^{2}(1-\mathrm{x})^{3} / 61\right] \\
& =-6 / l^{2}\left(1 \mathrm{x} / 6-\mathrm{x}^{3} / 61\right)+6 / l^{2}\left[1(1-x) / 6-(1-x)^{3} / 61\right] \\
& \left.=-6 / l^{2}\left(l^{2} x-x^{3} / 61\right)+6 / l^{2}\left(l^{2}-1 x\right) 1-(1-x)^{3} / 61\right) \\
& =1 / l^{3}\left[-x\left(l^{2}-x^{2}\right)+l^{2}(1-x)-(1-x)^{3}\right] \\
& =1 / l^{3}\left[-x(1+x)(1-x) l^{2}(1-x)-(1-x)^{2}\right] \\
& =(1-x) / l^{3}\left[-x(1+x) l^{2}-(1-x)^{2}\right] \\
& =(1-x) / l^{3}\left[-1 x-x^{2}+l^{2}-l^{2}+x^{2}+21 x\right] \\
& =(1-x) / l^{3}\left[-1 x-2 x^{2}\right] \\
& =(1-x) / l^{3}[x(1-2 x)] \\
& =x(1-x)(1-2 x) / l^{3}
\end{aligned}
$$

Influence line ordinate for reaction at support A

$$
\begin{aligned}
& =x / l-x(1-x)(1-2 x) / l^{3} \\
& =x .1^{2}-x(1-x)(1-2 x) / l^{3} \\
& =x l^{2}-\left[\left(x\left(1-x^{2}\right)(1-2 x)\right] / l^{3}\right. \\
& =x l^{2}-\left[x l^{2}-2 x^{2} 1-x^{2} 1+2 x^{3} / l^{3}\right.
\end{aligned}
$$

$$
\begin{aligned}
& =x l^{2}-\left[2 x^{3}-3 x^{2} 1+x^{2} 1 / l^{3}\right. \\
& =x l^{2}-2 x^{3}+3 x^{2} 1-x^{2} 1 / l^{3} \\
& =-2 x^{3}+3 x^{2} 1 / l^{3} \\
& =x^{2}(31-2 x) / l^{3}
\end{aligned}
$$



Fig. 2.4.7ILD for Moment at a distance $x$ from end $B$

