

Equilibrium of Particles in Two Dimensions

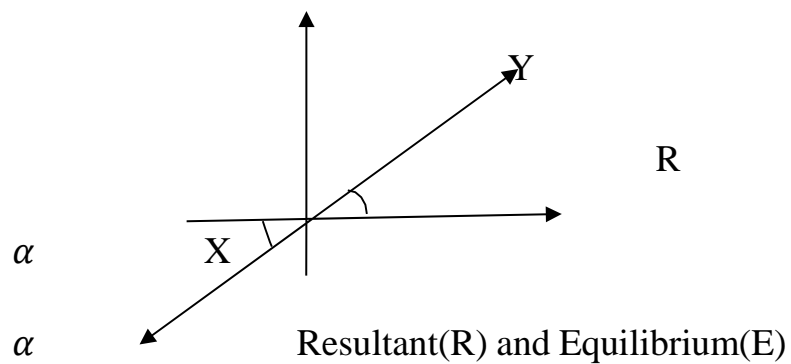
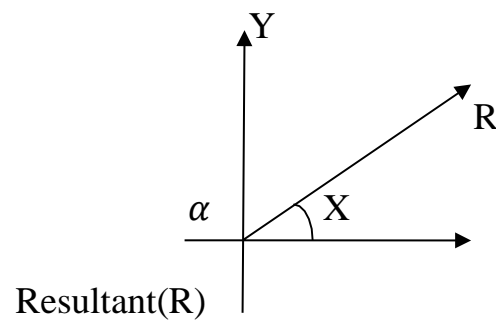
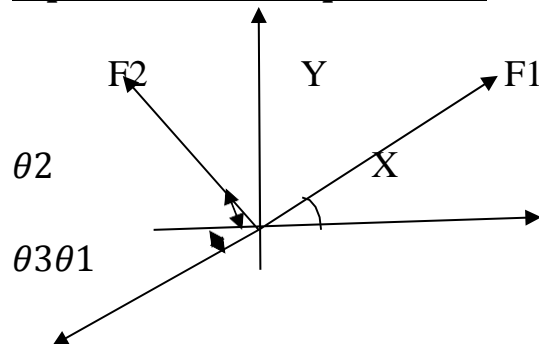
Equilibrium:

A body is said to be in a state of equilibrium, if the body is either at rest or moving at a constant velocity.

Equilibrium Force:

The set of forces where resultant is zero is called “Equilibrium Force”.

Equilibrium and Equilibrant:



Consider a particle subjected to three coplanar concurrent forces as shown in fig(1)

Let the resultant force of the force system R as shown in fig(2) with direction of α with horizontal. Due to this resultant force, the particle may start moving in the direction of resultant force.

But if we apply an additional force of same magnitude and direction as that of resultant force, on the same line of action, but in opposite direction, then the movement of the particle will be arrested or the particle is said to be in Equilibrium.

The force E, which brings the particle (or set of force) to equilibrium, is called equilibrant.

Hence, Equilibrant (E) is Equal to the resultant force(R) in magnitude and direction, collinear but opposite in nature.

Conditions of Equilibrium:

For equilibrium condition of force system, the resultant is Zero.

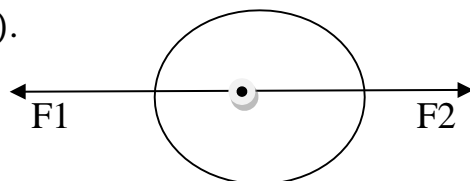
$$R=0$$

$$\text{But } R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2}$$

$$\sum F_H=0 \quad \sum F_V=0$$

Principle of Equilibrium:

Equilibrium principles are developed from the force Law of equilibrium ($\sum F = 0$).



1. Two force Equilibrium principle:

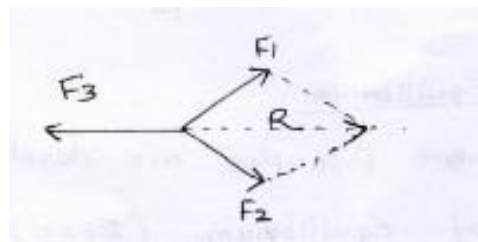
If a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite.

2. Three force equilibrium principle:

If a body is subjected to three forces, then the body will be in equilibrium, if the resultant of any two forces is equal, opposite and collinear with the third force.

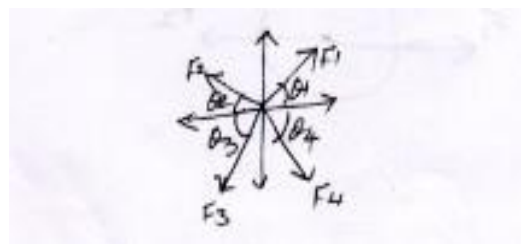
R is the resultant F_1

and F_2 also $R = F_3$



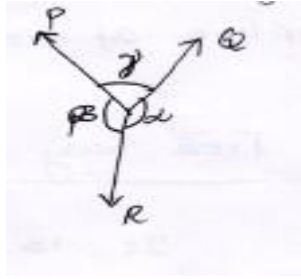
3. Four Force Equilibrium Principle:

If a body is in equilibrium, acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two.



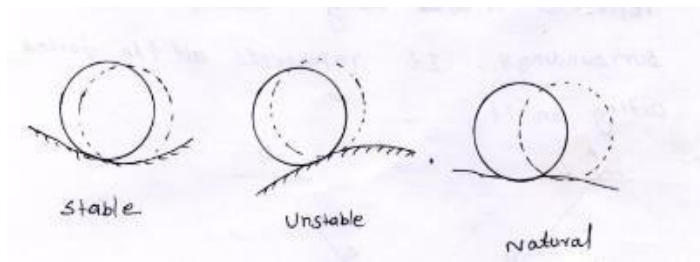
⇒ Lami's Theorem:

If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle b/w the other two.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Types of Equilibrium:



Stable Equilibrium:

A body is said to be in stable equilibrium, if it returns back to its original position after it is slightly displaced from its position.

Unstable Equilibrium:

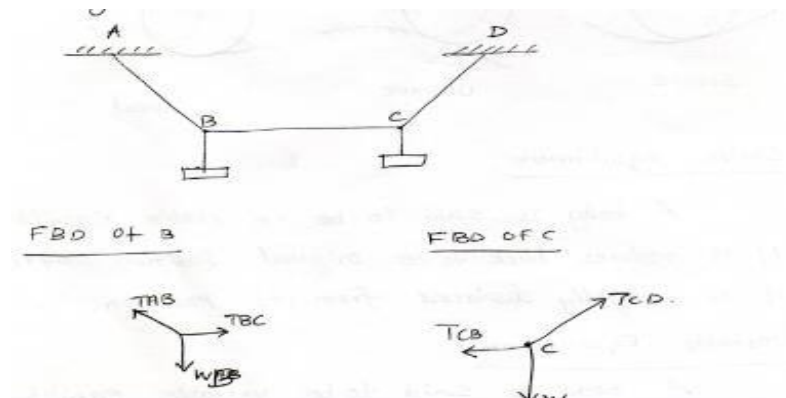
A body is said to be unstable equilibrium it does not return back to its original position and moves farther away after slightly displaced from its position of rest.

Natural Equilibrium:

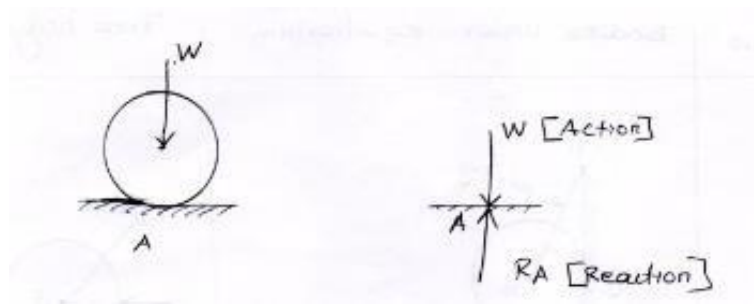
A body is said to be in natural equilibrium it occupies a new position (also remain at rest) after slightly displaced from its position of rest.

⇒Free body Diagram:

It is a sketch of the particle which represents it as being isolated from its surroundings. It represents all the forces acting on it.



