4. TEMPERATURE DISTRIBUTION IN I-D SYSTEMS

4.1 A Plane Wall

A plane wall is considered to be made out of a constant thermal conductivity material and extends to infinity in the Y- and Z-direction. The wall is assumed to be homogeneous and isotropic, heat flow is one-dimensional, under steady state conditions and losing negligible energy through the edges of the wall under the above mentioned assumptions the Eq. (2.2) reduces to

$d^2T / dx^2 = 0$; the boundary conditions are: at $x = 0, T = T_1$

Integrating the above equation, $x = L, T = T_2$

$T = C_1x + C_2$, where C_1 and C_2 are two constants.

Substituting the boundary conditions, we get $C_2 = T_1$ and $C_1 = (T_2 - T_1)/L$ The temperature distribution in the plane wall is given by

$$T = T_1 - (T_1 - T_2)\, x/L \qquad (2.3)$$

which is linear and is independent of the material.

Further, the heat flow rate, $\dot{Q}/A = -k\ dT/dx = (T_1 - T_2)k/L$, and therefore the temperature distribution can also be written as

$$T - T_1 = (\dot{Q}/A)(x/k) \qquad (2.4)$$

i.e., "the temperature drop within the wall will increase with greater heat flow rate or when k is small for the same heat flow rate,"

4.2 A Cylindrical Shell-Expression for Temperature Distribution

In the cylindrical system, when the temperature is a function of radial distance only and is independent of azimuth angle or axial distance, the differential equation (2.2) would be, (Fig. 1.4)

$$d^2T/dr^2 + (1/r)\, dT/dr = 0$$

with boundary conditions: at $r = r_1$, $T = T_1$ and at $r = r_2$, $T = T_2$.

The differential equation can be written as:

$$\frac{1}{r}\frac{d}{dr}\left(r\,dT/dr\right)=0, \text{ or, } \quad \frac{d}{dr}\left(r\ dT/dr\right)=0$$

upon integration, $T = C_1 \ln(r) + C_2$, where C_1 and C_2 are the arbitrary constants.

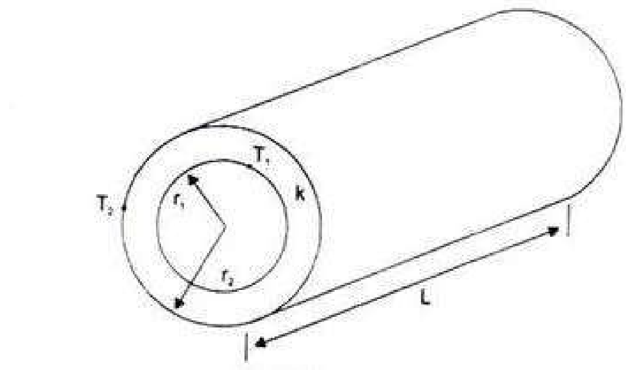


Fig 1.4: A Cylindrical shell

By applying the boundary conditions,

$$C_1 = \left(T_2 - T_1\right)/\ln\left(r_2/r_1\right)$$

and $$C_2 = T_1 - \ln\left(r_1\right).\left(T_2 - T_1\right)/\ln\left(r_2/r_1\right)$$

The temperature distribution is given by

$$T = T_1 + \left(T_2 - T_1\right).\ \ln\left(r/r_1\right)/\ln\left(r_2/r_1\right) \text{ and}$$

$$\dot{Q}/L = -kA\ dT/dr \quad = 2\pi k\left(T_1 - T_2\right)/\ln\left(r_2/r_1\right) \qquad (2.5)$$

From Eq (2.5) It can be seen that the temperature varies 10gantJunically through the cylinder wall In contrast with the linear variation in the plane wall .

If we write Eq. (2.5) as $\dot{Q} = kA_m\left(T_1 - T_2\right)/\left(r_2 - r_1\right)$, where

$$A_m = 2\pi\left(r_2 - r_1\right)L/\ln\left(r_2/r_1\right) \quad = \left(A_2 - A_1\right)/\ln\left(A_2/A_1\right)$$

where A_2 and A_1 are the outside and inside surface areas respectively. The term A_m is called ‘Logarithmic Mean Area' and the expression for the heat flow through a cylindrical wall has the same form as that for a plane wall.

4.3 Spherical and Parallelopiped Shells--Expression for Temperature Distribution

Conduction through a spherical shell is also a one-dimensional steady state problem if the interior and exterior surface temperatures are uniform and constant. The Eq. (2.2) in one-dimensional spherical coordinates can be written as

$$\left(1/r^2\right)\frac{d}{dT}\left(r^2 dT/dr\right)=0 \text{, with boundary conditions,}$$

at $\quad r=r_1,\ T=T_1;\ \text{at}\ r=r_2,\ T=T_2$

or, $\quad \frac{d}{dr}\left(r^2 dT/dr\right)=0$

and upon integration, T = $-C_1/r + C_2$, where c_1 and c_2 are constants. substituting the boundary conditions,

$$C_1=\left(T_1-T_2\right)r_1r_2/\left(r_1-r_2\right),\ \text{and}\ C_2=T_1+\left(T_1-T_2\right)r_1r_2/r_1\left(r_1-r_2\right)$$

The temperature distribution m the spherical shell is given by

$$T=T_1-\left\{\frac{\left(T_1-T_2\right)r_1r_2}{\left(r_2-r_1\right)}\right\}\times\left\{\frac{\left(r-r_1\right)}{r\ r_1}\right\} \tag{2.6}$$

and the temperature distribution associated with radial conduction through a sphere is represented by a hyperbola. The rate of heat conduction is given by

$$\dot{Q}=4\pi k\left(T_1-T_2\right)r_1r_2/\left(r_2-r_1\right)=k\left(A_1A_2\right)^{1/2}\left(T_1-T_2\right)/\left(r_2-r_1\right) \tag{2.7}$$

where $A_1=4\pi_1^2$ and $A_2=4\pi r_2^2$

If A_l is approximately equal to A_2 i.e., when the shell is very thin,

$$\dot{Q}=kA\left(T_1-T_2\right)/\left(r_2-r_1\right);\ \text{and}\ \dot{Q}/A=\left(T_1-T_2\right)/\Delta r/k$$

which is an expression for a flat slab.

The above equation (2.7) can also be used as an approximation for parallelopiped shells which have a smaller inner cavity surrounded by a thick wall, such as a small furnace surrounded by a large thickness of insulating material, although the h eat flow especially in the corners, cannot be strictly considered one-dimensional. It has been suggested that for $(A_2/A_1) > 2$, the rate of heat flow can be approximated by the above equation by multiplying the geometric mean area $A_m = (A_1 A_2)^{1/2}$ by a correction factor 0.725.]

4.4 Composite Surfaces

There are many practical situations where different materials are placed m layers to form composite surfaces, such as the wall of a building, cylindrical pipes or spherical shells having different layers of insulation. Composite surfaces may involve any number of series and parallel thermal circuits.

4.5 Heat Transfer Rate through a Composite Wall

Let us consider a general case of a composite wall as shown m Fig. 1.5 There are 'n' layers of different materials of thicknesses L_1, L_2, etc and having thermal conductivities k_l, k_2, etc. On one side of the composite wall, there is a fluid A at temperature T_A and on the other side of the wall there is a fluid B at temperature T_B. The convective heat transfer coefficients on the two sides of the wall are h_A and h_B respectively. The system is analogous to a series of resistances as shown in the figure.

