

SOLVED PROBLEMS ON STEAM PROPERTIES

1. A vessel of volume 0.04 m^3 contains a mixture of saturated water and steam at a temperature of 250°C . The mass of the liquid present is 9 kg . Find the pressure, mass, specific volume, enthalpy, entropy and internal energy. [April/May 2012,2015]

Given Data:

Volume, $V = 0.04 \text{ m}^3$

Temperature, $T = 250^\circ\text{C}$

Mass, $m = 9 \text{ kg}$

To find:

1) p , 2) m , 3) v , 4) h , 5) S , 6) ΔU

Solution:

From the Steam tables corresponding to 250°C , $v_f = v_1 = 0.001251 \text{ m}^3/\text{kg}$

$v_g = v_s = 0.050037 \text{ m}^3/\text{kg}$ $p = 39.776 \text{ bar}$

Total volume occupied by the liquid,

$$V_1 = m_1 \times v_1$$

$$= 9 \times 0.001251$$

$$= 0.0113 \text{ m}^3.$$

Total volume of the vessel,

$$V = \text{Volume of liquid} + \text{Volume of steam}$$

$$= V_1 + V_s$$

$$0.4 = 0.0113 + V_s$$

$$V_s = 0.0287 \text{ m}^3.$$

$$\begin{aligned}\text{Mass of steam, } m_s &= V_s / v_s \\ &= 0.0287 / 0.050037 \\ &= 0.574 \text{ kg.}\end{aligned}$$

$$\begin{aligned}\text{Mass of mixture of liquid and steam, } m &= m_l + m_s \\ &= 9 + 0.574 \\ &= 9.574 \text{ kg.}\end{aligned}$$

Total specific volume of the mixture,

$$\begin{aligned}v &= 0.04 / 9.574 \\ &= 0.00418 \text{ m}^3 / \text{kg.}\end{aligned}$$

We know that,

$$\begin{aligned}v &= v_f + x v_{fg} \\ 0.00418 &= 0.001251 + x (0.050037 - 0.001251) \\ x &= 0.06\end{aligned}$$

From Steam table corresponding to 250 °C,

$$\begin{aligned}h_f &= 1085.8 \text{ KJ / kg} \\ h_{fg} &= 1714.6 \text{ KJ / kg} \\ s_f &= 2.794 \text{ KJ / kg K} \\ s_{fg} &= 3.277 \text{ KJ / kg K.}\end{aligned}$$

Enthalpy of mixture,

$$\begin{aligned}h &= h_f + x h_{fg} \\ &= 1085.8 + 0.06 \times 1714.6\end{aligned}$$

$$= 1188.67 \text{ KJ / kg Entropy of mixture,}$$

$$s = s_f + x s_{fg}$$

$$= 2.794 + 0.06 \times 3.277$$

$$= 2.99 \text{ kJ / kg K. Internal energy, } u = h - p v$$

$$= 1188.67 - 39.776 \times 10^2 \times 0.00418$$

$$= 1172 \text{ KJ / kg.}$$

Result:

$$p = 39.776 \text{ bar}$$

$$m = 9.574 \text{ kg}$$

$$v = 0.00418 \text{ m}^3 / \text{kg}$$

$$h = 1188.67 \text{ KJ / kg}$$

$$S = 2.99 \text{ KJ /kg K}$$

$$\Delta U = 1172 \text{ KJ / kg.}$$

2). A steam power plant uses steam at boiler pressure of 150 bar and temperature of 550°C with reheat at 40 bar and 550 °C at condenser pressure of 0.1 bar. Find the quality of steam at turbine exhaust, cycle efficiency and the steam rate. [May/June 2014]

Given Data:

$$p_1 = 150 \text{ bar}$$

$$T_1 = 550^\circ\text{C}$$

$$p_2 = 40 \text{ bar}$$

$$T_3 = 550^\circ\text{C}$$

$$p_3 = 0.1 \text{ bar}$$

To find:

4. The quality of steam at turbine exhaust, (x_4)
5. cycle efficiency and
6. The steam rate.

Solution:**1. The quality of steam at turbine exhaust, (x_4):**

Properties of steam from steam tables at 150 bar & 550°C $h_1 = 3445.2$ KJ/kg.

$$S_1 = 6.5125 \text{ KJ/kg K}$$

At 40 bar & 550°C

$$h_3 = 3558.9 \text{ KJ/kg.}$$

$$S_3 = 7.2295 \text{ KJ/kg K}$$

At 40 bar

$$T_{\text{sat}} = 250.3^\circ\text{C} = 523.3 \text{ K}$$

$$\begin{aligned} h_f &= 1087.4 \text{ KJ/kg.} & h_{fg} &= 1712.9 \text{ KJ/kg.} \\ S_f &= 2.797 \text{ KJ/kg K} & S_{fg} &= 3.272 \text{ KJ/kg K} \end{aligned}$$

At 0.1 bar

$$\begin{aligned} h_f &= 191.8 \text{ KJ/kg.} & h_{fg} &= 2392.9 \text{ KJ/kg.} \\ S_f &= 0.649 \text{ KJ/kg K} & S_{fg} &= 7.502 \text{ KJ/kg K} \end{aligned}$$

1-2 = isentropic

$$S_1 = S = 6.5125 \text{ KJ/kg K}$$

$$S_2 = S_g \text{ at 40 bar}$$

Therefore,

Exit of HP turbine is superheat

$$T_{\text{sup}} = 332^{\circ}\text{C}$$

$$h_2 = 3047.18 \text{ KJ/kg}$$

$$S_3 = S_g \text{ at } 0.1 \text{ bar}$$

Steam is at wet condition.

$$S_4 = S_3 = 7.2295 \text{ KJ/kg K}$$

$$S_4 = S_{f4} + x_4 S_{fg4}$$

$$7.2295 = 0.649 + x_4 \times 7.502$$

$$x_4 = 0.877$$

$$h_4 = h_{f4} + x_4 h_{fg4}$$

$$= 191.8 + 0.877 \times 2392.9$$

$$h_4 = 2290.37 \text{ KJ/kg K}$$

2) Cycle efficiency:

$$\begin{aligned} D &= (h_1 - h_2) + (h_3 - h_4) / (h_1 - h_{f4}) + (h_3 - h_2) \\ &= (3445.2 - 3047.15) + (3558.9 - 2290.37) / (3445.2 - 191.8) + (3558.9 - 3047.18) \\ &= 0.4426 \times 100 \\ &= 44.26\% \end{aligned}$$

3) Steam rate:

$$\begin{aligned} &= 3600 / (h_1 - h_2) + (h_3 - h_4) \\ &= 3600 / (3445.2 - 3047.15) + (3558.9 - 2290.37) \\ &= 2.16 \text{ kg/Kw-hr.} \end{aligned}$$

Result:

The quality of steam at turbine exhaust, $(x_4) = 0.877$

cycle efficiency = 44.26%

The steam rate = 2.16 kg/Kw-hr.

3). Ten kg of water 45 °C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300°C. Find the change in volume, enthalpy, internal energy and entropy.

Given Data:

$$m = 10 \text{ kg} \quad t = 45^\circ\text{C}$$

$$p_1 = p_2 = 10 \text{ bar}$$

$$T_2 = 300^\circ\text{C}$$

To find:

Change in volume,

Change in Enthalpy,

Change in Internal energy,

Change in Entropy.