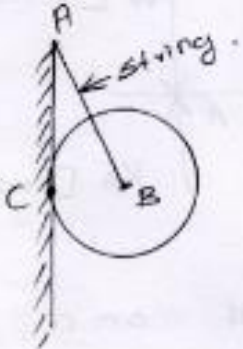
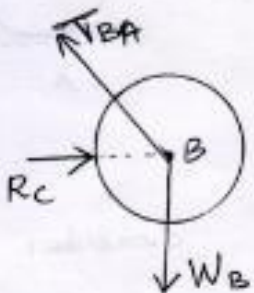
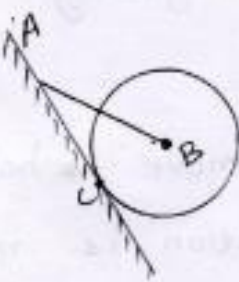

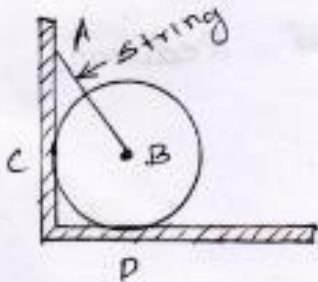
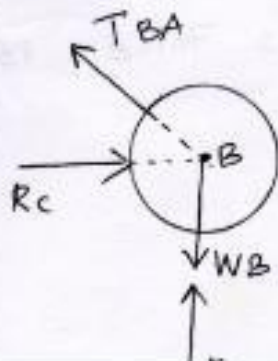
⇒Action and Reaction:

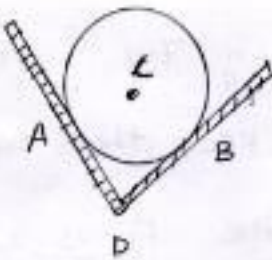

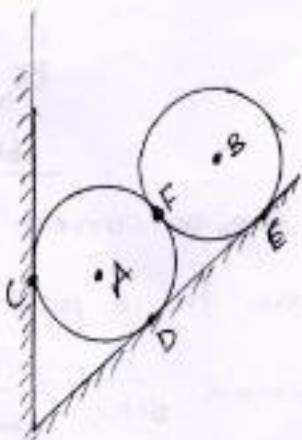
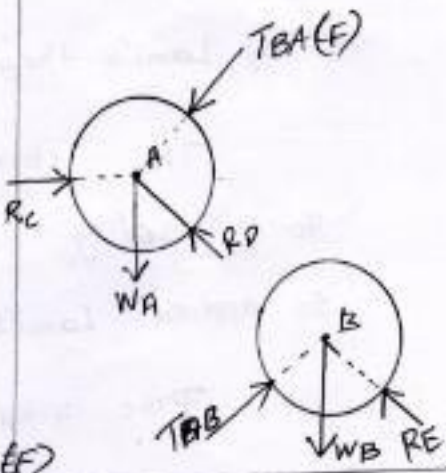
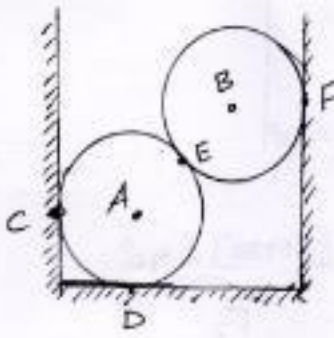
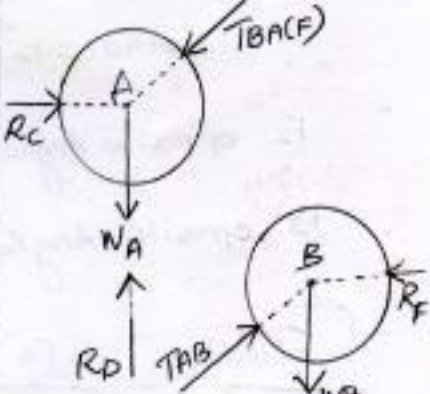
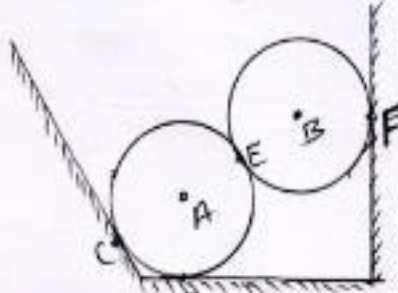
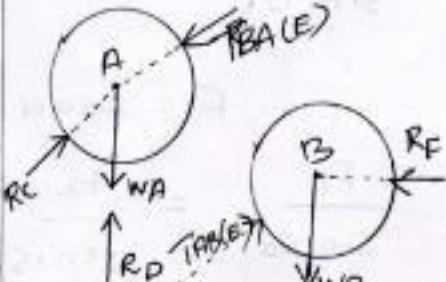


Consider a Ball placed on a horizontal surface shown in fig. The self weight of the ball (w) is acting vertically downwards through its centre of gravity. This force is called Action.

The ball can move horizontally, but its vertical downward motion is resisted due to resisting force developed at support (here, at the point of contact A) Acting vertically upwards. This force is called reaction.

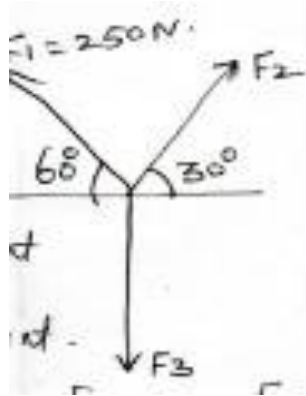
⇒Free body diagram:

SL NO	Bodies under equilibrium	Free body diagram.
10 ✓		
2		
3 ✓		

<p>4</p> <p>✓</p>		
<p>5</p> <p>✓</p>		 <p>$T_{BA(F)} = T_{AB(E)}$</p>
<p>6</p> <p>✓</p>		
<p>7</p>		 <p>$T_{BA(E)} = T_{AB(E)}$</p>

Problems:

1. The force shown in fig. is acting on a particle and keeps the particle in equilibrium. Then magnitude of force F_1 is 250 N. Find the magnitude of forces F_2 and F_3 .



Soln:

1. By Lami's theorem

The three concurrent force acting outwards from a point. So applied Lami's theorem. $\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$

First find the opposite angle of F_1 & F_2 & F_3

F_1 opposite angle

$$30 + 90 = 120^\circ$$

F_2 opposite angle $(90 + 60) = 150^\circ$

F_3 opposite angle $(180 - [60 + 30]) = 90^\circ$

$$\frac{F_1}{\sin 120} = \frac{F_2}{\sin 150} = \frac{F_3}{\sin 90}$$

$$F_1 = 250 \text{ N}$$

$$\frac{F_1}{\sin 120} = \frac{F_2}{\sin 150}$$

$$\frac{250}{\sin 120} = \frac{F_2}{\sin 150}$$

$$F_2 = 144.33 \text{ N}$$

$$\frac{F_1}{\sin 120} = \frac{F_3}{\sin 90}$$

$$F_3 = 288.67 \text{ N}$$

2) By Equations of Equilibrium

$$\sum F_H = 0 \quad \sum F_V = 0 \quad F_3 \text{ is No horizontal force}$$

$$1) \sum F_H = F_2 \cos 30 - F_1 \cos 60 = 0$$

$$F_2 \cos 30 - 250 \cos 60 = 0$$

$$F_2 = \frac{250 \cos 60}{\cos 30}$$

$$F_2 = 144.33 \text{ N}$$

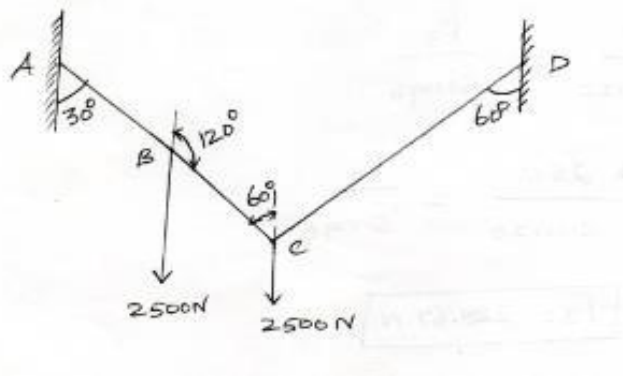
$$\sum F_H = F_1 \sin 30 + F_2 \sin 60 - F_3 = 0$$

$$250 \sin 30 + 144.33 \sin 60 - F_3 = 0$$

$$250 \sin 30 + 144.33 \sin 60 = F_3$$

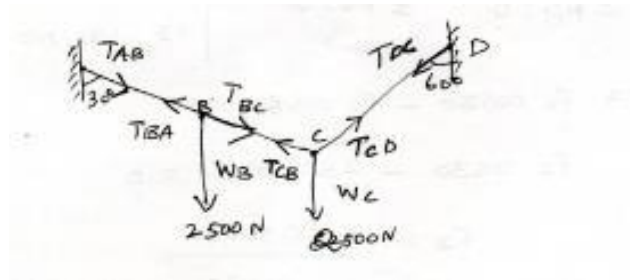
$$F_3 = 228.67 \text{ N}$$

2. Two equal loads of 2500N are supported by a flexible string ABCD at points A & D. Find the tension in the portions AB, BC & CD of string.



