ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY
AUTONOMOUS INSTITUTION
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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VII Semester

AU3008 Sensors and Actuators

UNIT – I - INTRODUCTION TO MEASUREMENTS AND SENSORS

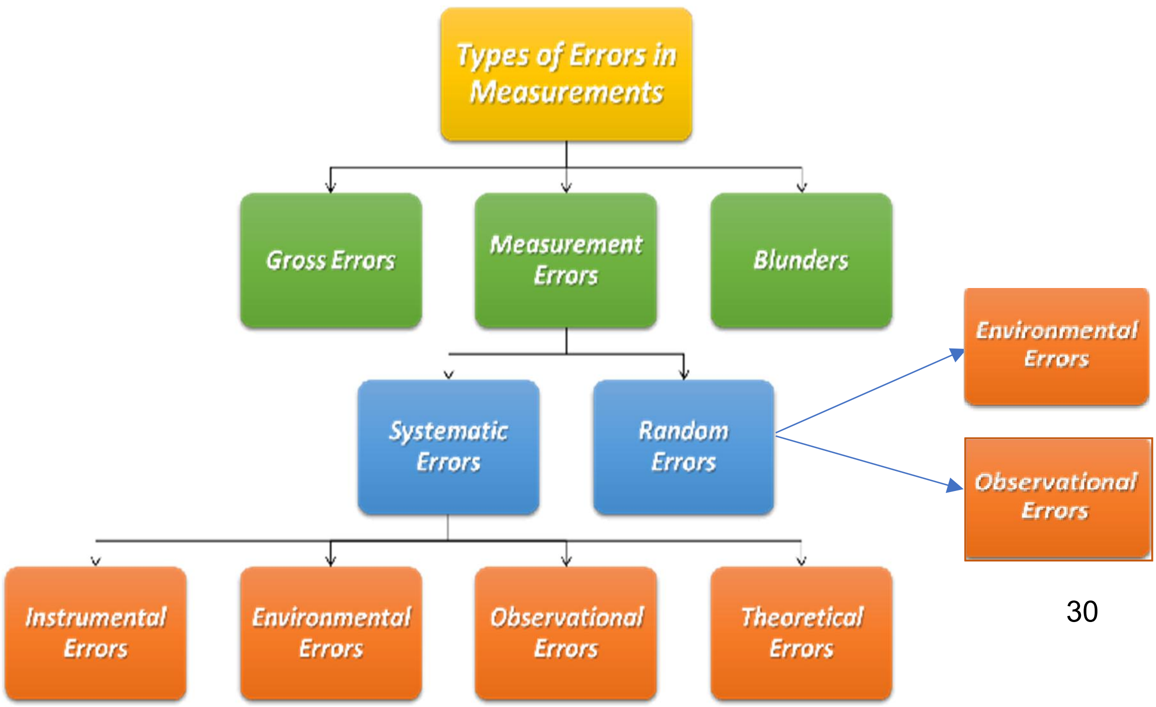
1.4 Classification of Errors- Error analysis

- Errors in measurement are unavoidable discrepancies between the measured value and the true value of a quantity.
- These errors can significantly *affect the accuracy* and *reliability* of experimental results.
- The *deviation of the measured quantity from the actual quantity* or true value is called **error**.

$$E = A_m - A_t$$

where E is the error, A_m is the measured quantity and A_t is the true value.

- **Classification of Errors**:



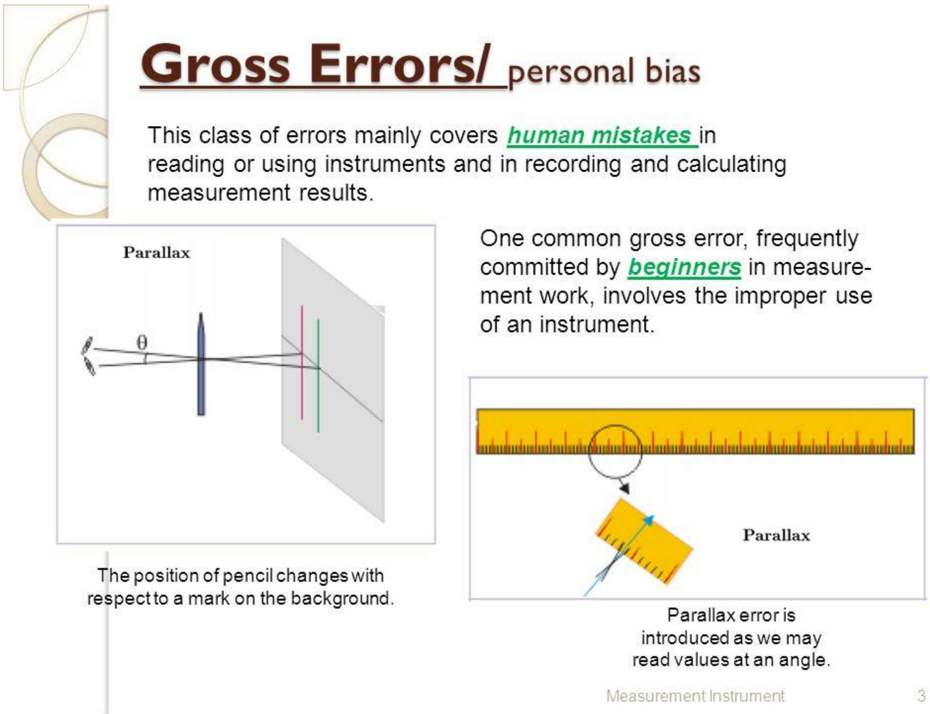
❒ **Gross Errors:**

❖ Human oversight and other mistakes while reading, recording, and readings.

❖ For example, the person taking the reading from the meter of the instrument may read 23 as 28

❖ Proper care should be taken in reading, recording the data. Also, the calculation of error should be done accurately.

❖ By increasing the number of experimenters, we can reduce the gross errors



❒ **Blunders:**

❖ A blunder is a significant, unpredictable mistake that is usually caused by *carelessness, ignorance, or stupidity*.

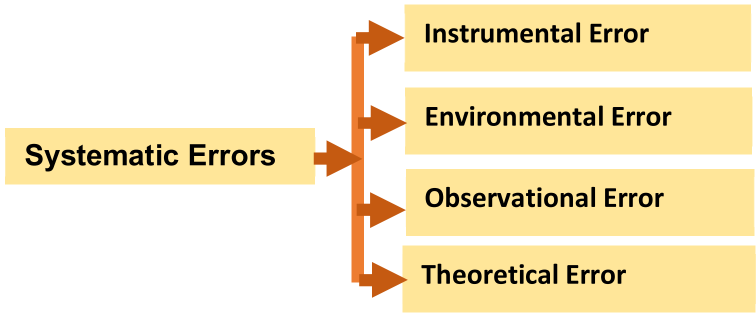
❖ Blunders can also be the result of miscommunication, fatigue, or poor judgment.

❖ Blunders are simply a clear mistake that causes an error in the experiment

❖ Example: such as dropping a beaker with the solution before measuring the final mass

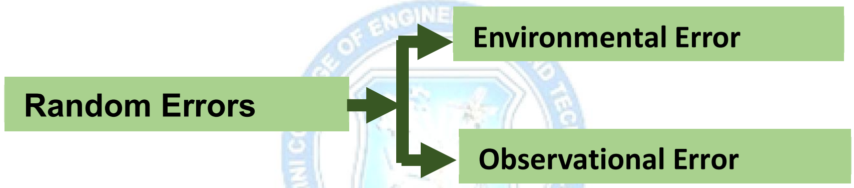
❒ **Systematic Error:**

❖ Systematic errors are errors that *have a clear cause* and can be eliminated for future experiments.



❒ **<u>Random Errors:</u>**

❖ Random errors *occur randomly*, and sometimes have *no source/cause.*



❒ **<u>Instrumental Error:</u>**

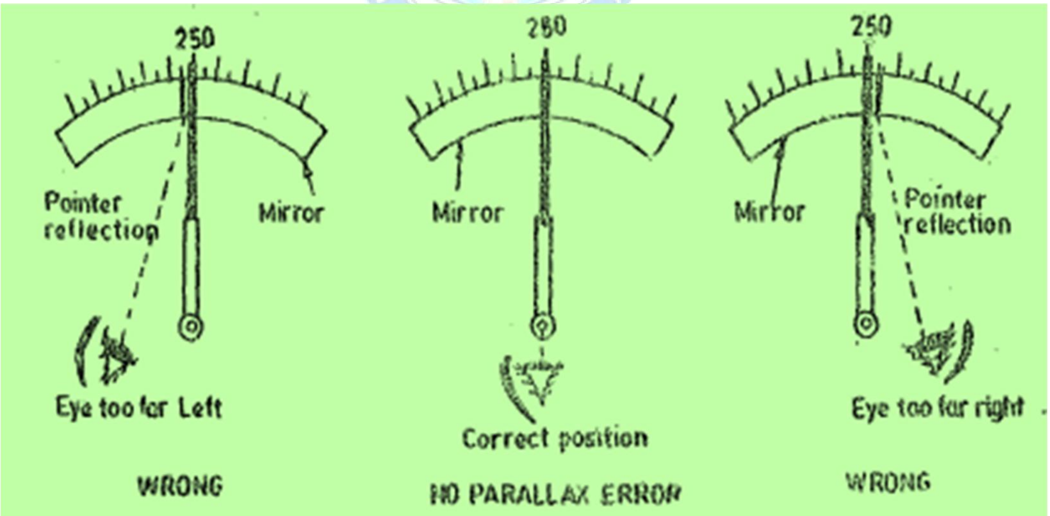
❖ Happen when an instrument gives the wrong reading.
❖ Most often, you can fix instrumental errors by recalibrating the instrument.