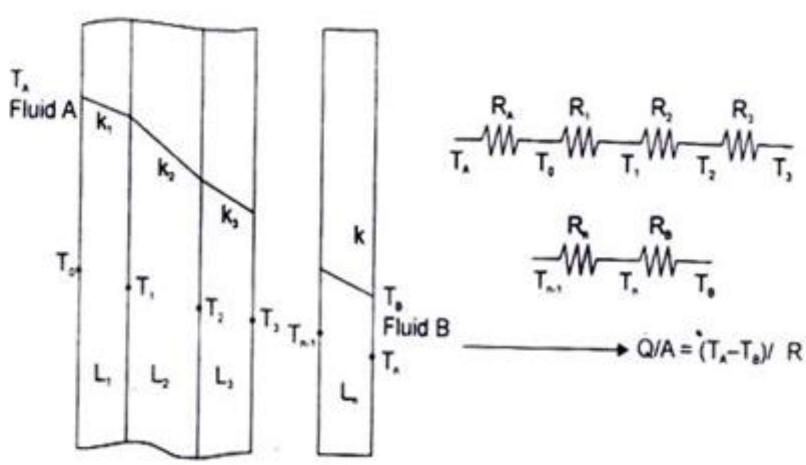


Fig 1.5 Heat transfer through a composite wall

4.6 The Equivalent Thermal Conductivity

The process of heat transfer through compos lie and plane walls can be more conveniently compared by introducing the concept of 'equivalent thermal conductivity', k_{eq}. It is defined as:

$$k_{eq} = \left(\sum_{i=1}^{n} L_i\right) \Big/ \sum_{i=1}^{n} (L_i / k_i) \qquad (2.8)$$

$$= \frac{\text{Total thickeness of the composite wall}}{\text{Total thermal resistance of the composite wall}}$$

And, its value depends on the thermal and physical properties and the thickness of each constituent of the composite structure.

Example 1.2 A furnace wall consists of 150 mm thick refractory brick (k = 1.6 W/mK) and 150 mm thick insulating fire brick (k = 0.3 W/mK) separated by an au gap (resistance 0 16 K/W). The outside walls covered with a 10 mm thick plaster (k = 0.14 W/mK). The temperature of hot gases is 1250°C and the room temperature is 25°C. The convective heat transfer coefficient for gas side and air side is 45 W/m2K and 20 W/m^2K. Calculate (i) the rate of heat flow per unit area of the wall surface (ii) the temperature at the outside and Inside surface of the wall and (iii) the rate of heat flow when the air gap is not there.

Solution: Using the nomenclature of Fig. 2.3, we have per m2 of the area, h_A = 45, and $R_A = 1/h_A = 1/45 = 0.0222$; h_B = 20, and R_B = 1120 = 0.05

Resistance of the refractory brick, $R_1 = L_1/k_1 = 0.15/1.6 = 0.0937$

Resistance of the insulating brick, $R_3 = L_3/k_3 = 0.15/0.30 = 0.50$

The resistance of the air gap, $R_2 = 0.16$

Resistance of the plaster, $R_4 = 0.01/0.14 = 0.0714$

Total resistance = 0.8973, m^2K/W

Heat flow rate = $\Delta T/\Sigma R = (1250\text{-}25)/0.8973 = 13662$ W/m^2

Temperature at the inner surface of the wall

$$= T_A - 1366.2 \times 0.0222 = 1222.25$$

Temperature at the outer surface of the wall

$$= T_B + 1366.2 \times 0.05 = 93.31\ °C$$

When the air gap is not there, the total resistance would be

$$0.8973 - 0.16 = 0.7373$$

and the heat flow rate = (1250 – 25)/0/7373 = 1661.46 W/m^2

The temperature at the inner surface of the wall

$$= 1250 – 1660.46 \times 0.0222 = 1213.12°C$$

i.e., when the au gap is not there, the heat flow rate increases but the temperature at the inner surface of the wall decreases.

The overall heat transfer coefficient U with and without the air gap is

$$U=\left(\dot{Q}/A\right)/\Delta T$$

$$= 13662 / (1250 – 25) = 1.115 \text{ Wm}^2 \text{ °C}$$

and 1661.46/l225 = 1356 W/m^{2o}C

The equivalent thermal conductivity of the system without the air gap

k_{eq} = (0.15 + 0.15 + 0.01)/(0.0937 + 0.50 + 0.0714) = 0.466 W/mK.

Example 1.2 A brick wall (10 cm thick, k = 0.7 W/m°C) has plaster on one side of the wall (thickness 4 cm, k = 0.48 W/m°C). What thickness of an insulating material (k = 0.065 W m^{o}C) should be added on the other side of the wall such that the heat loss through the wall IS reduced by 80 percent.

Solution: When the insulating material is not there, the resistances are:

$$R_1 = L_1/k_1 = 0.1/0.7 = 0.143$$

and $R_2 = 0.04/0.48 = 0.0833$

Total resistance = 0.2263

Let the thickness of the insulating material is L_3. The resistance would then be

$L_3/0.065 = 15.385\ L_3$

Since the heat loss is reduced by 80% after the insulation is added.

$$\frac{\dot{Q}\text{ with insulation}}{\dot{Q}\text{ without insulation}} = 0.2 = \frac{R\text{ without insulation}}{R\text{ with insulation}}$$

or, the resistance with insulation = 0.2263/0.2 = 01.1315

and, 15385 L_3 = I 1315 – 0.2263 = 0.9052

L_3 = 0.0588 m = 58.8 mm

Example 1.3 An ice chest IS constructed of styrofoam (k = 0.033 W/mK) having inside dimensions 25 by 40 by 100 cm. The wall thickness is 4 cm. The outside surface of the chest is exposed to air at 25°C with h = 10 W/m^{2}K. If the chest is completely filled with ice, calculate the time for ice to melt completely. The heat of fusion for water is 330 kJ/kg.

Solution: If the heat loss through the comers and edges are Ignored, we have three walls of walls through which conduction heat transfer Will occur.

(a) 2 walls each having dimensions 25 cm × 40 cm × 4 cm

(b) 2 walls each having dimensions 25 cm × 100 cm × 4 cm

(c) 2 walls each having dimensions 40 cm × 100 cm × 4 cm

The surface area for convection heat transfer (based on outside dimensions)

$2(33 \times 48 + 33 \times 108 + 48 \times 108) \times 10^{-4} = 2.0664\ m^2$.

Resistance due to conduction and convection can be written as

$$2\left(\frac{0.04}{0.033\times 0.25\times 0.4}+\frac{0.04}{0.033\times 0.25\times 1}+\frac{0.04}{0.033\times 0.4\times 1}\right)+\frac{1}{10\times 2.0664}$$

= 40 + 0.0484 = 40.0484 K/W

$\dot{Q} = \Delta T / \Sigma R$ = (25 – 0.0) / 40.0484 = 0.624 W

Inside volume of the container - 0.25 × 04 × 1 = 0.1 m^3

Mass of Ice stored = 800 × 0.1 = 80 kg; taking the density of Ice as 800 kg/m^3. The time required to melt 80 kg of ice is

$$t = \frac{80\times 330\times 1000}{0.624\times 3600\times 24} = 490\ \text{days}$$

Example1.4 A composite furnace wall is to be constructed with two layers of materials (k_1 =

2.5 W/m°C and k_2 = 0.25 W/m°C). The convective heat transfer coefficient at the inside and outside surfaces are expected to be 250 W/m^2°C and 50 W/m^2°C respectively. The temperature of gases and air are 1000 K and 300 K. If the interface temperature is 650 K, Calculate (i) the thickness of the two materials when the total thickness does not exceed 65 cm and (ii) the rate of heat flow. Neglect radiation.