Soln:

Free body diagram



$$T_{AB} = T_{BA}$$

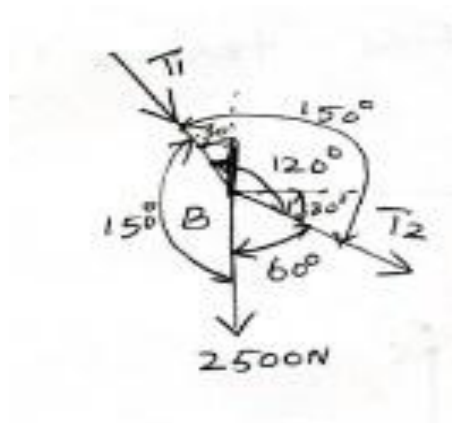
$$\text{III ly } T_{BC} = T_{CB} \text{ \& } T_{CD} = T_{DC}$$

\Rightarrow Let the tension in AB/BC & CD be T_1, T_2 & T_3 respectively

\Rightarrow Let us split up the string

ABCD into two parts.

Consider A and B



By Lami's Theorem

$$\frac{T_{BA}}{\sin 60} = \frac{T_{BC}}{\sin 150} = \frac{2500}{\sin 150}$$

$$\frac{T_{BA}}{\sin 60} = \frac{T_{BC}}{\sin 150} = \frac{2500}{\sin 150}$$

$$\frac{T_{BA}}{\sin 60} = \frac{2500}{\sin 150}$$

$$T_{BA} = \frac{2500}{\sin 150} \times \sin 60$$

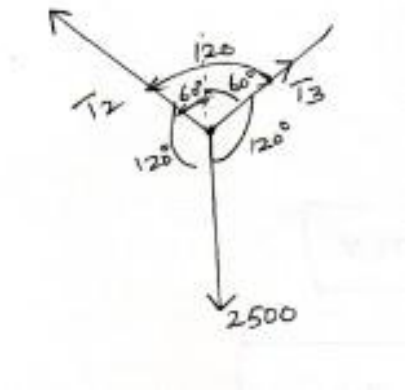
$$T_1 = 4330.13\text{ N}$$

$$\frac{T_{CB}}{\sin 150} = \frac{2500}{\sin 150}$$

$$T_2 = \frac{2500}{\sin 150} \times \sin 150$$

$$T_2 = 2500 \text{ N}$$

Consider a point c

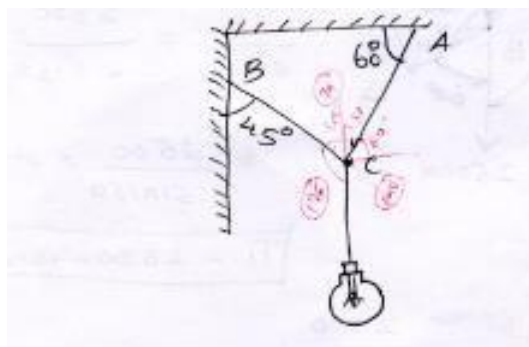


$$\frac{T_2}{\sin 120} = \frac{T_3}{\sin 120} = \frac{2500}{\sin 120}$$

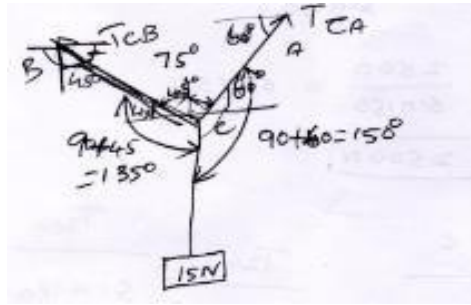
$$\frac{T_3}{\sin 120} = \frac{2500}{\sin 120}$$

$$T_3 = 2500 \text{ N}$$

3. An electric lamp weighting 15 N hangs from a point c, by two strings AC and BC as shown in fig. find tensions in string AC & BC



Soln:



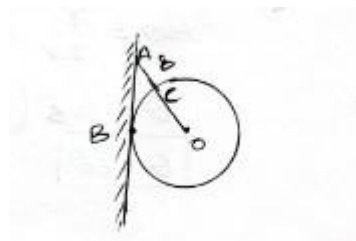
By Lami's theorem

$$\frac{TCB}{\sin 150} = \frac{TCA}{\sin 135} = \frac{15}{\sin 75}$$

$$\frac{TCB}{\sin 150} = \frac{15}{\sin 75} = TCB = 7.76 \text{ N}$$

$$\frac{TCA}{\sin 135} = \frac{15}{\sin 75} = TCA = 7.76 \text{ N}$$

4. A smooth sphere is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point B, on the wall as shown in fig. If the length of the string AC is equal to the radius of the sphere, find the tension in the string & reaction of the wall.



Given:

Radius of sphere $OB=OC=\text{radius length of string, } AC=\text{radius of sphere}=r$, weight of sphere $=w$

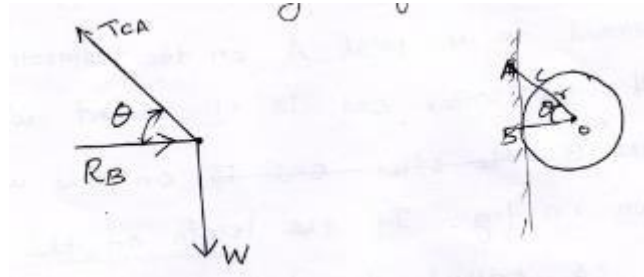
To find :

1. Tension in string

2. Reaction of the wall

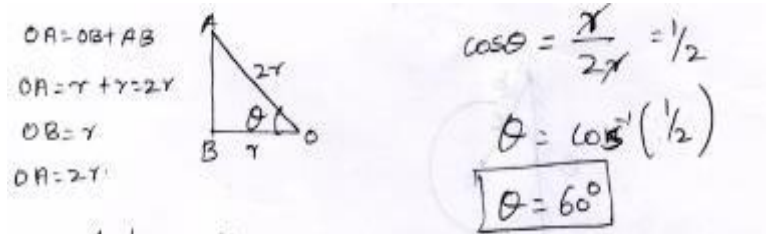
Soln:

Free body diagram



Find the angle b/w T_{CA} & R_B

From right angle triangle AOB



Apply $\sum F_H = 0$

$$R_B - T_{CA} \cos 60 = 0 \text{----- (1)}$$

$$\sum F_V = 0$$

$$T_{CA} \sin 60 - w = 0 \text{----- (2)}$$

$$T_{CA} \sin 60 = w$$

$$T_{CA} = \frac{w}{\sin 60}$$

$$T_{CA} = 1.155w$$

T_{CA} value sub in Eqn (1)

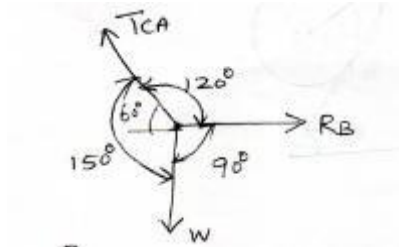
$$R_B - T_{CA} \cos 60 = 0 \text{----- (1)}$$

$$R_B - 1.155w \cos 60 = 0$$

$$R_B = 0.577W$$

Other method

By Lami's theorem

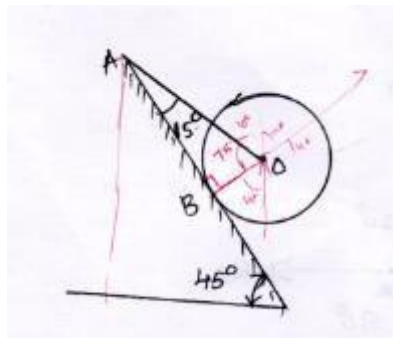


$$\frac{T_{CA}}{\sin 90^\circ} = \frac{R_B}{\sin 150^\circ} = \frac{W}{\sin 120^\circ}$$

$$\frac{T_{CA}}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} \Rightarrow R_B = \frac{W}{\sin 120^\circ} \times \sin 150^\circ$$

$$R_B = 0.577 W$$

5. String AO holds a smooth sphere on an inclined plane ABC as shown in fig. the weight of the sphere is 1000 N and the plane is smooth. Calculate the tension in the string and the reaction at the point of contact B.