1. Due to inherent shortcoming in the instrument.
   <u>Example</u>:- If the spring used in permanent magnet instrument has become weak then instrument will always read high. Errors may cause because of friction, hysteresis, or even gear backlash.
2. Due to misuse of the instruments.
   For <u>example</u>, these may be caused by failure to adjust zero of the instruments.
3. Due to Loading effects of instruments.

❒ **<u>Environmental Error:</u>**

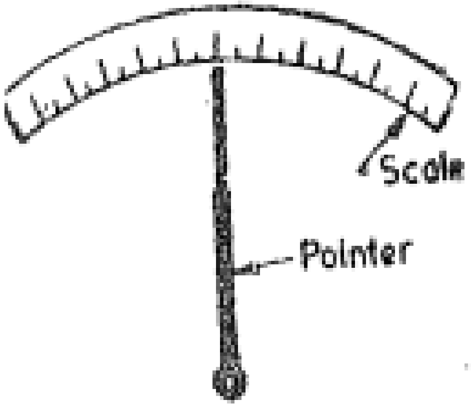
- These errors are due to conditions external to the measuring device including conditions in the area surrounding the instrument.
- These may be effects of temperature, pressure, humidity, dust, vibrations or of external magnetic or electrostatic field.
- These errors can be eliminated or reduced by using corrective measure such as:
  - Keep the condition as constant as possible.
  - Use instrument/equipment which is immune to these effects.
  - Employ technique which eliminates these disturbances.

- **<u>Observational Error</u>:**
  - As an example, the pointer of a voltmeter rests slightly above the surface of the scale.
  - Thus an error on account of PARALLAX will be incurred unless the line of vision of the observer is exactly above the pointer.
  - To minimize parallax errors, highly accurate meters are provided with mirrored scales



- Since the parallax errors arise on account of pointer and the scale not being in the same plane, we can eliminate this error by having the pointer and the scale in the same plane.
- No two persons observe the same situation in exactly the same way where small details are concerned,

- Example: **sound and light measurements**



Arrangement showing scale and pointer in the same plane

- **Theoretical Error:**

  - This type of error arises when the assumptions or simplifications made in a theoretical model do not perfectly reflect the real-world situation.

    Causes of Theoretical Error:

    - ✓ Simplifying Assumptions
    - ✓ Incomplete Theories
    - ✓ Numerical Approximations
    - ✓ Incorrect Parameters

Random Errors | **Environmental Error**

Changes in temperature, humidity, or other factors can affect the accuracy of measurements

Observational Error

Small mistakes made by observers, such as misreading instruments or recording data incorrectly.

1.4.2 Error Analysis:

Error analysis is a systematic way to identify and understand errors in processes, systems, or devices. It's used to improve quality, reduce costs, and prevent errors.

The experimental data is obtained ie. two forms of tests:

(i) Multisample test and

(ii) Single-sample test.

Multisample Test:

In this test, *repeated measurement* of a given quantity are done using different test conditions such as employing *different instruments*, *different ways* of measurement and by employing *different observers*.

Single Sample Test:

A single measurement (or succession of measurements) done under *identical conditions* excepting for time is known as single-sample test.