From steam tables, corresponding to 45°C,

$$v_1 = v_{f1} = 0.001010 \text{ m}^3/\text{kg}; \quad h_1 = h_{f1} = 188.4 \text{ KJ/kg};$$

$$s_1 = s_{f1} = 0.638 \text{ KJ/kg K}$$

From steam tables, corresponding to 10 bar and 300°C,

$$h_2 = 3052.1 \text{ KJ/kg}; \quad s_2 = 7.125 \text{ KJ/kg K};$$

$$v_2 = 0.258 \text{ m}^3/\text{kg};$$

Change in Volume, $\Delta V = m (v_2 - v_1)$

$$= 10 (0.258 - 0.001010)$$

$$= 2.5699 \text{ m}^3.$$

Change in Enthalpy, $\Delta h = m (h_2 - h_1)$

$$= 10 (3052.1 - 188.4)$$

$$= 28637 \text{ KJ}.$$

Change in Entropy, $\Delta S = m (s_2 - s_1)$

$$= 10 (7.125 - 0.638)$$

$$= 64.87 \text{ KJ/K}.$$

Result:

Change in volume, $\Delta V = 2.5699 \text{ m}^3$.

Change in Enthalpy, $= 8637 \text{ KJ}$.

Change in Internal energy, $\Delta U = 26067.1 \text{ KJ}$.

Change in Entropy, $\Delta S = 64.87 \text{ KJ/K}$.

4) A steam boiler generates steam at 30 bar, 300 °C at the rate of 2 kg/s. This steam is expanded isentropically in a turbine to a condenser pressure of 0.05 bar, condensed at constant pressure and pumped back to boiler.

- e) Find the heat supplied in the boiler per hour.**
- f) Determine the quality of steam after expansion.**
- g) What is the power generated by the turbine?**
- h) Estimate the Ranking efficiency considering pump work.**

Given Data:

$$p_1 = 30 \text{ bar}$$

$$p_2 = 0.05 \text{ bar}$$

$$T_1 = 300^\circ\text{C}$$

$$m = 2 \text{ kg / s}$$

To find:

Find the heat supplied in the boiler per hour (Q_S)

Determine the quality of steam after expansion (x_2)

What is the power generated by the turbine (W_T)

Estimate the Ranking efficiency considering pump work (η)

Solution:

2. Heat supplied in the boiler per hour (Q_S):

Properties of steam from the steam table

At 30 bar & 300°C

$$h_1 = 2995.1 \text{ KJ/kg}; \quad S_1 = 6.542 \text{ KJ/kg K};$$

At 0.05 bar

$$h_{f2} = 137.8 \text{ KJ/kg}; \quad h_{fg2} = 2423.8 \text{ KJ/kg};$$

$$S_{f2} = 0.476 \text{ KJ/kg K}; \quad S_{fg2} = 7.920 \text{ KJ/kg K};$$

$$V_{f2} = 0.001005 \text{ m}^3/\text{kg}.$$

1-2 = Isentropic expansion in the turbine

$$S_1 = S_2 = 6.542 \text{ KJ/ kg K}$$

$$S_2 = S_{f2} + x_2 \times S_{fg2}$$

$$6.542 = 0.476 + x_2 \times 7.92$$

$$= 0.766$$

Therefore, Quality of steam after expansion = 0.766 dry.

$$h_2 = h_{f2} + x_2 \times h_{fg2}$$

$$= 137.81 + 0.766 \times 2423.8$$

$$= 1994.43 \text{ KJ/kg}.$$

$$h_3 = h_{fg2} = 137.8 \text{ KJ/kg}.$$

Considering the pump work, $h_4 - h_3 = v_{f2} (p_1 - p_2)$

$$h_4 = h_3 + v_{f2} (p_1 - p_2)$$

$$= 137.8 + 0.001005 \times (30 - 0.05) \times 10^2$$

$$= 140.81 \text{ KJ/kg}.$$

Heat supplied in the boiler:

$$\begin{aligned}
 Q_S &= m \times (h_1 - h_4) \\
 &= 2 \times (2995.1 - 140.81) \\
 &= 5708.58 \text{ KJ/s} \\
 &= 20.55 \times 10^6 \text{ KJ /hr.}
 \end{aligned}$$

Power generated by the turbine:

$$\begin{aligned}
 W_T &= m \times (h_1 - h_2) \\
 &= 2 \times (2995.1 - 1994.43) \\
 &= 2001.34 \text{ KW.}
 \end{aligned}$$