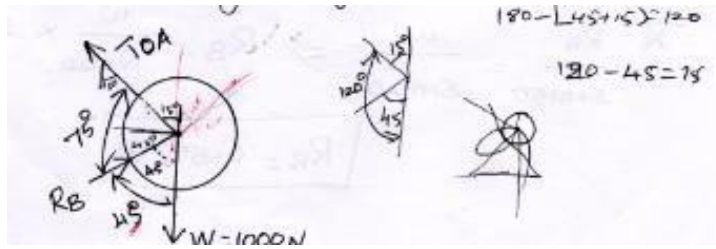
Given:

Weight of sphere $W = 1000\text{N}$

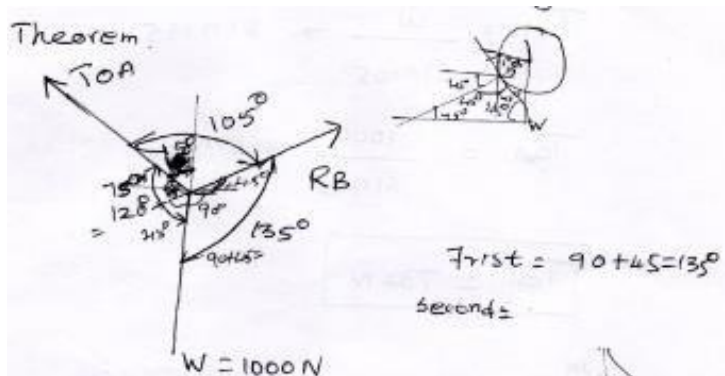
To find: Tension in string & Reaction

Soln:

Free body diagram



No of force is 3, so by using Lami's theorem



In right angled triangle OAB

$$\angle OAB + \angle ABO + \angle BOA = 180^\circ$$

$$\angle OAB = 15^\circ$$

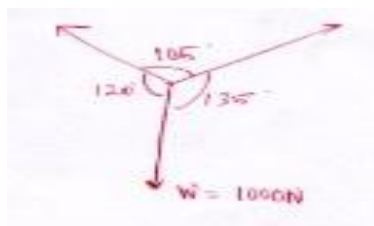
$$15 + 90 + \angle BOA = 180^\circ$$

$$\angle ABO = 90^\circ$$

$$\angle BOA = 180 - (15 + 90)$$

$$\angle BOA = 75^\circ$$

Apply Lami's Eqn



$$\frac{TOA}{\sin 135} = \frac{RB}{\sin 120} = \frac{w}{\sin 105}$$

$$\therefore \frac{TOA}{\sin 135} = \frac{w}{\sin 105}$$

$$T_{OA} = \frac{w}{\sin 105} \times \sin 135$$

$$T_{OA} = \frac{1000}{\sin 105} \times \sin 135$$

$$T_{OA} = 732 \text{ N}$$

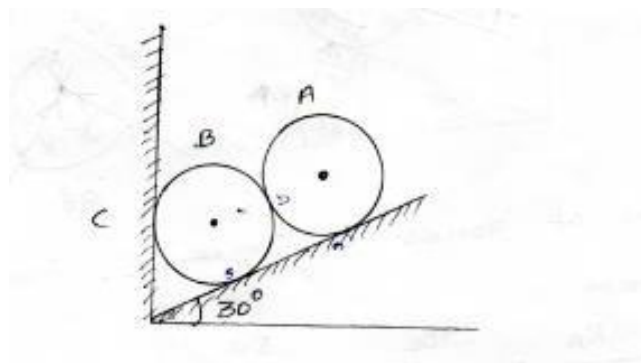
$$\text{III ly } \frac{R_B}{\sin 120} = \frac{w}{\sin 105}$$

$$\frac{R_B}{\sin 120} = \frac{1000}{\sin 105}$$

$$R_B = \frac{1000}{\sin 105} \times \sin 120$$

$$R_B = 896.57 \text{ N}$$

6. Two identical rollers, each of weight 50 N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B and C. assume all the surface to be smooth.



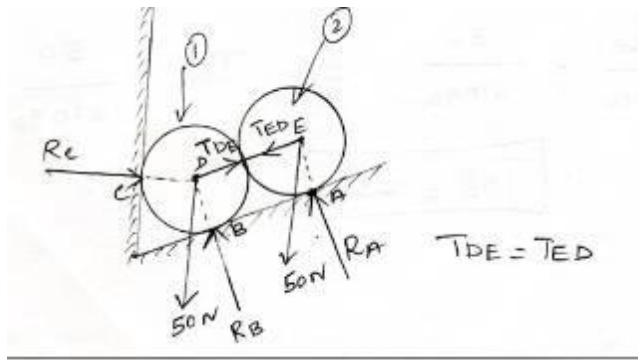
Given:

Weight of roller A & B = 50 N

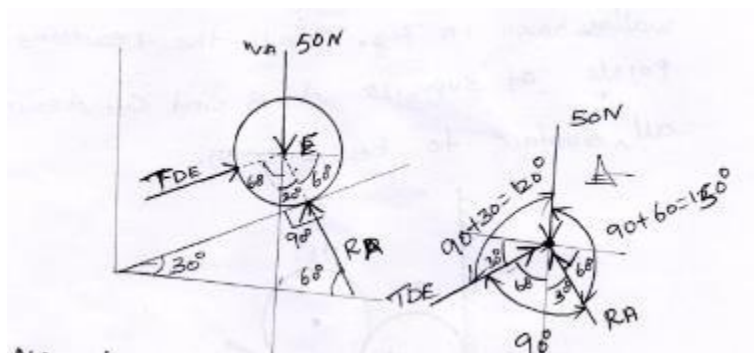
To find:

Reaction at the support A,B and C

Soln:



Free body diagram of roller 2



No of forces is three, apply Lami's Theorem

$$\frac{R_A}{\sin 120} = \frac{T_{DE}}{\sin 150} = \frac{50}{\sin 90}$$

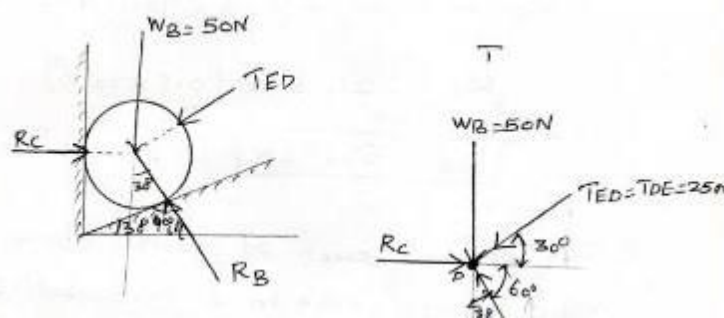
$$\frac{R_A}{\sin 120} = \frac{50}{\sin 90} \Rightarrow R_A = \frac{50}{\sin 90} \times \sin 120$$

$$R_A = 43.3 \text{ N}$$

$$\frac{50}{\sin 150} = \frac{T_{DE}}{\sin 90} \Rightarrow T_{DE} = 50 \times \sin 150$$

$$T_{DE} = 25 \text{ N}$$

Free body diagram of roller 1



All forces acting at point D.

In equilibrium condition

$$\sum F_H = 0 \text{ \& } \sum F_V = 0$$

$$\sum F_H = 0 \rightarrow + \quad - \leftarrow$$

$$R_C - T_{ED} \cos 30 - R_B \cos 60 = 0$$

$$R_C - 25 \cos 30 - R_B \cos 60 = 0$$

$$R_C - 21.65 - 0.5 R_B = 0 \text{ ----- (1)}$$

$$\sum F_V = 0 \downarrow - \quad \uparrow +$$

$$R_B \sin 60 - T_{ED} \sin 30 - 50 = 0$$

$$R_B \sin 60 - 25 \sin 30 - 50 = 0$$

$$R_B \sin 60 = 25 \sin 30 + 50 = 62.5$$

$$R_B = \frac{62.5}{\sin 60}$$

$$R_B = 72.17 \text{ N}$$

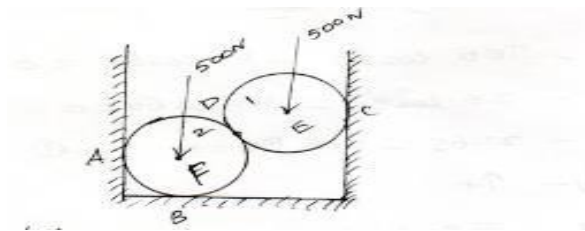
R_B value substitute Eqn(1)

$$R_C - 21.65 - (0.5 \times 72.17) = 0$$

$$R_C = 21.65 + (0.5 \times 72.17)$$

$$R_C = 57.73 \text{ N}$$

7. Two spheres each of weight 500 N and of radius 100mm rest in a horizontal channel of width of 360mm as shown in fig. find the reactions on the points of contact A ,B and C. Assume all the surface of contact are smooth.