1. Histogram

- ❒ When a number of multisample observations are taken experimentally there is a scatter of the data about some central value. One method presenting test results in the form of a Histogram

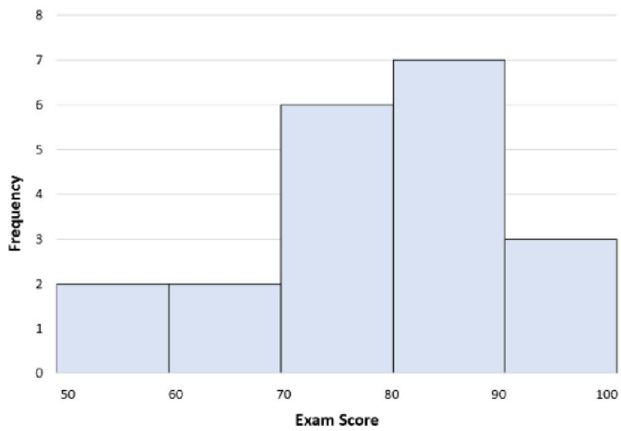
Example Make a Histogram from a Frequency Table:

Suppose we collect the following data that shows the exam scores of 20 students in some class:

Scores: 50, 58, 62, 65, 70, 71, 72, 74, 74, 78, 81, 82, 82, 85, 87, 88, 89, 92, 94, 96

We can create the following frequency table using a bin range of 10 to summarize the frequency of each range of scores:

Score	Frequency
50-59	2
60-69	2
70-79	6
80-89	7
90-99	3



The x-axis of the histogram displays bins of data values and the y-axis tells us how many observations in a dataset fall in each bin.

2. Arithmetic Mean:

- The most probable value of measured variable (variate) is the arithmetic mean of the number of readings taken.
- The best approximation is made when the number of readings of the same quantity are very large.
- Theoretically, an infinite number of readings would give the best result, although in practice, only a finite number of measurements can be made.

The arithmetic mean is given by,

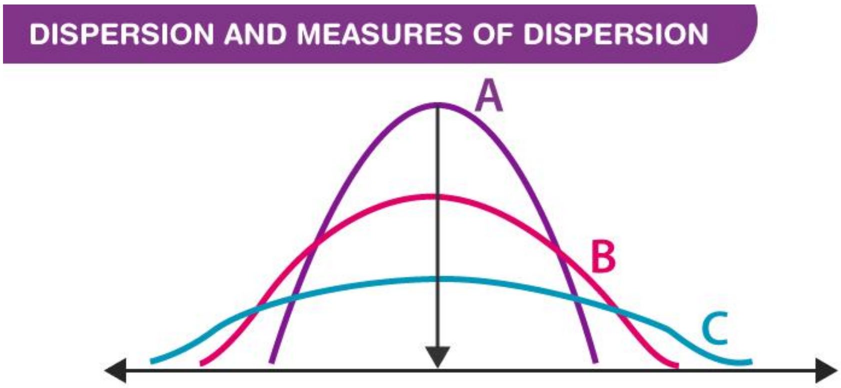
$$\bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + \dots\dots\dots + x_n}{n} = \frac{\sum x}{n}$$

$\bar{X}$ - arithmetic mean

x1, x2 ... Xn - readings or variates or samples

N - number of readings

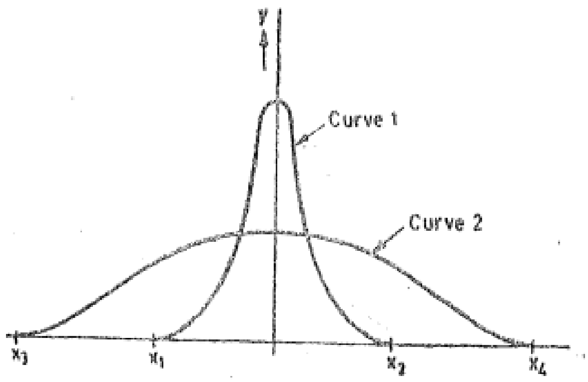
3. Measure of Dispersion from the Mean



- ❖ The property which denotes the extent to which the values are dispersed about the central value is termed as dispersion. The other names used for dispersion are spread of scatter.

4. Range:

- ❖ The simplest possible measure of dispersion is the range which is the difference between greatest and least values of data.
- ❖ For example, in Figure, the range of curve 1 is (x2- x1) and that of curve 2 is (X4 – X3).



5. Deviation:

- ❐ Deviation is departure of the observed reading from the arithmetic mean of the group of readings.

- Let the deviation of reading X_1 be d_1 and that of reading X_2 be d_2, etc.

$$d_1 = x_1 - \bar{X}$$
$$d_2 = x_2 - \bar{X}$$
$$\cdots\cdots\cdots\cdots$$
$$d_n = x_n - \bar{X}$$

$$\bar{X} = \frac{\Sigma(x_n - d_n)}{n}$$

- Algebraic sum of the deviation = d1 + d2 + d3 + ……dn
- Therefore, the algebraic, sum of deviations is zero.