Rankine efficiency by the plant:

$$\begin{aligned}
 (\eta) &= (h_1 - h_2) - (h_4 - h_3) / (h_1 - h_4) \\
 &= (2995.1 - 1994.43) - (140.81 - 137.8) / (2995.1 - 140.81) \\
 &= 35 \%
 \end{aligned}$$

Result:

Find the heat supplied in the boiler per hour (Q_S) = 20.55×10^6 KJ /hr

Determine the quality of steam after expansion (x_2) = 0.766 dry

What is the power generated by the turbine (W_T) = 2001.34 KW.

Estimate the Rankine efficiency considering pump work (η) = 35%

5) Determine the state of steam at a pressure of 12 bar with its specific volume of 0.175 m³/kg.

Given data:

$$p = 12 \text{ bar}$$

$$v = 0.175 \text{ m}^3/\text{kg}$$

To find:

State of steam (whether dry, wet or superheated)

Solution:

From saturated water table of pressure scale at 12 bar,

Specific volume of dry steam, $v_g = 0.16321 \text{ m}^3/\text{kg}$, $T_s = 188^\circ\text{C} = 188 + 273 = 461 \text{ K}$

Since $v > v_g$, the steam is in superheated condition.

Note:

$v > v_g \Rightarrow$ Superheated steam

$v = v_g \Rightarrow$ Dry steam

$v < v_g \Rightarrow$ Wet steam

We know that
$$v_{sup} = v_g \left(\frac{T_{sup}}{T_s} \right)$$

$$T_{sup} = \frac{v_{sup}}{v_g} \times T_s = \left(\frac{0.175}{0.16321} \right) \times 461 = 494.3 \text{ K} = 221.3^\circ\text{C}$$

The steam is **superheated to 221.3°C**

Ans. 

6) Determine the condition of steam at a temperature of 220°C and enthalpy of 2750 kJ/kg.

Given data:

$T = 220^\circ\text{C}$

$h = 2750 \text{ kJ/kg}$

To find:

State of steam

Solution:

From saturated water table of temperature scale at 220°C,

$h_f = 943.7 \text{ kJ/kg}$

$$h_{fg} = 1856.2 \text{ kJ/kg}$$

$$h_g = 2799.9 \text{ kJ/kg}$$

Since $h < h_g$, the steam is in wet condition.

Note:

$h > h_g \Rightarrow$ Superheated steam

$h = h_g \Rightarrow$ Dry steam

$h < h_g \Rightarrow$ Wet steam

$$h_{wet} = h_f + x h_{fg}$$

$$x = \frac{h_{wet} - h_f}{h_{fg}} = \frac{2750 - 943.7}{1856.2} = 0.973 \quad \text{Ans.}$$

7) Find the specific volume and enthalpy of steam at 9 bar when the condition of steam is (a) wet with dryness fraction 0.98 (b) dry saturated and (c) superheated and the temperature of steam 240°C.

Given data:

$$p = 9 \text{ bar}$$

(i) $x = 0.95$, (ii) $x = 1$, (iii) Superheated with $T_{sup} = 240^\circ\text{C} = 240 + 273 = 513 \text{ K}$

Solution:

From saturated water table of pressure scale at 9 bar,

$$T_s = 175.4^\circ\text{C} = 175.4 + 273 = 448.4 \text{ K}$$

$$v_f = 0.001121 \text{ m}^3/\text{kg}$$

$$v_g = 0.21482 \text{ m}^3/\text{kg}$$

$$h_f = 742.6 \text{ kJ/kg}$$

$$h_{fg} = 2029.5 \text{ kJ/kg}$$

$$h_g = 2772.1 \text{ kJ/kg}$$

(a) *Wet steam, $x = 0.95$:*

$$v_{wet} = v_f + x (v_g - v_f) \\ = 0.001121 + 0.95 \times (0.21482 - 0.001121) = \mathbf{0.204 \text{ m}^3/\text{kg}}$$