Solution: Let the thickness of one material (k = 2.5 W / mK) is xm, then the thickness of the other material (k = 0.25 W/mK) will be (0.65 –x)m.

For steady state condition, we can write

$$\frac{\dot{Q}}{A} - \frac{1000-650}{\frac{1}{250}+\frac{x}{2.5}} = \frac{1000-300}{\frac{1}{250}+\frac{x}{2.5}+\frac{(0.65-x)}{0.25}+\frac{1}{50}}$$

$$\therefore\ 700(0.004+0.4x) = 350\{0.004+0.4x+4(0.65-x)+0.02\}$$

(i) 6x = 3.29 and x= 0.548 m.

and the thickness of the other material = 0.102 m.

(ii) $\dot{Q}/A$ = (350) / (0.004 + 0.4 × 0.548) = 1.568 kW/m^2

Example 1.5 A composite wall consists of three layers of thicknesses 300 rum, 200 mm and 100 mm with thermal conductivities 1.5, 3.5 and is W/mK respectively. The inside surface is exposed to gases at 1200°C with convection heat transfer coefficient as 30W/m^{2}K. The temperature of air on the other side of the wall is 30°C with convective heat transfer coefficient 10 Wm^{2}K. If the temperature at the outside surface of the wall is 180°C, calculate the temperature at other surface of the wall, the rate of heat transfer and the overall heat transfer coefficient.

Solution: The composite wall and its equivalent thermal circuits is shown in the figure.

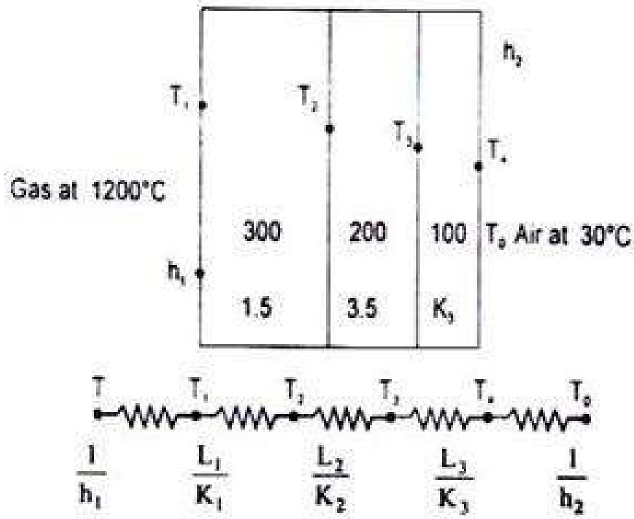


Fig 1.6

The heat energy will flow from hot gases to the cold air through the wall.

From the electric Circuit, we have

$\dot{Q}/A = h_2(T_4 - T_0) = 10\times(180-30) = 1500\,W/m^2$

also, $\dot{Q}/A = h_1(1200 - T_1)$

$T_1 = 1200 - 1500/30 = 1150^oC$

$\dot{Q}/A = (T_1 - T_2)/L_1/k_1$

$T_2 = T_1 - 1500\times 0.3/1.5 = 850$

Similarly, $\dot{Q}/A = (T_2 - T_3)/(L_2/k_2)$

$T_3 = T_2 - 1500\times 0.2/3.5 = 764.3^oC$

and $\dot{Q}/A = (T_3 - T_4)/(L_3/k_3)$

$L_3/k_3 = (764.3-180)/1500$ and $k_3 = 0.256$ W/mK

Check:

$\dot{Q}/A = (1200-30)/\Sigma R;$

where $\Sigma R = 1/h_1 + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_2$

$$\Sigma R = 1/30 + 0.3/1.5 + 0.2/3.5 + 0.1/0.256 + 1/10 = 0.75$$

and $\dot{Q}/A = 1170/0.78 = 1500\ W/m^2$

The overall heat transfer coefficient, $U = 1/\Sigma R = 1/0.78 = 1.282\ W/m^2K$

Since the gas temperature is very high, we should consider the effects of radiation also. Assuming the heat transfer coefficient due to radiation = 3.0 W/m^2K the electric circuit would be:

The combined resistance due to convection and radiation would be

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{1}{h_c}} + \frac{1}{\frac{1}{h_r}} = h_c + h_r = 60 W/m^{2o}C$$

$$\therefore\ \dot{Q}/A = 1500 = 60(T - T_1) = 60(1200 - T_1)$$

$$\therefore\ T_1 = 1200 - \frac{1500}{60} = 1175^oC$$

again, $\therefore\ \dot{Q}/A = (T_1 - T_2)/L_1/k_1 \Rightarrow T_2 = T_1 - 1500 \times 0.3/1.5 = 875^oC$

and $T_3 = T_2 - 1500 \times 0.2/3.5 = 789.3^oC$

$$L_3/k_3 = (789.3 - 180)/1500;\ \therefore k_3 = 0.246\ W/mK$$

$$\Sigma R = \frac{1}{60} + \frac{0.3}{1.5} + \frac{0.2}{1.5} + \frac{0.2}{3.5} + \frac{0.1}{0.246} + \frac{1}{10} = 0.78$$

and $U = 1/\Sigma R = 1.282\ W/m^2K$

Example 1.6 A flat roof (12 m x 20 m) of a building has a composite structure It consists of a 15 cm lime-khoa plaster covering (k = 0 17 W/m°C) over a 10 cm cement concrete (k = 0.92 W/m°C). The ambient temperature is 42°C. The outside and inside heat transfer coefficients are 30 W/m2°C and 10 W/m2 0C. The top surface of the roof absorbs 750 W/m2 of solar radiant energy. The temperature of the space may be assumed to be 260 K. Calculate the temperature of the top surface of the roof and the amount of water to be sprinkled uniformly over the roof surface such that the inside temperature is maintained at 18°C.

Solution: The physical system is shown in Fig. 1.7 and it is assumed we have one-dimensional flow, properties are constant and steady state conditions prevail.

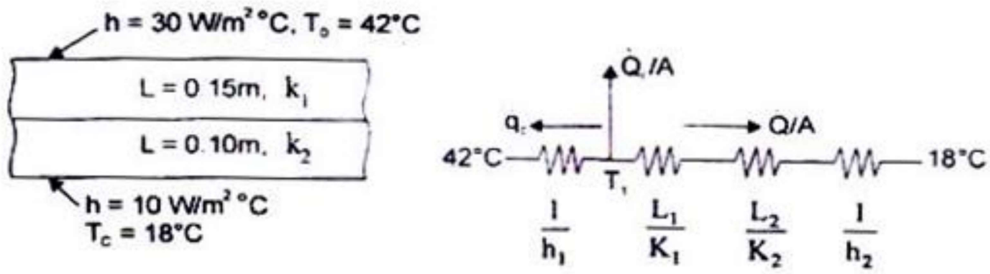


Fig 1.7

Let the temperature of the top surface be T_1°C.