6. Average Deviation:

- The average deviation is an indication of the precision of the instruments used in making the measurements. Highly precise instruments yield a low average deviation between readings.
- Average deviation is defined as the *sum of the absolute values of deviations divided by the number of readings*. The absolute value of deviation is the value without respect to its sign.
- $$D = \frac{|d_1| + |d_2| + |d_3| + \ldots + |d_n|}{n} = \frac{\Sigma |d|}{n}$$

7. Standard Deviation

- A standard deviation (or σ) is a measure of how dispersed the data is in relation to the mean.
- Defined as the square root of the sum of the individual deviations squared, divided by the number of readings.

$$S.D=\sigma=\sqrt{\frac{d_1^2+d_2^2+d_3^2+...+d_4^2}{n}}=\sqrt{\frac{\sum d^2}{n}}(>20observation)$$

$$S.D=s=\sqrt{\frac{d_1^2+d_2^2+d_3^2+...+d_4^2}{n-1}}=\sqrt{\frac{\sum d^2}{n-1}}(<20observation)$$

8. Variance:

- The variance is the *mean square deviation*, which is the same as S.D., except that square root is not extracted.

$$V=(\text{Standard Deviation})^2$$

$$=(S.D.)^2=\sigma^2=\frac{d_1^2+d_2^2+d_3^2+...+d_n^2}{n}$$

$$=\frac{\Sigma d^2}{n}$$

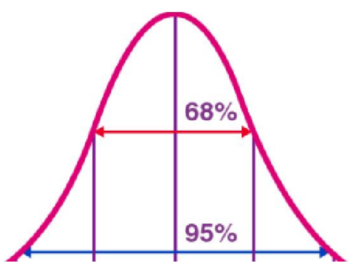
But when the number of observations is less than 20,

$$V=s^2=\Sigma\ \frac{d^2}{n-1}$$

9. Normal Distribution:

What Is a Normal Distribution?

- Normal distribution, also known as the Gaussian distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.
- The normal distribution appears as a "bell curve" when graphed.

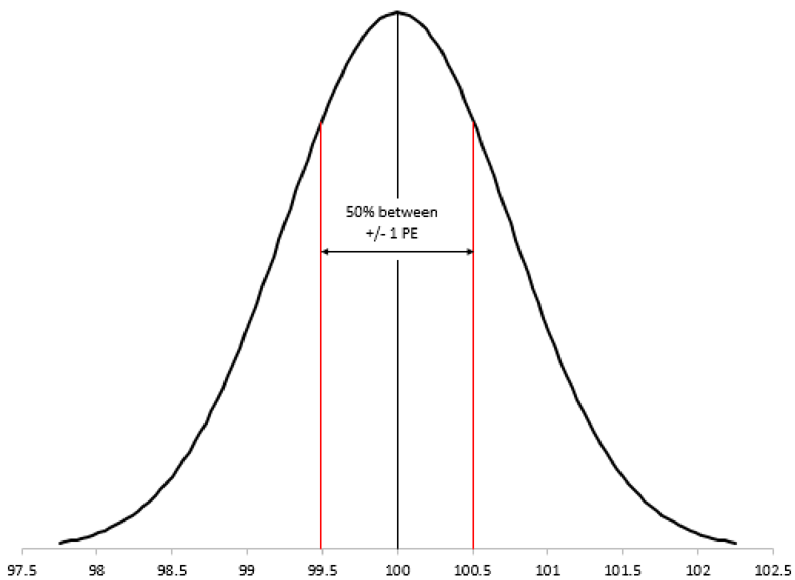


Using 1 standard deviation, the Empirical Rule states that,

- Approximately 68% of the data falls within one standard deviation of the mean. (i.e., Between Mean- one Standard Deviation and Mean + one standard deviation)
- Approximately 95% of the data falls within two standard deviations of the mean. (i.e., Between Mean- two Standard Deviation and Mean + two standard deviations)
- Approximately 99.7% of the data fall within three standard deviations of the mean. (i.e., Between Mean- three Standard Deviation and Mean + three standard deviations)

10. Probable Error:

- Probable error is a statistical term that describes the half-range of an interval around a central point in a distribution
- What is special about the value of PE? The PE defines the middle 50% of the normal distribution as shown in Figure.



