DEPARTMENT OF MATHEMATICS

UNIT II – FOURIER SERIES

2.3 Harmonic Analysis

The process of finding the Fourier series for a function given by numerical values is known as harmonic analysis.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx), \text{ where}$$

ie, $f(x) = (a_0/2) + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + (a_3 \cos 3x + b_3 \sin 3x) + \cdots$ …(1)

$$\text{Here } a_0 = 2\,[\text{mean values of } f(x)] = \frac{2\sum f(x)}{n}$$

$$a_n = 2\,[\text{mean values of } f(x)\cos nx] = \frac{2\sum f(x)\cos nx}{n}$$

$$\& \quad b_n = 2\,[\text{mean values of } f(x)\sin nx] = \frac{2\sum f(x)\sin nx}{n}$$

In (1), the term $(a_1 \cos x + b_1 \sin x)$ is called the **fundamental or first harmonic**, the term $(a_2 \cos 2x + b_2 \sin 2x)$ is called the **second harmonic** and so on.

Problem 1.

Compute the first three harmonics of the Fourier series of f(x) given by the following table.

x:	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x):	1.0	1.4	1.9	1.7	1.5	1.2	1.0

We exclude the last point $x = 2\pi$.
Let $f(x) = (a_0/2) + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \ldots\ldots\ldots\ldots$
To evaluate the coefficients, we form the following table.

x	f(x)	cosx	sinx	cos2x	sin2x	cos3x	sin3x
0	1.0	1	0	1	0	1	0
π/3	1.4	0.5	0.866	-0.5	0.866	-1	0
2π/3	1.9	-0.5	0.866	-0.5	-0.866	1	0
π	1.7	-1	0	1	0	-1	0
4π/3	1.5	-0.5	-0.866	-0.5	0.866	1	0
5π/3	1.2	0.5	-0.866	-0.5	-0.866	-1	0

$$\text{Now, } a_0 = \frac{2\sum f(x)}{6} = \frac{2\,(1.0 + 1.4 + 1.9 + 1.7 + 1.5 + 1.2)}{6} = 2.9$$

$$a_1 = \frac{2\sum f(x)\cos x}{6} = -0.37$$

$$a_2 = \frac{2\sum f(x)\cos 2x}{6} = -0.1$$

$$a_3 = \frac{2\sum f(x)\cos 3x}{6} = 0.033$$

$$b_1 = \frac{2\sum f(x)\sin x}{6} = 0.17$$

$$b_2 = \frac{2\sum f(x)\sin 2x}{6} = -0.06$$

$$b_3 = \frac{2\sum f(x)\sin 3x}{6} = 0$$

$\therefore\ f(x) = 1.45 - 0.37\cos x + 0.17 \sin x - 0.1\cos 2x - 0.06 \sin 2x + 0.033 \cos 3x + \ldots$

Problem 2

Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table:

x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

Taking the interval as 60^o, we have

θ:	0^o	60^o	120^o	180^o	240^o	300^o
x:	0	1	2	3	4	5
y:	4	8	15	7	6	2

∴ Fourier cosine series in the interval (0, 2π) is $y = (a_0/2) + a_1\cos\theta + a_2\cos2\theta + a_3\cos3\theta + \ldots$..

To evaluate the coefficients, we form the following table.

θ^o	cosθ	cos2θ	cos3θ	y	y cosθ	y cos2θ	y cos3θ
0^o	1	1	1	4	4	4	4
60^o	0.5	-0.5	-1	8	4	-4	-8
120^o	-0.5	-0.5	1	15	-7.5	-7.5	15
180^o	-1	1	-1	7	-7	7	-7
240^o	-0.5	-0.5	1	6	-3	-3	6
300^o	0.5	-0.5	-1	2	1	-1	-2
			Total	42	-8.5	-4.5	8

Now, $a0 = 2\,(42/6) = 14$

$a1 = 2\,(-8.5/6) = -2.8$

$a2 = 2 \quad (-4.5/6) =$

$a3 = 2\,(8/6) = 2.7$

$y = 7 - 2.8\cos\theta - 1.5\cos2\theta + 2.7\cos3\theta + \ldots$..

Problem 3

The values of x and the corresponding values of f(x) over a period T are given below. Show that $f(x) = 0.75 + 0.37\cos\theta + 1.004\sin\theta$, where $\theta = (2\pi x\ \ )/T$

x:	0	T/6	T/3	T/2	2T/3	5T/6	T
y:	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

We omit the last value since f(x) at x = 0 is known.

Here $\theta = 2\pi x / T$

When x varies from 0 to T, θvaries from 0 to 2π with2π/6. an incre

Let $f(x) = F(\theta) = (a_0/2) + a_1\cos\theta + b_1\sin\theta$.

To evaluate the coefficients, we form the following table.

θ	y	cosθ	sinθ	y cosθ	y sinθ
0	1.98	1.0	0	1.98	0
π/3	1.30	0.5	0.866	0.65	1.1258
2π/3	1.05	-0.5	0.866	-0.525	0.9093
Π	1.30	-1	0	-1.3	0
4π/3	-0.88	-0.5	-0.866	0.44	0.762
5π/3	-0.25	0.5	-0.866	-0.125	0.2165
	4.6			1.12	3.013

Now, $a_0 = 2 (\sum f(x)/6)=1.5$

$a_1 = 2 (1.12 /6) = 0.37$

$a_2 = 2 (3.013/6) = 1.004$

Therefore, $f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$