Ans. ↗

$$h_{wet} = h_f + x h_{fg} \\ = 742.6 + 0.95 \times 2029.5 = \mathbf{2670.63 \text{ kJ/kg}}$$

Ans. ↗

(b) *Dry steam:*

$$v_g = \mathbf{0.21482 \text{ m}^3/\text{kg}}$$

Ans. ↗

$$h_g = \mathbf{2772.1 \text{ kJ/kg}}$$

Ans. ↗

(c) *Superheated steam, $T_{sup} = 240^\circ\text{C}$:*

$$v_{sup} = v_g \left(\frac{T_{sup}}{T_s} \right) = 0.21482 \times \left(\frac{513}{448.4} \right) = \mathbf{0.246 \text{ m}^3/\text{kg}}$$

Ans. ↗

Corresponding to $p = 9 \text{ bar}$ and $T = 240^\circ\text{C}$ from the superheated enthalpy table,

$$h_{sup} = \mathbf{2923.29 \text{ kJ/kg}}$$

Ans. ↗

8) Find the internal energy of unit mass of steam at a pressure of 7 bar (i) when its quality is 0.8 (ii) when it is dry and saturated and (iii) superheated, the degree of superheat being 65°C . The specific heat of superheated steam at constant pressure is 2.277 kJ/kg K .

Given data

$$p = 7 \text{ bar} = 700 \text{ kPa}$$

(i) $x = 0.80$

(ii) dry saturated and

(iii) Superheated, $(T_{sup} - T_s) = 65^\circ\text{C}$

To find

Internal energy

Solution

From saturated water table of pressure scale at 7 bar,

$$T_1 = 165^\circ\text{C}, v_f = 0.001108 \text{ m}^3/\text{kg}, v_g = 0.27268 \text{ m}^3/\text{kg}$$

$$h_{fg} = 2064.9 \text{ kJ/kg}, h_f = 697.1 \text{ kJ/kg}, h_g = 2762 \text{ kJ/kg},$$

$$S_f = 1.992 \text{ kJ/kg KS}, S_{fg} = 4.713 \text{ kJ/kg KS}, S_g = 6.705 \text{ kJ/kg K}$$

(i) When $x = 0.8$:

$$h_{wet} = h_f + x h_{fg} = 697.1 + 0.8 \times 2064.9 = 2349.02 \text{ kJ/kg}$$

$$v_{wet} = v_f + x (v_g - v_f)$$

$$= 0.001108 + 0.8 \times (0.27268 - 0.001108) = 0.218 \text{ m}^3/\text{kg}$$

$$\text{Work done, } W = p v_{wet} = 700 \times 0.218 = 152.6 \text{ kJ/kg}$$

$$\text{Internal energy, } u = h - W = 2349.02 - 152.6 = \mathbf{2196.42 \text{ kJ/kg}} \quad \text{Ans.}$$

(ii) When it is dry and saturated:

$$h_g = 2762 \text{ kJ/kg}$$

$$W_{dry} = p v_g = 700 \times 0.27268 = 190.88 \text{ kJ/kg}$$

$$u_g = h_g - W = 2762 - 190.88 = \mathbf{2571.12 \text{ kJ/kg}} \quad \text{Ans.}$$

(iii) When superheated:

$$h_{sup} = h_g + C_p (T_{sup} - T_{sat}) = 2762 + 2.277 \times 65 = 2910 \text{ kJ/kg}$$

$$W_{sup} = p v_{sup}$$

$$= p \times v_g \times \left(\frac{T_{sup}}{T_s} \right)$$

$$= 700 \times 0.27268 \times \left(\frac{230 + 273}{165 + 273} \right)$$

$$= 219.2 \text{ kJ/kg}$$

$$u_{sup} = h - W = 2910 - 219.2 = \mathbf{2690.8 \text{ kJ/kg}} \quad \text{Ans.}$$

$$\begin{aligned} \because T_{sup} - T_s &= 65 \\ T_{sup} &= T_s + 65 \\ &= 165 + 65 = 230^\circ\text{C} \end{aligned}$$

9) Determine the condition of steam whether it is wet, dry or superheated for the following cases by using Steam tables only.