Heat lost by thee top surface by convection to the surroundings is

$$\dot{Q}_c / A = h(\Delta T) = 30 \times (T_1 - 42) = (30T_1 - 1260)$$

Heat energy conducted inside through the roof = $(\Delta T / \Sigma R)$

or, $$\frac{\dot{Q}}{A} = \frac{T_1 - 18}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}} = (T_1 - 18) / \left(\frac{0.15}{0.17} + \frac{0.1}{0.92} + \frac{1}{10} \right) = 0.918\ (T_1 - 18)$$

Assuming that the top surface of the roof behaves like a black body, energy lost by radiation.

$$\dot{Q}_r / A = \sigma\left[(T_1 + 273)^4 - 260^4\right] \quad = 5.67 \times 10^{-8} (T_1 + 273)^4 - 259.1$$

By making an energy balance on the top surface of the roof,

Energy coming in = Energy going out

$750 = (30T, -1260) + 0.918\ (T_1 - 18) + 5.67 \times 10^{-8}\ (T_1 + 273)^4 - 259.1$

or, $2285.624 = 30.918\ T_1 + 5.67 \times 10^{-8}\ (T_1 + 273)^4$

Solving by trial and error, T_1 = 53.4°C, and the total energy conducted through the roof per hour is

0.918 (53.4 – 18) × (12 × 20) × 3600 = 28077.58 kJ/hr

Assuming the latent heat of vaporization of water as 2430 kJ/kg, the quantity of water to be sprinkled over the surface such that it evaporates and consumes 28077.58 kJ/hr, is

$\dot{M}_w$ = 28077.58/2430 = 11.55 kg/hr.

Example 1.7 An electric hot plate is maintained at a temperature of 350°C and is used to keep a solution boiling at 95°C. The solution is contained in a cast iron vessel (wall thickness 25 mm, k = 50 W/mK) which is enamelled inside (thickness 0.8 mm, k = 1.05 WmK) The heat transfer coefficient for the boiling solution is 5.5 kW/m1K. Calculate (i) the overall heat transfer coefficient and (ii) heat transfer rate.

If the base of the cast iron vessel is not perfectly flat and the resistance of the resulting air film is 35 m2K1kW, calculated the rate of heat transfer per unit area. (Gate'93)

Solution: The physical system is shown in the figure below.

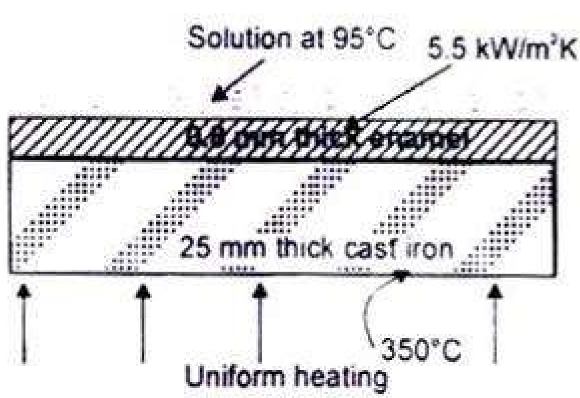


Fig 1.8

Under steady state conditions,

$$\dot{Q}/A = U(\Delta T) = \frac{(\Delta T)}{1/U}$$, where U is the overall heat transfer coefficient.

$$= \frac{(\Delta T)}{R} = \frac{(\Delta T)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h}}$$

Therefore,

$$1/U = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h} = \left(\frac{0.025}{50} + \frac{0.0008}{1.05} + \frac{1}{5500} \right) = 0.00144$$

$U = 692.65 \text{ W/m}^2\text{K}$

$\dot{Q}/A = U(\Delta T) = 692.65 \times (350 - 95) = 176.65 \text{ kW/m}^2$.

With the presence of air film at the base, the total resistance to heat flow would be:

$0.00144 + 0.035 = 0.03644 \text{ m}^2\text{K/W}$

and the rate of heat transfer, $\dot{Q}/A = 255/0.03644 = 7 \text{ kW/m}^2$.

(Fig. 1.9 shows a combination of thermal resistance placed in series and parallel for a composite wall having one-dimensional steady state heat transfer. By drawing analogous electric circuits, we can solve such complex problems having both parallel and series thermal resistances.)

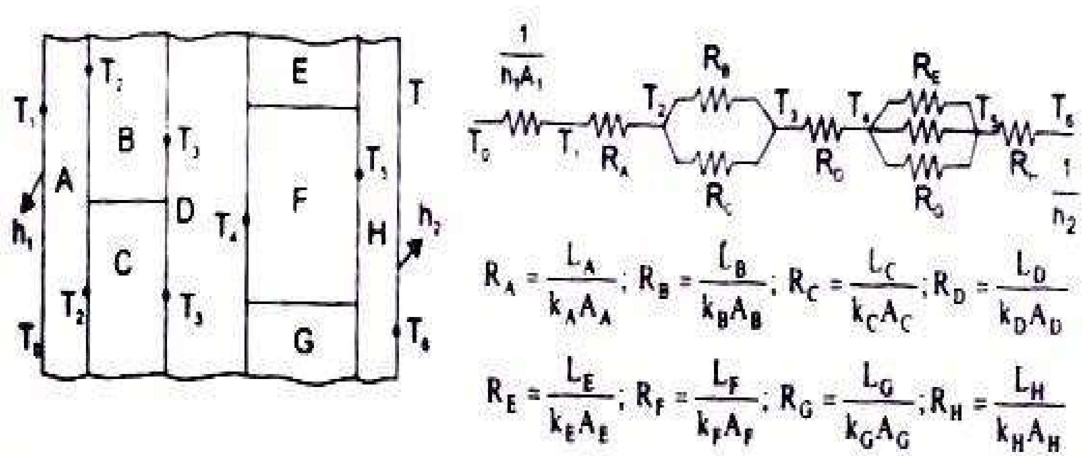


Fig. 1.9 Series and parallel one-dimensional heat transfer through a composite wall with convective heat transfer and its electrical analogous circuit

Example 1.8 A door (2 m x I m) is to be fabricated with 4 cm thick card board (k = 0.2 W /mK) placed between two sheets of fibre glass board (each having a thickness of 40 mm and k = 0.04 W/mK). The fibre glass boards are fastened with 50 steel studs (25 mm diameter, k = 40 W/mK). Estimate the percentage of heat transfer flow rate through the studs.

Solution: The thermal circuit with steel studs can be drawn as in Fig. 1.10.

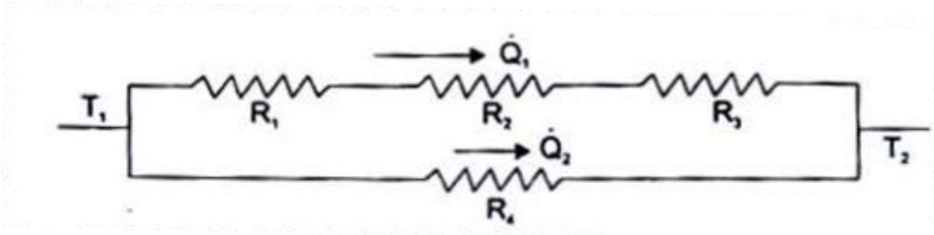


Fig 1.10

The cross-sectional area or the surface area of the door for the heat transfer is $2m^2$. The cross-sectional area of the steel studs is:

$50 \times \square/4\ (0.025)^2 = 0.02455\ m^2$

and the area of the door – area of the steel studs = 2.0 – 0.02455 = 1.97545

R_1, the resistance due to fibre glass board on the outside

= L/kA = 0.04/(0.04 × 1.97545) = 0.506.