Given data:

Weight of each Roller $w = 500 \text{ N}$

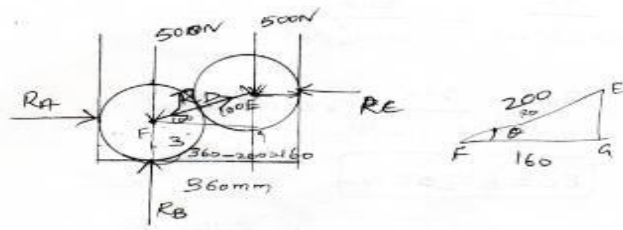
Width of channel $= 360 \text{ mm}$

Radius of rollers $r = 100\text{mm}$

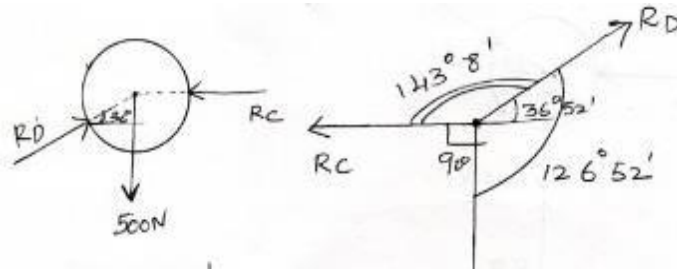
To find:

Reaction on the points of A,B & C.

Soln



Free body diagram of roller (1)



$$\cos\theta = \frac{FG}{EF}$$

$$\theta = \cos^{-1}\left(\frac{FG}{EF}\right)$$

$$\theta = \cos^{-1}\left(\frac{160}{200}\right)$$

$$\theta = 36^\circ 52'$$

By Lami's Theorem

$$\frac{R_D}{\sin 90} = \frac{R_C}{\sin 126^\circ 52'} = \frac{500}{\sin 143^\circ 8'}$$

$$\frac{R_D}{\sin 90} = \frac{500}{\sin 143^\circ 8'}$$

$$R_D = \frac{500}{\sin 143^\circ 8'} \times \sin 90$$

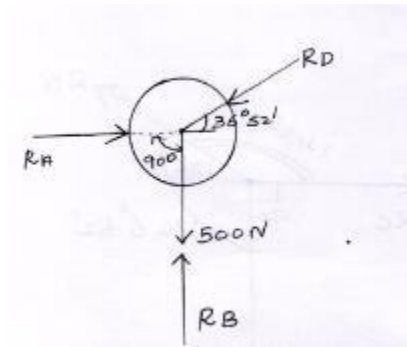
$$R_D = 833.39 \text{ N}$$

$$\frac{R_C}{\sin 126^\circ 52'} = \frac{500}{\sin 143^\circ 8'}$$

$$R_C = \frac{500}{\sin 143^\circ 8'} \times \sin 126^\circ 52'$$

$$R_C = 673.08 \text{ N}$$

Free body diagram of roller(2)



$$\sum F_H = R_A - R_D \cos 36^\circ 52' = 0$$

$$\rightarrow + \quad - \leftarrow R_A = R_D \cos 36^\circ 52'$$

$$R_A - 833.39 \times \cos 36^\circ 52'$$

$$R_A = 666.74 \text{ N}$$

$$\sum F_V = 0 \quad \downarrow - \quad \uparrow +$$

$$R_B - R_D \sin 36^\circ 52' - 500 = 0$$

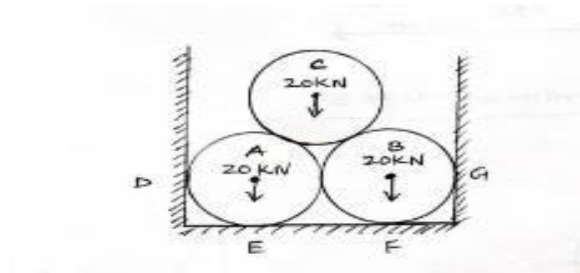
$$R_B - 833.39 \sin 36^\circ 52' - 500 = 0$$

$$R_B - 499.99 - 500 = 0$$

$$R_B = 999.99 \text{ N}$$

8. Three smooth pipes each weighting 20 kN and of diameter 60 cm are to be placed in a rectangular channel with horizontal base as shown in fig.

Calculate the reactions at the points of contact b/w the pipes and b/w the channel and the pipes. Take width of the channel as 160 cm.



Given:

Weight of each pipe = $w_A = w_B = w_C = 20 \text{ kN}$

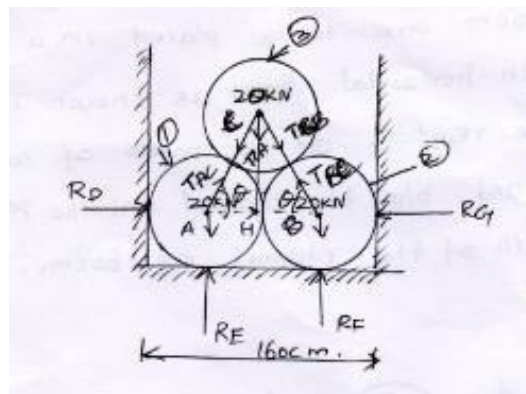
Diameter of each pipe = $D_A = D_B = D_C = 60 \text{ cm}$

Width of channel = 160 cm

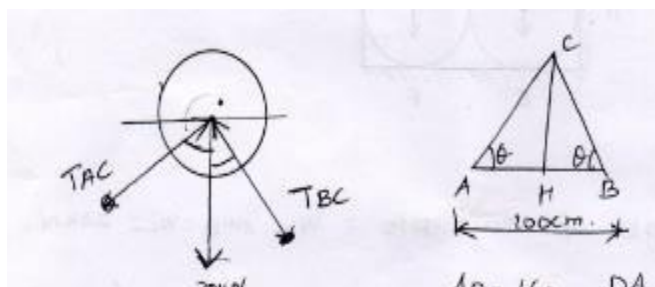
To find:

Reaction at the points : D, E, f, G

Soln:



Free body diagram of pipe 3



$$AB = 160 - DA - BG$$

diameter

From triangle HAC

$$AB = 160 - 30 - 30$$

$$DA = DB = 60 \text{ cm}$$

$$\cos \theta = \frac{AH}{AC}$$

$$AB=100\text{cm}$$

$$\theta = \cos^{-1} \left[\frac{AH}{AC} \right]$$

$$AC=BC$$

$$BG=DB/2$$

$$= \cos^{-1} \left[\frac{50}{60} \right]$$

$$AC=BC=2 \times \text{radius}$$

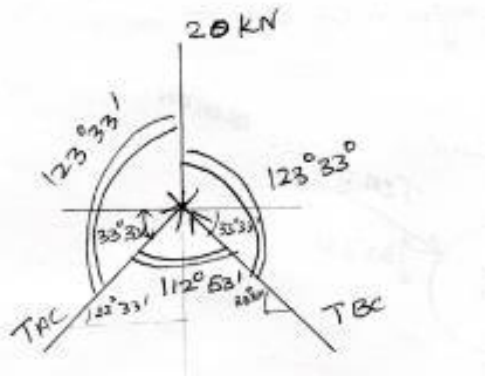
$$DA=BG=\text{radius}$$

$$AC=BC=2 \times 30$$

$$\theta = 33^\circ 33'$$

$$AC=BC=60 \text{ cm}$$

$$AH = \frac{AB}{2} = \frac{100}{2} = 50 \text{ cm}$$



By Lami's theorem

$$\frac{T_{AC}}{\sin 123^\circ 33'} = \frac{T_{BC}}{\sin 123^\circ 33'} = \frac{20}{\sin 112^\circ 53'}$$

$$\frac{T_{AC}}{\sin 123^\circ 33'} = \frac{20}{\sin 112^\circ 53'}$$

$$T_{AC} = \frac{20}{\sin 112^\circ 53'} \times \sin 123^\circ 33'$$

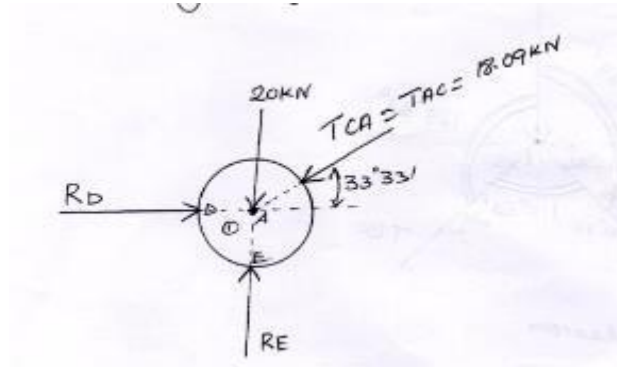
$$T_{AC} = 18.09 \text{ KN}$$

$$\frac{T_{BC}}{\sin 123^\circ 33'} = \frac{20}{\sin 112^\circ 53'}$$

$$T_{BC} = \frac{20}{\sin 112^\circ 53'} \times \sin 123^\circ 33'$$

$$T_{BC} = 18.09KN$$

Free body diagram of pipe(1)



