5.4 SOLUTION OF STATE EQUATIONS

Solution of homogeneous state equations (solution of state equations without input or excitation)

Consider a first order differential equation, with initial condition, $x(0) = x_0$.

$= ax; \%(0) = x_0$

On rearranging equation we get $\frac{ctx}{} = a\,dt$ integrating the equation we get, 1 o gx = at —

$$X = 3^{(dt''I^{-c)}} = e^{dt} Q_C$$

When t=0, X — x(0) — e^c

Given that,x (0) — x_0 • e^c — X_0

The solution of first order differential equation as,

$$x = e^{at}x_0$$

We know that,e^x — $\left[1 — x — —x^2 — —I- -x^1\right]$ L 2! i! J

$$x — [1 — at — — (a t)^2 — —- — (a\,t)\,x_0$$

Consider the state equations without input vector,

$X(t)$— A X(t); $x(0) = X_0$.

The solution of the matrix or vector equation can be assumed

$$X(t) — A_0 — A_1 t — A_2 t^2 — \cdots$$

Where $A_0, A_1, A_2, \ldots A_i$ are matrices and the elements of the matrices are constant.

$$X(t) — [I — A t + \frac{1}{2!}A^2t^2 — \frac{1}{3!}A^3t^3 — \cdots \frac{1}{i!}A^i t^i] A$$

Where I is the unit matrix.

Matrix exponential may be written as,

$$I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots \frac{1}{i!}A^i t^i$$

Hence the solution of the state equation is ( ) The matrix e^{At} is alled state transition matrix and denoted by p(t).

PROPERTIES OF STATE TRANSITION MATRIX

1. *( )*
2. *( ) ( ) [( )]*
3. *( )$^{( )}$ ( )( ) ( ) ( )*

COMPUTATION OF STATE TRANSISTION MATRIX

Method 1: Computation of $e^{A c}$ using matrix exponential

In this method the $e^{A c}$ is computed using the matrix exponential of equation

$$e^{At} = \left[I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \cdots \frac{1}{i!}A^it^i \right]$$

Where,

$e^{A L}$ state transition matrix of order n x n

A = System matrix of order n x n

I = Unit matrix of order n x n

Method 2: Computation of state transition matrix by Laplace transforms method

Consider the state equation without input vector *X.(t) = A X(t)*

On taking Laplace transform of above equation,

$$X(s) = (sI\text{-}A)_{\sim}^{1}X(0)$$

On taking inverse Laplace transform

$$X(t) = L_{-}^{1}[(sI — A)_{-}^{1}]\ X(0)$$

On comparing above equation with state equation

$$X(t) = e^{A\,l}x_0$$

$$e^{A\,t} = L_{\sim}\ \hat{}(sI\text{-}A)_{-}^{1}]$$

$$L^{[}e^{A\,t]} = L^{[}p(t)^{]} = p(s)$$

Where ( ) ( ) it is called resolvant matrix