R_2, the resistance due to card board = 0.101

R_3, the resistance due to fibre glass board on the inside = 0.506

R_4, the resistance due to steel studs = L/kA = 0.121 (40 × 0.2455) = 0 1222

With reference to Fig 2.9, $\dot{Q}_1 = (T_1 - T_2)/\Sigma R = (T_1 - T_2)/1.113$

and $\dot{Q}_2 = (T_1 - T_2)/0.1222$

Therefore, $\dot{Q}_2/(\dot{Q}_1 + \dot{Q}_2) = 8.1833/9.0818 = 0.9$

ie, 90 percent of the heat transfer will take place through the studs.

Example 1.9 Find the heat transfer rate per unit depth through the composite wall sketched. Assume one dimensional heat flow.

Solution: The analogous electric circuit has been drawn in the figure.

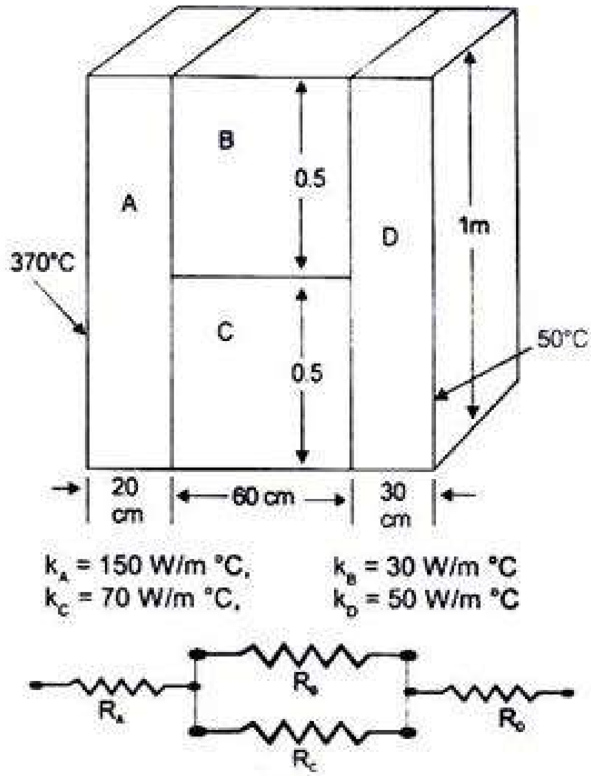


Fig 1.11

$R_A = 0.2/150 = 0.00133$

$R_B = 0.6/(30 \times 0.5) = 0.04$

$R_C = 0.6/(70 \times 0.5) = 0.017$

$R_D = 0.3/50 = 0.006$

$1/R_B + 1/R_C = 1/R_{BC} = 83.82$

Therefore, $R_{BC} = 1/83.82 = 0.0119$

Total resistance to heat flow = 0.00133 + 0.0119 + 0006 = 0.01923

Rate of heat transfer per unit depth = (370–50)/ 0.01923 = 16.64 kW m.

The Significance of Biot Number

Let us consider steady state conduction through a slab of thickness L and thermal conductivity k. The left hand face of the wall is maintained at T constant temperature T_1 and the right hand face is exposed to ambient air at T_o, with convective heat transfer coefficient h. The analogous electric circuit will have two thermal resistances: R_1 = L/k and R_2 = l/h. The drop in temperature across the wall and the air film will be proportional to their resistances, that is, (L/k)/(1/h) = hL/k.

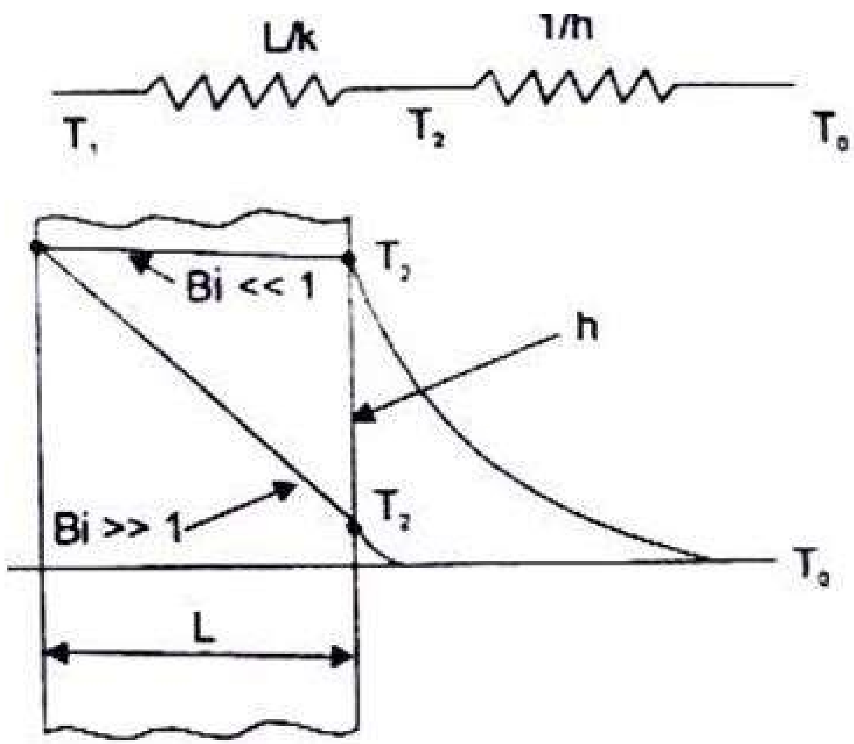


Fig 1.12: Effect of Biot number on temperature profile

This dimensionless number is called 'Biot Number' or,

$$B_i = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

When Bi >> 1, the temperature drop across the air film would be negligible and the temperature at the right hand face of the wall will be approximately equal to the ambient temperature. Similarly, when Bi «I, the temperature drop across the wall is negligible and the transfer of heat will be controlled by the air film resistance.

5. The Concept of Thermal Contact Resistance

Heat flow rate through composite walls are usually analysed on the assumptions that - (i) there is a perfect contact between adjacent layers, and (ii) the temperature at the interface of the two plane surfaces is the same. However, in real situations, this is not true. No surface, even a so-called 'mirror-finish surface', is perfectly smooth ill a microscopic sense. As such, when two surfaces are placed together, there is not a single plane of contact. The surfaces touch only at limited number of spots, the aggregate of which is only a small fraction of the area of the surface or 'contact area'. The remainder of the space between the surfaces may be filled with air or other fluid. In effect, this introduces a resistance to heat flow at the interface. This resistance IS called 'thermal contact resistance' and causes a temperature drop between the materials at the interfaces as shown In Fig. 2.12. (That is why, Eskimos make their houses having double ice walls separated by a thin layer of air, and in winter, two thin woolen blankets are more comfortable than one woolen blanket having double thickness.)