(i) Steam has a pressure of 10 bar and specific volume $0.22 \text{ m}^3/\text{kg}$.

(ii) Steam has a pressure 15 bar and temperature 225°C .

(iii) Steam has a temperature 200°C and enthalpy 2790.9 kJ/kg .

(iv) Steam has a temperature of 120°C and entropy 7 kJ/kgK .

Given data:

(i) $p = 10 \text{ bar}$ and $v = 0.22 \text{ m}^3/\text{kg}$

(ii) $p = 15 \text{ bar}$ and $T = 225^\circ\text{C}$

(iii) $T = 200^\circ\text{C}$ and $h = 2790.9 \text{ kJ/kg}$

(iv) $T = 120^\circ\text{C}$ and $s = 7 \text{ kJ/kg K}$

To find:

Condition of steam

Solution:

(i) $p = 10 \text{ bar}$ and $v = 0.22 \text{ m}^3/\text{kg}$

From saturated water table of pressure scale at 10 bar , $v_g = 0.1943 \text{ m}^3/\text{kg}$

Since $v > v_g$, the **steam is superheated.**

(ii) $p = 15 \text{ bar}$ and $T = 225^\circ\text{C}$

From saturated water table of pressure scale at 15 bar , $T_s = 198.3^\circ\text{C}$

Since $T > T_s$, the **steam is superheated.**

(iii) $T = 200^\circ\text{C}$ and $h = 2790.9 \text{ kJ/kg}$

From saturated water table of temperature scale at 200°C , $h_g = 2790.9 \text{ kJ/kg}$

Since $h = h_g$, the **steam is dry saturated.**

(iv) $T = 120^\circ\text{C}$ and $s = 7 \text{ kJ/kgK}$

From saturated water table of temperature scale at 120°C , $s_g = 7.129 \text{ kJ/kg K}$

Since $s < s_g$, the **steam is in wet condition.**

10) One kg of steam contains $1/3$ liquid and $2/3$ vapour by volume. The temperature of the steam is 150°C . Find the quality, specific volume and specific enthalpy of mixture.

Given data:

$m = 1 \text{ kg}$, $V_f = 1/3$, $V_g = 2/3$, $T_3 = 150^\circ\text{C}$

To find:

Quality x , specific volume, v and specific enthalpy, h

Solution:

$$p = 4.76 \text{ bar}, \quad v_f = 0.001091 \text{ m}^3/\text{kg}, \quad v_g = 0.39245 \text{ m}^3/\text{kg},$$

$$h_f = 632.2 \text{ kJ/kg}, \quad h_{fg} = 2113.2 \text{ kJ/kg}$$

$$\frac{v_f}{v_g} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$\frac{v_f}{v_g} = \frac{m_w v_f}{m_s v_g} = \frac{1}{2}$$

$$\frac{m_w}{m_s} \times \frac{0.001091}{0.39245} = \frac{1}{2}$$

$$\frac{m_w}{m_s} = 179.86$$

Dryness fraction,

$$x = \frac{m_s}{m_w + m_s}$$

$$= \frac{1}{\left(\frac{m_w}{m_s}\right) + 1} = \frac{1}{179.86 + 1} = \mathbf{0.0055}$$

Ans.

Volume of mixture,

$$\begin{aligned} v &= v_f + x(v_g - v_f) \\ &= 0.001091 + 0.0055 \times (0.39245 - 0.001091) \\ &= \mathbf{0.00324 \text{ m}^3/\text{kg}} \end{aligned}$$

Ans.

Enthalpy of mixture,

$$\begin{aligned} h &= h_f + x h_{fg} \\ &= 632.2 + 0.0055 \times 2113.2 \\ &= \mathbf{643.82 \text{ kJ/kg}} \end{aligned}$$

Ans.