STRESS DISTRIBUTION IN SOIL MEDIA:

Stress in soil is caused by the following factors.

i. Self weight of soil

ii. Structural loads.

The most widely used theories regarding distribution of stresses in soil are there of Boussinesq and waster guard.

Geostatic Stress:

The vertical stress in soil due to its self weight is called geostatic stress. [OR]

When the ground surface is horizontal and the properties of soil do not change along ahorizontal plane, the stresses due to self weight are known as geostatic stresses.

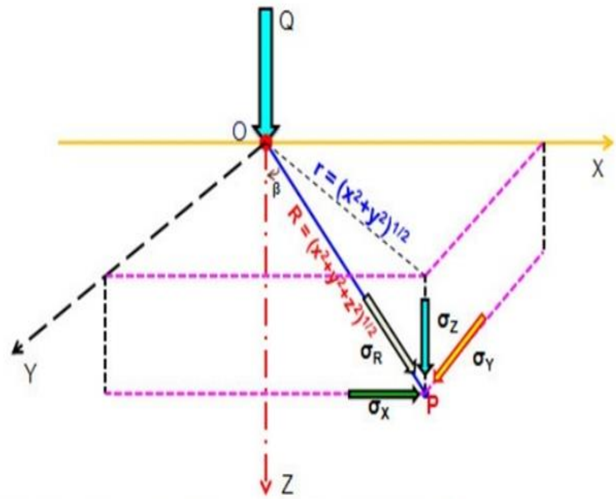
BOUSSINESQ FORMULAE:

This formula is useful for point load (or) concentrated load.

Assumptions:

- The soil is elastic, homogeneous and Isotropic.
- The soil mass obeys hook's law.
- The self weight of the soil is ignored.
- The soil is initially unstressed.
- The change in volume is neglected.
- Top surface of soil is subjected to point load.
- The stress distribution is symmetrical.
- Not applicable for layered soils.

1)<u>Normal stress at any point in a soil mass is subjected to point load:</u>



Let a point load act at a ground surface at a point O. which may be taken as a origin of x,y,z direction.

Let us find the stress component at a point P in the soil mass having coordinate x,y,z.

A radial horizontal distance(r) and the vertical distance (z) from O.

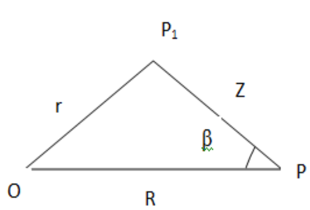
Using the logarithmic stress function, Boussinesq shows that the polar radial stress may be expressed as

$$\sigma_R = \frac{3QCOS\beta}{2\pi R^2}$$

Here ,

R=polar radial coordinates of point p

Consider a triangle opp$_1$



$$R = \sqrt{r^2 + z^2}$$

$$cos\beta = \frac{z}{R}$$

Calculation of σ_z

$$\sigma_z = \sigma_R COS^2\beta \text{------------(1)}$$

Sub σ_R in eqn (1)

$$\sigma_z = \sigma_R COS^2\beta$$

$$\sigma_z = \frac{3QCOS\beta}{2\pi R^2} COS^2\beta$$

$$\sigma_z = \frac{3QCOS^3\beta}{2\pi R^2} - - - - - - (2)$$

Sub $cos\beta = \frac{z}{R}$ in eqn (2)

$$\sigma_z = \frac{3Q\left(\frac{z}{R}\right)^3}{2\pi R^2}$$

$$\sigma_z = \frac{3Q(Z)^3}{2\pi R^5} \text{-------------(3)}$$

Sub R in eqn (3)

$$\sigma_z = \frac{3Q(Z)^3}{2\pi(\sqrt{r^2 + z^2})^5}$$

$$= \frac{3Q(Z)^3}{2\pi(r^2 + z^2)^{5/2}}$$

$$= \frac{3Q(Z)^3}{2\pi}\left[\frac{1}{(r^2 + z^2)^{5/2}}\right]$$

$$= \frac{3Q(Z)^3}{2\pi}\left[\frac{1}{(z^2(\frac{r^2}{z^2} + 1)^{5/2)}}\right]$$

$$= \frac{3Q(Z)^3}{2\pi z^5}\left[\frac{1}{((\frac{r^2}{z^2}+1)^{5/2})}\right]$$

$$= \frac{3Q}{2\pi z^2}\left[\frac{1}{(\frac{r^2}{z^2}+1)^{5/2})}\right]$$

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\sigma_z = K_B \frac{Q}{Z^2}$$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