Fig. 2.12 Temperature profile with and without contact resistance when two solid surfaces are joined together

Example 1.10 A furnace wall consists of an inner layer of fire brick 25 cm thick k = 0.4 W/mK and a layer of ceramic blanket insulation, 10 cm thick k = 0.2 W/mK. The thermal contact resistance between the two walls at the interface is 0.01 m^2K/w. Calculate the temperature drop at the interface if the temperature difference across the wall is 1200K.

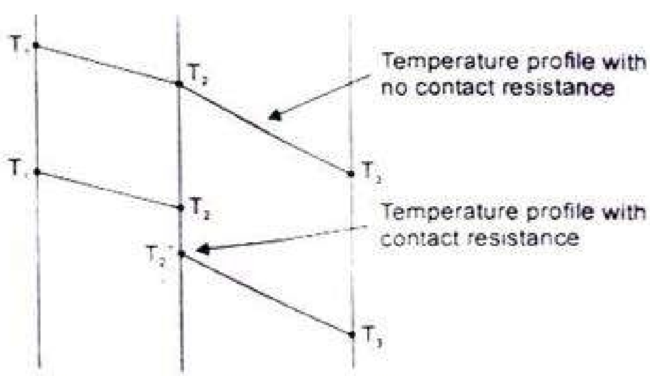


Fig 1.13: temperature profile with and without contact resistance when two solid surfaces are joined together

Solution: The resistance due to inner fire brick = L/k = 0.25/0.4 = 0.625.

The resistance of the ceramic insulation = 0.1/0.2 = 0.5

Total thermal resistance = 0.625 + 0.01 + 0.5 = 1 135

Rate of heat flow, $\dot{Q}/A$ = □t /□□ =1200/1 135 =1057.27 W/m^2

Temperature drop at the interface,

$$\Delta T = \left(\dot{Q}/A\right) \times R = 1057.27 \times 0.01 = 10.57 \text{ K}$$

Example 1.11 A 20 cm thick slab of aluminium (k = 230 W/mK) is placed in contact with a 15 cm thick stainless steel plate (k = 15 W/mK). Due to roughness, 40 percent of the area is in direct contact and the gap (0.0002 m) is filled with air (k = 0.032 W/mK). The difference in temperature between the two outside surfaces of the plate is 200°C Estimate (i) the heat flow rate, (ii) the contact resistance, and (iii) the drop in temperature at the interface.

Solution: Let us assume that out of 40% area m direct contact, half the surface area is occupied by steel and half is occupied by aluminium.

The physical system and its analogous electric circuits is shown in Fig. 2.13.

$$R_1 = \frac{0.2}{230 \times 1} = 0.00087, \qquad R_2 = \frac{0.0002}{230 \times 0.2} = 4.348 \times 10^{-6}$$

$$R_3 = \frac{0.0002}{0.032 \times 0.6} = 1.04 \times 10^{-2}, \qquad R_4 = \frac{0.0002}{15 \times 0.2} = 6.667 \times 10^{-5}$$

and $R_5 = \dfrac{0.15}{(15 \times 1)} = 0.01$

Again $1/R_{2,3,4} = 1/R_2 + 1/R_3 + 1/R_4$

$$= 2.3 \times 10^5 + 96.15 + 1.5 \times 10^4 = 24.5 \times 10^4$$

Therefore, $R_{2,3,4} = 4.08 \times 10^{-6}$

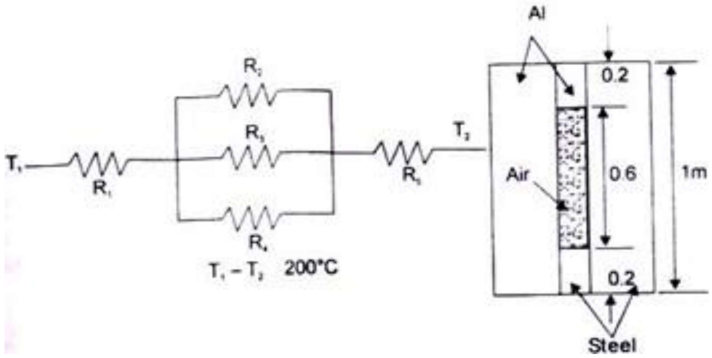


Fig 1.14

Total resistance, $\Sigma R = R_1 + R_{2,3,4} + R_5$

$= 870 \times 10^{-6} + 4.08 \times 10^{-6} + 1000 \times 10^{-6} = 1.0874 \times 10^{-2}$

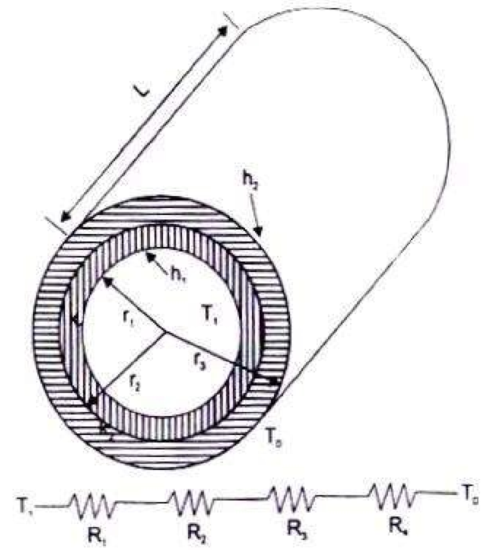
Heat flow rate, $\dot{Q}$ = 200/1.087 × 10^{-2} = 18.392 kW per unit depth of the plate.

Contact resistance, $R\,R_{2,\,3,\,4} = 4.08 \times 10^{-6}$ mK / W

Drop in temperature at the interface, □T = 4.08 × 10^{-6} × 18392 = 0.075°C

6. An Expression for the Heat Transfer Rate through a Composite Cylindrical System

Let us consider a composite cylindrical system consisting of two coaxial cylinders, radii r_1, r_2 and r_2 and r_3, thermal conductivities k_l and k_2 the convective heat transfer coefficients at the inside and outside surfaces h_1 and h_2 as shown in the figure. Assuming radial conduction under



steady state conditions we have:

Fig 1.15

$$R_1 = 1/h_1A_1 = 1/2\ \pi_1\ Lh_1$$

$$R_2 = \ln(r_2/r_1)2\pi Lk_1$$

$$R_3 = \ln(r_3/r_2)2\pi Lk_2$$

$$R_4 = 1/h_2A_2 = 1/2\pi_3 h_2 L$$

And $\dot{Q}/2\pi L = (T_1 - T_0)/\Sigma R$

$$= (T_1 - T_0)/\left[\left(1/h_1r_1 + \ln(r_2/r_1)/k_1 + \ln(r_3 + r_2)/k_2 + 1/h_2r_3\right)\right]$$

Example 1.12 A steel pipe. Inside diameter 100 mm, outside diameter 120 mm (k 50 W/mK) IS Insulated with a 40 mm thick high temperature Insulation (k = 0.09 W/mK) and another Insulation 60 mm thick (k = 0.07 W/mK). The ambient temperature IS 25°C. The heat transfer coefficient for the inside and outside surfaces are 550 and 15 W/m^{2}K respectively. The pipe carries steam at 300^{o}C. Calculate (1) the rate of heat loss by steam per unit length of the pipe (11) the temperature of the outside surface

Solution: I he cross-section of the pipe with two layers of insulation is shown 111 Fig. 1.16. with its analogous electrical circuit.

