## ERGODIC PROCESS:

In the event that the distributions and statistics are not available we can avail ourselves of the time averages from the particular sample function. The mean of the sample function Xλo(t) is referred to as the sample mean of the process X(t) and is defined as

$$\langle \mu(X)T \rangle = (\frac{1}{T}) \int_{-T/2}^{T/2} X\lambda o(t)dt$$

This quantity is actually a random-variable by itself because its value depends on the parameter sample function over it was calculated. the sample variance of the random process is defined as

$$\langle \sigma^2(X)T \rangle = (\frac{1}{T}) \int_{-T/2}^{T/2} \left| |X\lambda o(t)| - \langle \mu x \rangle T \right|^2 dt$$

The time-averaged sample ACF is obtained via the relation is

$$\langle RXX \rangle T = (\frac{1}{T}) \int_{-T/2}^{T/2} x(t) * x(t-T)dt$$

These quantities are in general not the same as the ensemble averages described before. A random process X(t) is said to be ergodic in the mean, i.e., first-order ergodic if the mean of sample average asymptotically approaches the ensemble mean

$$\lim_{T\to\infty} E\{\langle \mu X \rangle T\} = \mu x(t)$$

$$\lim_{T\to\infty} var\{\langle \mu X \rangle T\} = 0$$

In a similar sense a random process X(t) is said to be ergodic in the ACF, i.e, second-order ergodic if

$$\lim_{T\to\infty} E\{\langle RXX(\tau)\rangle\} = RXX(\tau)$$

$$\lim_{T\to\infty} var\{\langle RXX(\tau)\rangle\} = 0$$

The concept of ergodicity is also significant from a measurement perspective because in Practical situations we do not have access to all the sample realizations of a random process. We therefore have to be content in these situations with the time-averages that we obtain from a single realization. Ergodic processes are signals for which measurements based on a single sample function are sufficient to determine the ensemble statistics. Random signal for which this property does not hold are referred to as non-ergodic processes. As before the Gaussian random signal is an exception where strict sense ergodicity implies wide sense ergodicity.

**GUASSIAN PROCESSS:**

A random process X(t) is a Gaussian process if for all n and all (t1 ,t2 ,…,tn ), the random variables have a jointly Gaussian density function. For Gaussian processes, knowledge of the mean and autocorrelation; i.e., mX (t) and Rx (t1 ,t2 ) gives a complete statistical description of the process. If the Gaussian process X(t) is passed through an LTI system, then the output process Y(t) will also be a Gaussian process. For Gaussian processes, WSS and strict stationary are equivalent.

A Gaussian process is a stochastic process Xt, t ∈ T, for which any finite linear combination of samples has a joint Gaussian distribution. More accurately, any linear functional applied to the sample functionXt will give a normally distributed result. Notation-wise, one can write X ~ GP(m,K), meaning the random function X is distributed as a GP with mean function m and covariance function K.[1] When the input vector t is two- or multi-dimensional a Gaussian process might be also known as a Gaussian random field.

A sufficient condition for the ergodicity of the stationary zero-mean Gaussian process X(t) is that

$$\int_{-\infty}^{\infty} RX(\tau)d\tau < \infty.$$

**Jointly Gaussian processes:**

The random processes X(t) and Y(t) are jointly Gaussian if for all n, m and all (t1 ,t2 ,…,tn ), and (τ1 , τ2 ,…, τm ), the random vector (X(t1 ),X(t2 ),…,X(tn ), Y(τ1 ),Y( τ2 ),…, Y(τm )) is distributed according to an n+M dimensional jointly Gaussian distribution.

For jointly Gaussian processes, uncorrelatedness and independence are equivalent.

**Linear Filtering Of Random Processes:**

A random process X(t) is applied as input to a linear time-invariant filter of impulse response h(t),

• It produces a random process Y (t) at the filter output as

$$X(t) \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow h(t) \rightarrow\rightarrow\rightarrow Y(t)$$

• Difficult to describe the probability distribution of the output random process Y (t), even when the probability distribution of the input random process X(t) is completely specified for $-\infty \leq t \leq +\infty$.

- Estimate characteristics like mean and autocorrelation of the output and try to analyse its behaviour.

- Mean The input to the above system X(t) is assumed stationary. The mean of the output random process Y (t) can be calculated

$$
\begin{aligned}
mY\,(t) = E[Y\,(t)] &= E[\textstyle\int_{-\infty}^{\infty} h(\tau)X(t-\tau)\,d\tau] \\
&= \textstyle\int_{-\infty}^{\infty} h(\tau)E[X(t-\tau)]\,d\tau \\
&= mX \textstyle\int_{-\infty}^{\infty} h(\tau)\,d\tau \\
&= mXH(0)
\end{aligned}
$$

where H(0) is the zero frequency response of the system.

**Autocorrelation:**

The autocorrelation function of the output random process Y (t). By definition, we have

$$RY\,(t, u) = E[Y\,(t)Y\,(u)]$$

where t and u denote the time instants at which the process is observed. We may therefore use the convolution integral to write

$$
\begin{aligned}
RY\,(t, u) &= E\,[\textstyle\int_{-\infty}^{\infty} h(\tau 1)X(t-\tau 1)\,d\tau 1]\ \textstyle\int_{\infty}^{\infty} h(\tau 2)X(t-\tau 2)\,d\tau 2\,] \\
&= \textstyle\int_{-\infty}^{\infty} h(\tau 1)\,d\tau 1 \textstyle\int_{-\infty}^{\infty} h(\tau 2)E\,[X(t-\tau 1)X(t-\tau 2)]\,d\tau 2
\end{aligned}
$$

When the input X(t) is a wide-stationary random process, autocorrelation function of X(t) is only a function of the difference between the observation times $t - \tau 1$ and $u - \tau 2$.

Putting $\tau = t - u$, we get

$$RY(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau 1)h(\tau 2)RX(\tau - \tau 1 + \tau 2)\, d\tau 1\, d\tau 2$$

$$RY(0) = E[Y^2(t)]$$

The mean square value of the output random process Y (t) is obtained by putting τ = 0 in the above equation.

$$E[Y^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau 1)h(\tau 2)RX(\tau 2 - \tau 1)\, d\tau 1\, d\tau 2$$

The mean square value of the output of a stable linear time-invariant filter in response to a wide-sense stationary random process is equal to the integral over all frequencies.

of the power spectral density of the input random process multiplied by the squared magnitude of the transfer function of the filter.

**APPLICATION AND ITS USES:**

• A Gaussian process can be used as a prior probability distribution over functions in Bayesian inference.

• Wiener process (aka Brownian motion) is the integral of a white noise Gaussian process. It is not stationary, but it has stationary increments.