Problem 1:
A set of independent current measurements were taken by six observers and were recorded as 12.8 A, 12.2 A, 12.5 A, 13.1 A. 12.9 A, and 12.4 A. Calculate (a) the arithmetic mean,
(b) the deviations from the mean,
(c) the average deviation,
(d) the· standard deviation, and
(e) variance.

Solution:

(a). Arithmetic Mean = 12. 65 A.

(b). The deviations from the mean = d1 =x1-X=12·8-12·65 =+0·15 A
d2 =x2-X=I2.2-12.65 = -0.45 A
d3=x3-X=12.5-12.65= -0.15 A
d4=X4-X=13.1-12.65 =. +0·45 A
d5=x5-X=12.9-12.65= +0·25 A
d6=X6-X=12.4 -12.65= -0.25 A

(c) the average deviation D =0. 283 A

(d) the· standard deviation s = =0.399 A.

(e) Variance = **-0·115** A^2**.**

Problem: 2

In an experiment, ten observations of pressure are made which are given below:

Trial no.	1	2	3	4	5
Scale reading (K P_a)	10.02	10.20	10.26	10.20	10.22
Trial no.	6	7	8	9	10
Scale reading (K P_a)	10.13	9.97	10.12	10.09	9.9

Calculate (i) arithmetic mean (ii) average deviation (iii) standard deviation and (iv) variance.

Solution:

(i) Arithmetic mean

$$\bar{x} = \frac{10.02+10.02+10.26+10.20+10.22+10.13+9.97+10.12+10.09+9.99}{10}$$

$$= 10.111\ KP_a$$

(ii) Average deviation absolute

$$d = |d_1| + |d_2| + \ldots + |d_n|$$

$$d = \frac{0.091+0.089+0.149+0.089+0.109+0.019+0.141+0.009+0.021+0.211}{10}$$

$$= 0.0928\ KP_a$$

(iii) $[\text{Standard deviation}]^2 = \sigma^2$

$$= \frac{10^{-3}[8.281+7.921+22201+7.921+11.881+0.361+19.881+0.081+0.441+44.521]}{9}$$

$$= -0.12349 / 9 = 0.0137$$

$$\sigma = \text{standard deviation} = 0.117\ KP_a$$

(iv) Variance $V = \sigma^2 = 0.0137$

11. Median:

Median is also used to indicate the most probable value of the measured quantity when a set of readings are taken. When the readings are arranged in ascending or descending order of magnitude, the middle value of the set is taken as the median.

For example, the temperature of a bath is noted by eleven observers as follows:

66.5°C, 63.8°C, 65.7°C, 66.1°C, 64.8°C, 67.0°C, 65.3°C, 63.9°C, 64.4°C, 65.9°C, 66.5°C

It is rearranged in ascending order as follows:

63.8°C, 63.9°C, 64.4°C, 64.8°C, 65.3°C, 65.7°C, 65.9°C, 66.1 °C, 66.5°C, 66.5°C, 67.0°C

Now the median is the sixth reading, i.e. **65.7°C.**

12. Mode:

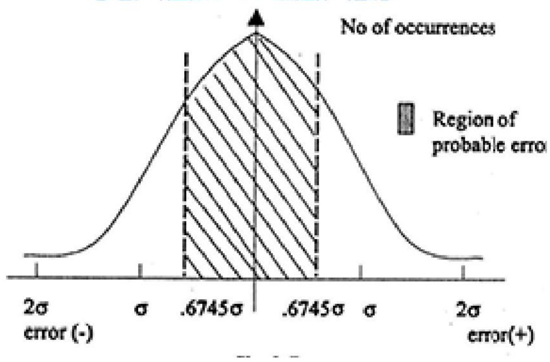
- Mode is the value which occurs most frequently in a set of observations and around which other items of the set cluster densely.
- For example, the frequency distributions of a set of 101 observations is given as follows

Pressure readings KP_a	50	51	52	53	54	55	56	57
No. of readings	4	9	16	25	22	15	7	3

- The value of pressure reading corresponding to maximum number of occurences is 53 KPa. hence mode is 53 KPa.

13. <u>Probable error:</u>

- It is quite often useful to specify the probable error in a measurement due to random error.
- If the central value of a Gaussian curve is assumed as the true value, then error Vs number of occurences in the measurement can be plotted as shown in Fig.



- For a Gaussian distribution of data, it is found that about 68% of the total number of observations have errors lying within ± σ. It is also found that about 50% of the total number of observations have errors lying within ± 0.6745 σ and this is taken as the probable value of error because there is an even chance for any one observation to have a random error more than this value. Hence probable error, r = ± 0.6745 σ .

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