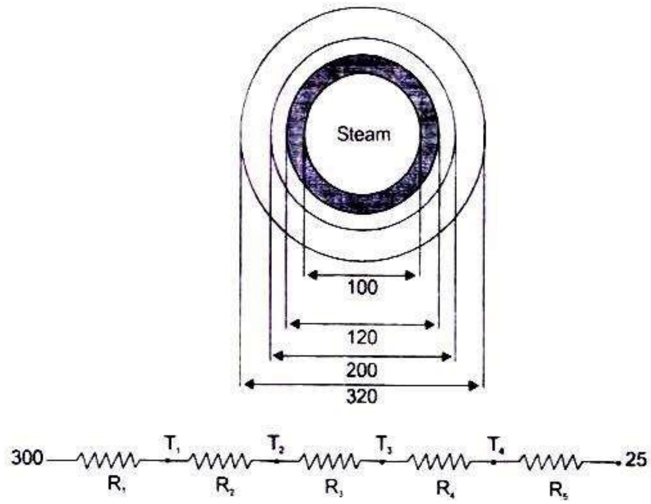


Fig1.16 Cross-section through an insulated cylinder, thermal resistances in series.

For L = 1.0 m. we have

R_1, the resistance of steam film = 1/hA = $1/(500 \times 2 \times 3.14 \times 50 \times 10^{-3}) = 0.00579$

R_2, the resistance of steel pipe = $\ln(r_2/r_1) / 2\,\pi\,k$

$= \ln(60/50)/2\,\pi \times 50 = 0.00058$

R_3, resistance of high temperature Insulation

$\ln(r_3/r_2) / 2\,\pi\,k = \ln(100/60) / 2\,\pi \times 0.09 = 0.903$

$R_4 = 1n(r_4/r_3)/2\,\pi\,k = \ln(160/100)/2\,\pi \times 0.07 = 1.068$

R_5 = resistance of the air film = $1/(15 \times 2\,\pi \times 160 \times 10^{-3}) = 0.0663$

The total resistance = 2.04367

and $\dot{Q} = \Delta T / \Sigma R$ = (300 – 25) / 204367 = 134.56 W per metre length of pipe.

Temperature at the outside surface. $T_4 = 25 + R_5$,

$\dot{Q} = 25 + 134.56 \times 0.0663 = 33.92^{o}$ C

When the better insulating material (k = 0.07, thickness 60 mm) is placed first on the steel pipe, the new value of R_3 would be

$R_3 = \ln(120/60) / 2\,\pi \times 0.07 = 1.576$; and the new value of R_4 will be

$R_4 = \ln(160/120)\ 2\,\pi \times 0.09 = 0.5087$

The total resistance = 2.15737 and Q = 275/2.15737 = 127.47 W per m length (Thus the better insulating material be applied first to reduce the heat loss.) The overall heat transfer coefficient, U, is obtained as U = $\dot{Q} / A \, \Delta T$

The outer surface area = $\pi \times 320 \times 10^{-3} \times 1 = 1.0054$

and U = 134.56/(275 × 1.0054) = 0.487 W/m^2 K.

Example 1.13 A steam pipe 120 mm outside diameter and 10m long carries steam at a pressure of 30 bar and 099 dry. Calculate the thickness of a lagging material (k = 0.99 W/mK) provided on the steam pipe such that the temperature at the outside surface of the insulated pipe does not exceed 32°C when the steam flow rate is 1 kg/s and the dryness fraction of steam at the exit is 0.975 and there is no pressure drop.

Solution: The latent heat of vaporization of steam at 30 bar = 1794 kJ/kg.

The loss of heat energy due to condensation of steam = 1794(0.99 – 0.975)

= 26.91 kJ/kg.

Since the steam flow rate is 1 kg/s, the loss of energy = 26.91 kW.

The saturation temperature of steam at 30 bar IS 233.84°C and assuming that the pipe material offers negligible resistance to heat flow, the temperature at the outside surface of the uninsulated steam pipe or at the inner surface of the lagging material is 233.84°C. Assuming one-dimensional radial heat flow through the lagging material, we have

$\dot{Q} = (T_1 - T_2)/[\ln(r_2/ r_1)]\ 2\ \pi\ Lk$

or, 26.91 × 1000 (W) = (233.84 – 32) × 2 π × 10 × 0.99/1n(r/60)

ln (r/60) = 0.4666

$r_2/60$ = exp (0.4666) = 1.5946

r_2 = 95.68 mm and the thickness = 35.68 mm

Example 1.14 A Wire, diameter 0.5 mm length 30 cm, is laid coaxially in a tube (inside diameter 1 cm, outside diameter 1.5 cm, k = 20 W/mK). The space between the wire and the inside wall of the tube behaves like a hollow tube and is filled with a gas. Calculate the thermal conductivity of the gas if the current flowing through the wire is 5 amps and voltage across the two ends is 4.5 V, temperature of the wire is 160°C, convective heat transfer coefficient at the outer surface of the tube is 12 W/m^2K and the ambient temperature is 300K.

Solution: Assuming steady state and one-dimensional radial heat flow, we can draw the thermal circuit as shown In Fig. 1 17.

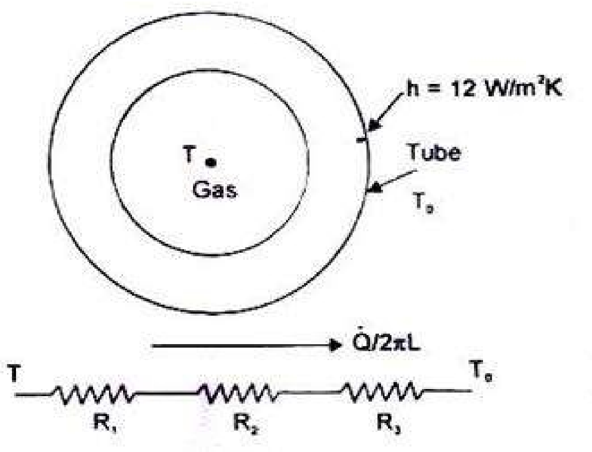


Fig 1.17

The rate of heat transfer through the system,

$\dot{Q}/2\pi L = VI/2\pi L = (4.5 \times 5)/(2 \times 3.142 \times 0.3) = 11.935$ (W/m)

R_1, the resistance due to gas = $\ln(r_2/r_l)$, k = ln(0.01/0.0005)/k = 2.996/k.

R_2, resistance offered by the metallic tube = $\ln(r_3 / r_2)$ k

= ln(1.5 /1.0) / 20 = 0.02

R_3, resistance due to fluid film at the outer surface

$l/hr_3 = 1/(l2 \times 1.5 \times I0^{-2}) = 5.556$

and $\dot{Q} / 2\pi L$ = □L/□□R = [(273 + 160) – 300]/□R

Therefore, □R = 133/11.935 = 11.l437, and

R_1 = 2.9996/k = 11.1437 – 0.02 – 5.556 = 5.568

or, k = 2.996/5.568 = 0.538 W/mK.

Example 1.15 A steam pipe (inner diameter 16 cm, outer diameter 20 cm, k = 50 W/mK) is covered with a 4 cm thick insulating material (k = 0.09 W/mK). In order to reduce the heat loss, the thickness of the insulation is Increased to 8mm. Calculate the percentage reduction in heat transfer assuming that the convective heat transfer coefficient at the Inside and outside surfaces are 1150 and 10 W/m^2K and their values remain the same.