$$\sum F_H = 0 \quad \rightarrow + \quad - \leftarrow$$

$$R_D - T_{CA} \cos 33^\circ 33' = 0$$

$$R_D = 18.09 \cos 33^\circ 33' = 15.07$$

$$R_D - 15.07 = 0 \quad T_{CA} = T_{AC}$$

$$R_D = 15.07 \text{ KN}$$

$$\sum F_V = 0 \quad \downarrow - \quad \uparrow +$$

$$R_E - 20 - T_{CA} \sin 33^\circ 33' = 0$$

$$R_E - 20 - 18.09 \sin 33^\circ 33' = 0$$

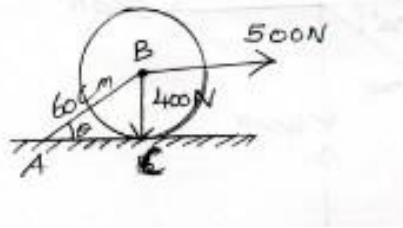
$$R_E - 29.99 = 0$$

$$R_E = 29.99 \text{ KN}$$

Similarly the free body diagram of pipe (2) is analyzed for pipe (1)

$$\therefore R_E = 29.99 \text{ KN} \quad \& R_D = 15.07 \text{ KN}$$

9. A circular roller of radius 20 cm and of weight 400 N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 60 cm as shown in fig. a horizontal force of 500 N is acting at B. Find the Tension in bar AB and the reaction at C.



Given:

Radius of circular roller $r = 20 \text{ cm}$

Weight of roller $w = 400 \text{ N}$

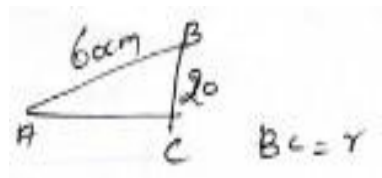
Horizontal force $= 500 \text{ N}$

To find:

- 1) Tension in AB
- 2) Reaction at C.

Soln:

From triangle ABC

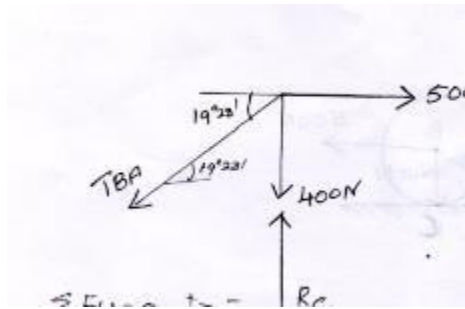


$$\sin \theta = \frac{BC}{AB} \Rightarrow \theta = \sin^{-1} \left(\frac{BC}{AB} \right)$$

$$\theta = \sin^{-1} \left(\frac{20}{60} \right)$$

$$\theta = 19^\circ 28'$$

Free body diagram



$$\sum F_H = 0 \quad \rightarrow + \quad - \leftarrow$$

$$\sum F_H = -T_{BA} \cos 19^\circ 28' + 500 = 0$$

$$-T_{BA} \cos 19^\circ 28' = -500$$

$$-T_{BA} = \frac{-500}{\cos 19^\circ 28'}$$

$$-T_{BA} = 530.31 \text{ N}$$

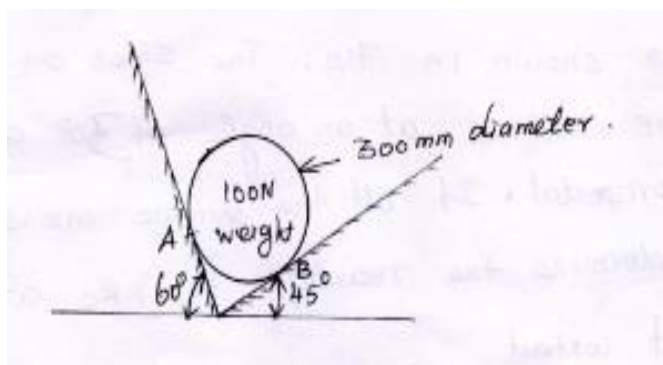
$$\sum F_V = 0 \quad \downarrow - \quad \uparrow +$$

$$-400 + R_c - T_{BA} \sin 19^\circ 28' = 0$$

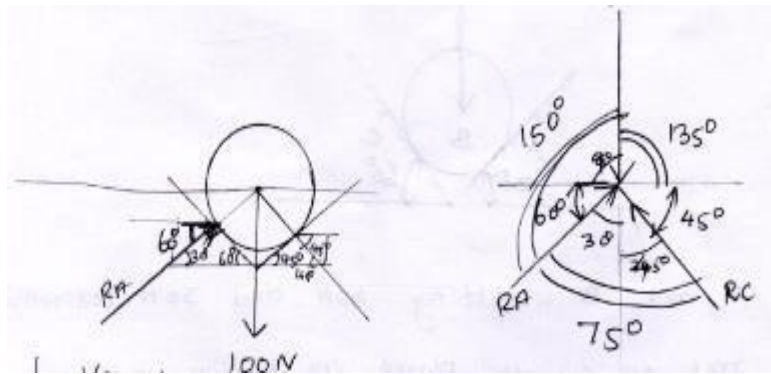
$$R_c = T_{BA} \sin 19^\circ 28' + 400 = 530.31 \times \sin 19^\circ 28' + 400$$

$$R_c = 576.62 \text{ N}$$

10. Determine the reaction at a and B



Soln:



By Lami's theorem

$$\frac{R_A}{\sin 135^\circ} = \frac{R_C}{\sin 150^\circ} = \frac{100}{\sin 75^\circ}$$

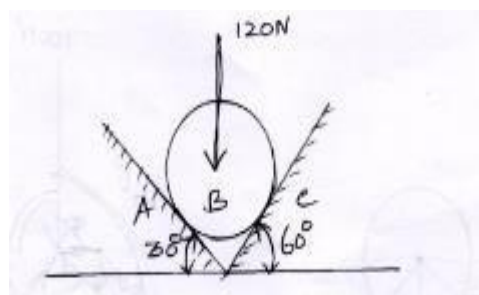
$$\frac{R_A}{\sin 135^\circ} = \frac{100}{\sin 75^\circ} \Rightarrow R_A = \frac{100 \times \sin 135^\circ}{\sin 75^\circ}$$

$$R_A = 73.2N$$

$$\frac{R_C}{\sin 135^\circ} = \frac{100}{\sin 75^\circ} \Rightarrow R_C = \frac{100 \times \sin 150^\circ}{\sin 75^\circ}$$

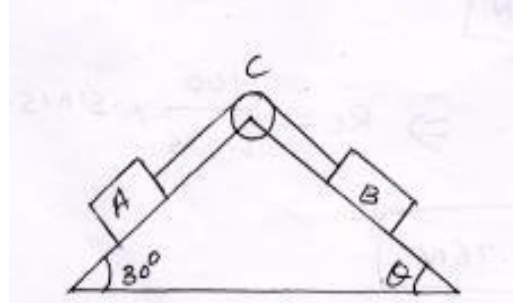
$$R_C = 51.76N$$

- 11.** A Ball weight 120N in a right angle groove as shown in fig. The sides of the groove are inclined at an angle of 30° and 60° to the horizontal. If all the surface are smooth, then determine the reaction R_A & R_C at the point of contact



12. A and B weighting 40 N and 30 N respectively rest on smooth planes as shown in fig. They are connected by a weight less chord passing over

a friction less pulley. Determine the angle θ & the tension in the chord for equilibrium. Also find the reaction of Block A & B