Solution: Assuming one-dimensional radial conduction under steady state,

$$\dot{Q} / 2\square L = \square T/\square R$$

R_1, resistance due to steam film = 1/hr = 1/(1150 × 0.08) = 0.011

R_2, resistance due to pipe material = $\ln (r_2/r_1)/k = \ln (10/8)/50 = 0.00446$

R_3, resistance due to 4 cm thick insulation

$= \ln(r_3/r_2)/k = \ln(14/10)/0.09 = 3.738$

R_4, resistance due to air film = 1/hr = 1/(10 × 0.14) = 0.714.

Therefore, $\dot{Q}/2\pi L = \Delta T / (0.011 + 0.00446 + 3.738 + 0.714) = 0.2386\square T$

When the thickness of the insulation is increased to 8 cm, the values of R_3 and R_4 will change.

$R_3 = \ln(r_3/r_2)/k = \ln(18/10)/0.09 = 6.53$; and

$R_4 = 1/hr = 1/(10 \times 0.18) = 0.556$

Therefore, $\dot{Q}/2\pi L = \Delta T \,/\, (0.011 + 0.00446 + 6.53 + 0.556)$

$= \Delta T \,/\, 7.1 = 0.14084 \ \Delta T$

Percentage reduction in heat transfer = $\dfrac{(0.22386 - 0.14084)}{0.22386} = 0.37 = 37\%$

Example 1.16 A small hemispherical oven is built of an inner layer of insulating fire brick 125 mm thick (k = 0.31 W/mK) and an outer covering of 85% magnesia 40 mm thick (k = 0.05 W/mK). The inner surface of the oven is at 1073 K and the heat transfer coefficient for the outer surface is 10 W/m^2K, the room temperature is 20^oC.

Calculate the rate of heat loss through the hemisphere if the inside radius is 0.6 m.

Solution: The resistance of the fire brick

$$= (r_2 - r_1)/2\pi k r_1 r_2 = \frac{0.725 - 0.6}{2\pi \times 0.31 \times 0.6 \times 0.725} = 0.1478$$

The resistance of 85% magnesia

$$= (r_3 - r_2)/2\pi k r_2 r_3 = \frac{0.765 - 0.725}{2\pi \times 0.05 \times 0.725 \times 0.765} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.0272$$

Rate of heat flow, $\dot{Q} = \Delta T / \Sigma R = \frac{800 - 20}{0.1478 + 0.2295 + 0.272} = 1930 W$

Example 1.17 A cylindrical tank with hemispherical ends is used to store liquid oxygen at – 180°C. The diameter of the tank is 1.5 m and the total length is 8 m. The tank is covered with a 10 cm thick layer of insulation. Determine the thermal conductivity of the insulating material so that the boil off rate does not exceed 10 kg/hr. The latent heat of vapourization of liquid oxygen is 214 kJ/kg. Assume that the outer surface of insulation is at 27°C and the thermal resistance of the wall of the tank is negligible. (ES-94)

Solution: The maximum amount of heat energy that flows by conduction from outside to inside = Mass of liquid oxygen × Latent heat of vapourisation.

= 10 × 214 = 2140 kJ/hr = 2140 × 1000/3600 = 594.44 W

Length of the cylindrical part of the tank = 8 – 2r = 8 – 1.5 = 6.5m

since the thermal resistance of the wall does not offer any resistance to heat flow, the temperature at the inside surface of the insulation can be assumed as - 183°C whereas the temperature at the outside surface of the insulation is 27°C.

Heat energy coming in through the cylindrical part, $\dot{Q}_1 = \dfrac{\Delta T}{\dfrac{\ln(r_2/r_1)}{2\pi Lk}}$

or, $\dot{Q}_1 = \dfrac{(27+183)\times 2\pi\times 6.5k}{\ln(8.5/7.5)} = 68531.84\ k$

Heat energy coming in through the two hemispherical ends,

$\dot{Q}_2 = 2\times(\Delta T\times 2\pi k\ r_2 r_1)/(r_2 - r_1) = \dfrac{2\times 210\times 2\pi k\times 0.85\times 0.75}{0.10} = 16825.4\ k$

Therefore, $594.44 = (68531.84 + 16825.4)\ k$; or, $k = 6.96\times 10^{-3}$ W/mK.

Example 1.18 A spherical vessel, made out of2.5 em thick steel plate IS used to store 10m3 of a liquid at 200°C for a thermal storage system. To reduce the heat loss to the surroundings, a 10 cm thick layer of insulation (k = 0.07 W/rnK) is used. If the convective heat transfer coefficient at the outer surface is W/m^{2}K and the ambient temperature is 25°C, calculate the rate of heat loss neglecting the thermal resistance of the steel plate.

If the spherical vessel is replaced by a 2 m diameter cylindrical vessel with flat ends, calculate the thickness of insulation required for the same heat loss.

Solution: Volume of the spherical vessel = $10m^3 = \dfrac{4\pi r^3}{3}$ $\therefore\ r = 1.336$ m

Outer radius of the spherical vessle, $r_2 = 1.3364 + 0.025 = 1.361$ m

Outermost radius of the spherical vessel after the insulation = 1.461 m.

Since the thermal resistance of the steel plate is negligible, the temperature at the inside surface of the insulation is 200°C.

Thermal resistance of the insulating material = $(r_3 - r_2)/4\pi\ k\ r_3 r_2$

$$= \frac{0.1}{4\pi\times 0.07\times 1.461\times 1.361} = 0.057$$

Thermal resistance of the fluid film at the outermost surface = 1/hA

$$=1/\left[10\times 4\pi\times(1.461)^2\right]=0.00373$$

Rate of heat flow = $\Delta T/\Sigma R=(200-25)/(0.057+0.00373)=2873.8\ \text{W}$

Volume of the insulating material used = $(4/3)\pi\left(r_3^3-r_2^3\right)=2.5\ \text{m}^3$

Volume of the cylindrical vessel $=10\ \text{m}^3=\dfrac{\pi}{4}(d)^2\,L;\ \therefore L=10/\pi=3.183\,\text{m}$

Outer radius of cylinder without insulation = 1.0 + 0.025 = 1.025 m.

Outermost radius of the cylinder (with insulation) = r_3.

Therefore, the thickness of insulation = r_3 – 1.025 = □

Resistance, the heat flow by the cylindrical element

$$=\frac{\ln(r_3/1.025)}{2\pi Lk}+1/hA=\frac{\ln(r_3/1.025)}{2\pi\times 3.183\times 0.07}+\frac{1}{10\times 2\pi\times r_3\times 3.183}$$

= 0.714 ln (r_3 / 1.025) + 0.005/r_3

Resistance to heat flow through sides of the cylinder

$$=2\delta/kA+1/hA=\frac{2(r_3-1.025)}{0.07\times\pi\times 1}+\frac{1}{10\times 2\times\pi}$$

$$=9.09(r_3-1.025)+0.0159$$

For the same heat loss, $\Delta T/\Sigma R$ would be equal in both cases, therefore,

$$\frac{1}{0.06073}=\frac{1}{0.714\ \ln(r_3/1.025)+0.005/r_3}+\frac{1}{9.09(r_3-1.025)+0.0159}$$

Solving by trial and error, (r – 1.025j) = □ = 9.2 cm.

and the volume of the insulating material required = 2.692 m^3.