Given:

$$W_A = 40\text{N}$$

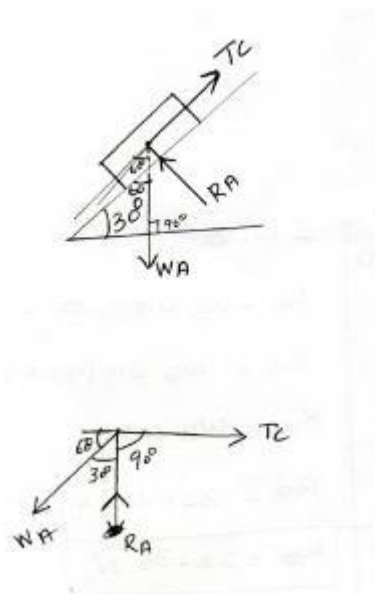
$$W_B = 30\text{N}$$

To find

1. θ
2. Reaction of Block A & B

Soln

F B D of Block A



$$\sum FH = 0$$

$$TC - WA \cos 60 = 0$$

$$TC = WA \cos 60$$

$$TC = 40 \cos 60$$

$$TC = 20N$$

$$\sum FV = 0$$

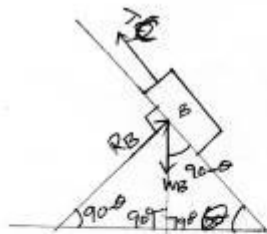
$$+RA - WA \sin 60 = 0$$

$$+RA = WA \sin 60$$

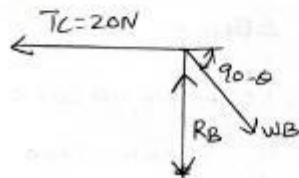
$$RA = 40 \sin 60$$

$$RA = 34.64N$$

FBD of Block B



$$T_C = T_D$$



$$\sum FH = 0$$

$$-T_C + W_B \times \cos (90 - \theta) = 0$$

$$-T_C = -W_B \cos (90 - \theta)$$

$$\sum FV = 0$$

$$R_B - W_B \sin (90 - \theta) = 0$$

$$R_B = W_B \sin (90 - \theta)$$

$$T_C = W_B \sin \theta$$

$$R_B = W_B \cos \theta$$

$$20 = 30 \sin \theta$$

$$R_B = 30 \times \cos 41^\circ 48'$$

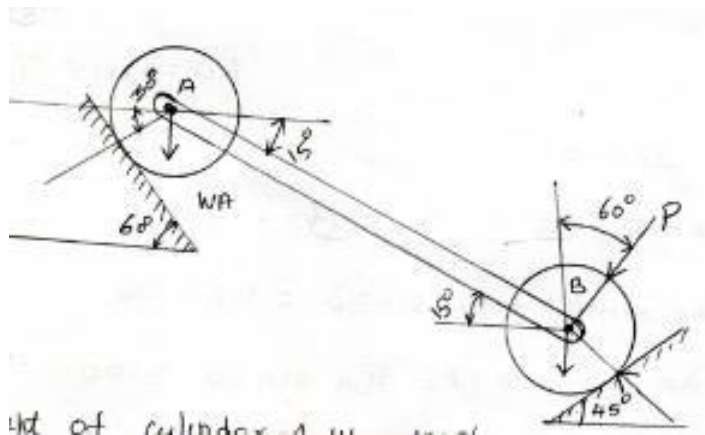
$$\sin \theta = \frac{20}{30}$$

$$R_B = 22.36 \text{ N}$$

$$\theta = \sin^{-1} \left(\frac{20}{30} \right)$$

$$\theta = 41^\circ 48'$$

13. The following fig shows cylinders, A of weight 100 N and B weight 50 N resting on smooth inclined planes. They are connected by a bar of negligible weight hinged to each cylinder at their geometric centers by smooth pins. Find the force P, that can hold the system in the given position.



Given:

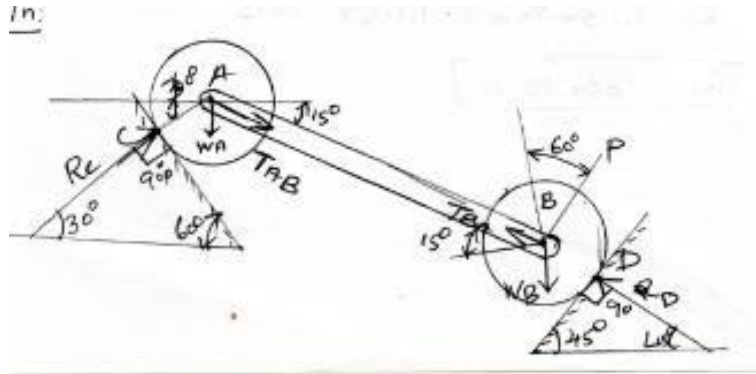
Weight of cylinder A $W_A = 100 \text{ N}$

Weight of cylinder B $W_B = 50 \text{ N}$

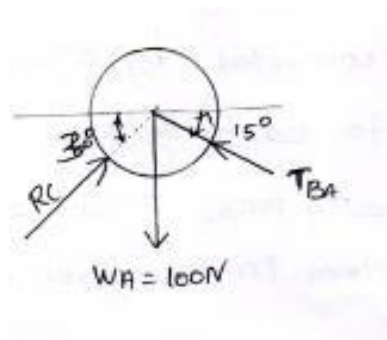
To find:

Force 'P'

Soln:



Free body diagram for cylinder A



$$\sum F_H = 0$$

$$\sum F_H = R_C \cos 30 - T_{BA} \cos 15 = 0$$

$$R_C \cos 30 = T_{BA} \cos 15$$

$$R_C = \frac{T_{BA} \cos 15}{\cos 30}$$

$$R_C = 1.115 T_{BA} \text{----- (1)}$$

$$\sum F_V = 0$$

$$\sum F_V - W_A + T_{BA} \sin 15 + R_C \sin 30 = 0$$

$$T_{BA}\sin 15 + R_C\sin 30 = W_A = 0$$

$$T_{BA}\sin 15 + 1.115 T_{BA}\sin 30 = 0$$

$$[\sin 15 + 1.115\sin 30] = 100$$

$$T_{BA} = \frac{100}{\sin 15 + 1.115\sin 30}$$

$$T_{BA} = 122.5 \text{ N}$$

$$R_C = 1.115 \times T_{BA} = 1.115 \times 122.5 R_C = 136.58 \text{ N}$$

Free body diagram of cylinder B

$$\Sigma F_H = 0 \quad \rightarrow + \quad - \leftarrow$$

$$T_{AB}\cos 15 - P\cos 30 - R_D\cos 45 = 0$$

$$122.5 \cos 15^\circ - P\cos 30 - R_D\cos 45 = 0$$

$$118.32 - 0.866P - 0.707R_D = 0$$

$$-0.866P - 0.707R_D = -118.32 \text{----- (1)}$$

$$\Sigma F_V = 0 \quad \downarrow - \quad \uparrow +$$

$$-W_B - P\sin 30 - T_{AB}\sin 15 + R_D\sin 45 = 0$$

$$-50 - P \times \sin 30 - 122.5\sin 15 + R_D\sin 45 = 0$$

$$-50 - 0.5P - 31.7 + 0.707 R_D = 0$$

$$-81.7 - 0.5P + 0.707 R_D = 0$$

$$0.707 R_D - 0.5P = 81.7$$

$$-0.5P + 0.707R_D = 81.7 \text{----- (2)}$$

$$(1) \Rightarrow -0.866P - 0.707R_D = -118.32$$

$$(2) \Rightarrow -0.5P + 0.707R_D = 81.7$$

$$-0.366P = -36.62$$

$$P = \frac{36.62}{0.366}$$

$$P = 100 \text{ N}$$

Substitute in (1)

$$-0.866P - 0.707R_D = -118.32$$

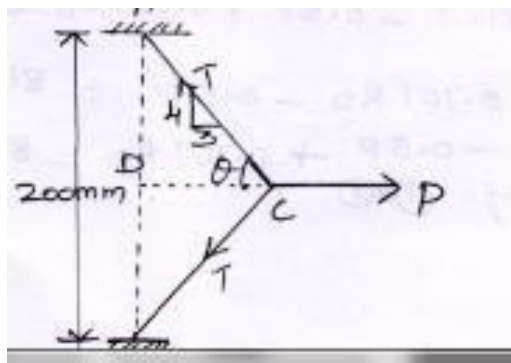
$$-86.6 - 0.707 \times R_D = -118.32$$

$$-0.707R_D = -118.32 + 86.6 = -32$$

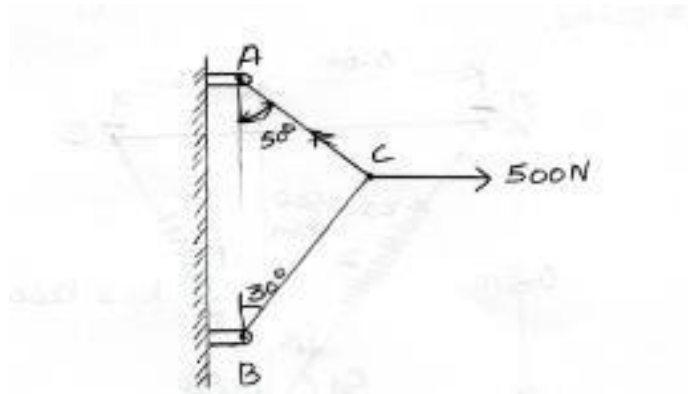
$$R_D = \frac{-32}{-0.707}$$

$$R_D = 45.26 \text{ N}$$

- 14.** A rubber band has an unstretched length of 200mm. It is pulled until its length is 250mm. as shown in fig. the horizontal force P is 1.75 n. what is the tension in the band (HW)



- 15.** Two cables are tied together at c and are loaded as shown in fig below. Determine the tension in the cable AC and BC



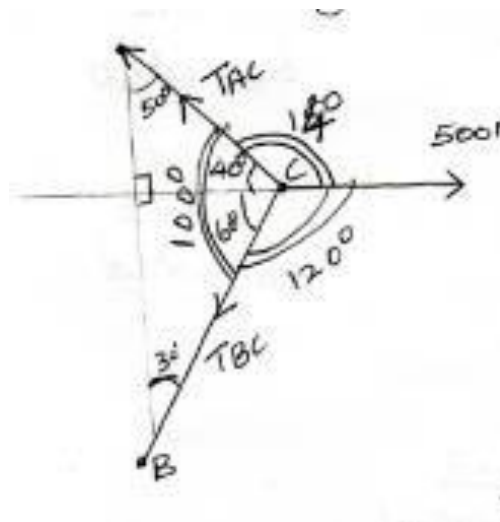
Given:

Force on C = 500 N

To find: Tension of cable AC & BC

Soln:

Free body Diagram



By using Lami's Theorem

$$\frac{T_{AC}}{\sin 120} = \frac{T_{BC}}{\sin 140} = \frac{500}{\sin 100}$$

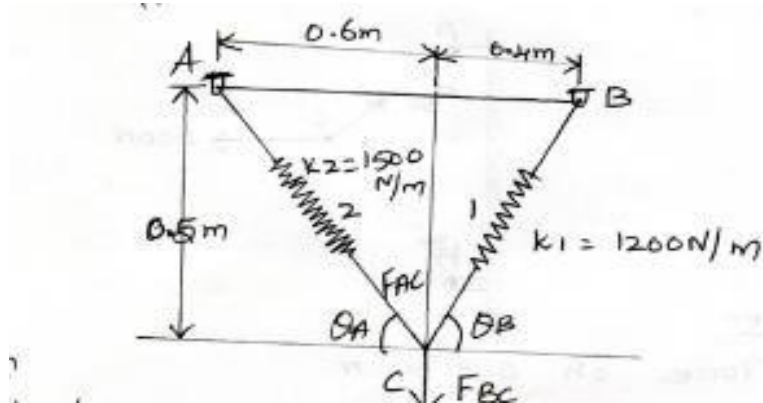
$$\frac{T_{AC}}{\sin 120} = \frac{500}{\sin 100} \Rightarrow T_{AC} = \frac{500}{\sin 100} \times \sin 120$$

$$T_{AC} = 439.69 \text{ N}$$

$$\frac{T_{BC}}{\sin 140} = \frac{500}{\sin 100} \Rightarrow T_{BC} = \frac{500}{\sin 100} \times \sin 140$$

$$T_{BC} = 326.35 \text{ N}$$

16. A 30kg block is suspended by two springs having stiffness as shown. Determine the unstretched length of each spring after the block is removed.

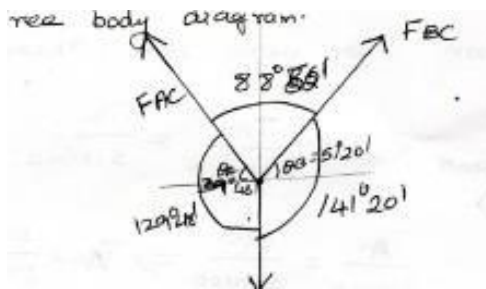


Unknown

Length of each spring L_1 & L_2

Soln:

Free body diagram



$$\tan \theta_B = \frac{0.5}{0.4}$$

$$\theta_B = 51^\circ 20'$$

$$\tan \theta_A = \frac{0.5}{0.6}$$

$$\theta_A = 39^\circ 48'$$

$$\begin{aligned} F &= 30 \text{ kg} \times 9.81 \\ &= 294.3 \text{ N} \end{aligned}$$

By Lami's theorem
$$\frac{F_{AC}}{\sin 141^\circ 20'} = \frac{F_{BC}}{\sin 129^\circ 48'} = \frac{294.3}{\sin 88^\circ 52'}$$

$$\frac{F_{AC}}{\sin 141^\circ 20'} = \frac{294.3}{\sin 88^\circ 52'} \Rightarrow F_{AC} = \frac{294.3}{\sin 88^\circ 52'} \times \sin 141^\circ 20'$$

$$F_{AC} = 183.91N$$

$$\frac{F_{BC}}{\sin 129^\circ 48'} = \frac{294.3}{\sin 88^\circ 52'}$$

$$F_{BC} = \frac{294.3}{\sin 88^\circ 52'} \times \sin 129^\circ 48'$$

$$F_{BC} = 226.15N$$

$$\text{Stiffness} = \frac{\text{Force}}{\text{Deflection}}$$

$$k_2 = \frac{F_{AC}}{\delta_2}$$

$$1500 = \frac{183.91}{\delta_2}$$

$$\delta_2 = 0.122m$$

$$k_1 = \frac{F_{BC}}{\delta_1}$$

$$1200 = \frac{226.15}{1}$$

$$\delta_2 = 0.188m$$

$$L_1 = \sqrt{(0.4)^2 + (0.5)^2} = 0.6403m$$

$$L_2 = \sqrt{(0.6)^2 + (0.5)^2} = 0.7810m$$

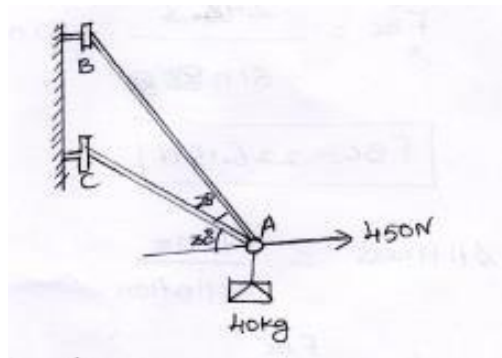
$$l_1 = L_1 - \delta_1 = 0.6403 - 0.188$$

$$l_1 = 0.452m$$

$$l_2 = L_2 - \delta_2 = 0.7810 - 0.122$$

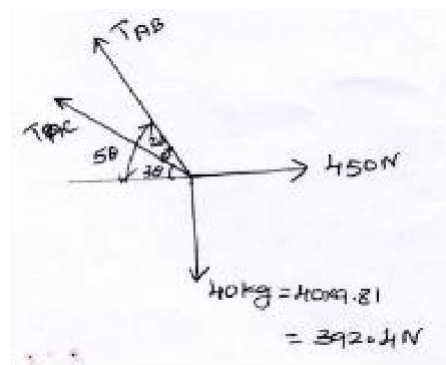
$$l_2 = 0.728m$$

17. Determine the tension in cables AB and AC required to hold the 40 kg crate shown in fig. below.



To find: T_{AB} & T_{AC}

Soln:



$$\sum F_H = 0 \quad \rightarrow + \quad - \leftarrow$$

$$450 - T_{AB} \cos 50 - T_{AC} \cos 30 = 0$$

$$T_{AB} \cos 50 + T_{AC} \cos 30 = 450$$

$$0.64T_{AB} + 0.86T_{AC} = 450 \text{-----(1)}$$

$$\sum F_V = 0 \quad \downarrow - \quad \uparrow +$$

$$-392.4 + T_{AB}\sin 50 + T_{AC}\sin 30 = 0$$

$$T_{AB}\sin 50 + T_{AC}\sin 30 = 392.4$$

$$0.76T_{AB} + 0.5T_{AC} = 392.4 \text{----- (2)}$$

Solving Eq(1)&(2)

$$(1) \quad \Rightarrow 0.64T_{AB} + 0.86 = 450$$

$$(2) \times 1.72 \quad \Rightarrow 0.76T_{AB} + 0.85T_{AC} = 392.4$$

$$-0.46T_{AB} = -224.92$$

$$T_{AB} = \frac{224.92}{0.46}$$

$$T_{AB} = 488.95N$$

T_{AB} sub in (1)

$$0.64 \times 488.95 + 0.86T_{AC} = 450$$

$$T_{AC} = 159.38N$$