Calculation of τ_{rz}

$$\tau_{rz} = \frac{1}{2}\sigma_R \sin 2\beta \text{---------(4)}$$

sub σ_R in above equation

$$\tau_{rz} = \frac{1}{2} x \frac{3xQxcos\beta}{2x\pi x R^2} \sin 2\beta \qquad [\sin 2\beta = 2\sin\beta\cos\beta]$$

$$\tau_{rz} = \frac{1}{2} x \frac{3xQxcos\beta}{2x\pi x R^2} x2\sin\beta\cos\beta$$

$$\tau_{rz} = \frac{3xQxcos^2\beta}{2x\pi x R^2} x\sin\beta \text{---------(5)}$$

We know that

$$\text{Sin}\beta = \frac{r}{R} \quad , \quad \cos\beta = \frac{z}{R}$$

sub $\sin\beta$ and $\cos\beta$ in eqn (5)

$$\tau_{rz} = \frac{3xQx\left(\frac{Z}{R}\right)^2}{2x\pi x R^2} X \frac{r}{R}$$

$$\tau_{rz} = \frac{3xQx(Z)^2}{2x\pi x R^4} X \frac{r}{R}$$

$$\tau_{rz} = \frac{3xQx(Z)^2 r}{2x\pi x R^5} - - - - - (6)$$

Sub R in eqn (6)

$$\tau_{rz} = \frac{3xQx(Z)^2 r}{2x\pi x(\sqrt{r^2+z^2})^5}$$

$$\tau_{rz} = \frac{3xQx(Z)^2 r}{2x\pi x(r^2+z^2)^{5/2}}$$

$$\tau_{rz} = \frac{3xQx(Z)^2 r}{2x\pi x(z^2(\frac{r^2}{z^2}+1)^{5/2})}$$

$$\tau_{rz} = \frac{3xQx(Z)^2 r}{2x\pi x(z^2(\frac{r^2}{z^2}+1)^{5/2})}$$

$$= \frac{3Q(Z)^2 r}{2\pi z^5}\left[\frac{1}{((\frac{r^2}{z^2}+1)^{5/2})}\right]$$

$$\tau_{rz} = \frac{3Qr}{2\pi z^3}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

K_B = Influence factor (Boussinesq)

Q = Point load, Z = Depth,

r = horizontal (or) radial distance

Based on Boussinesq equation the following pressure distribution .Diagrams are drawn.

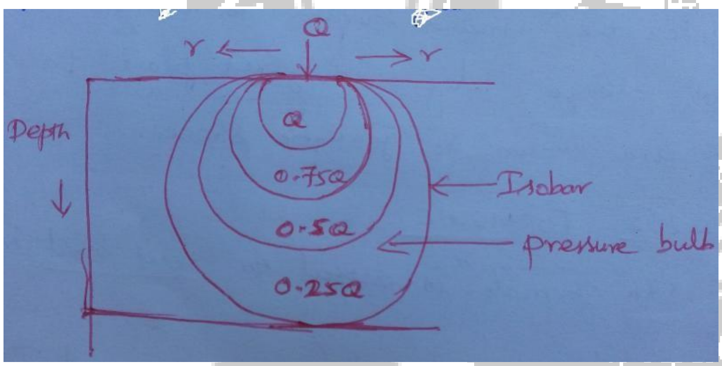
- Stress Isobar (or) Isobar diagram.
- Vertical Pressure Distribution on a horizontal plane.
- Vertical Pressure distribution on a vertical plane.

Stress Isobar:

An Isobar is a cure (or) contours connecting all points below the ground surface of equalvertical pressure.

The Zone bounded by an isobar of a given vertical pressure intensity is called as pressure bulb.

A Number of isobars corresponding to various intensity of vertical pressure forms isobar diagram.



Vertical Pressure Distribution on a horizontal plane:

$$\sqrt{z} = K_B \frac{Q}{z^2}$$

$\sqrt{z}$ is maximum, when $r/z = 0$, $K_B = 0.4775$.

$$\sqrt{z} = 0.4775 \frac{Q}{z^2}$$

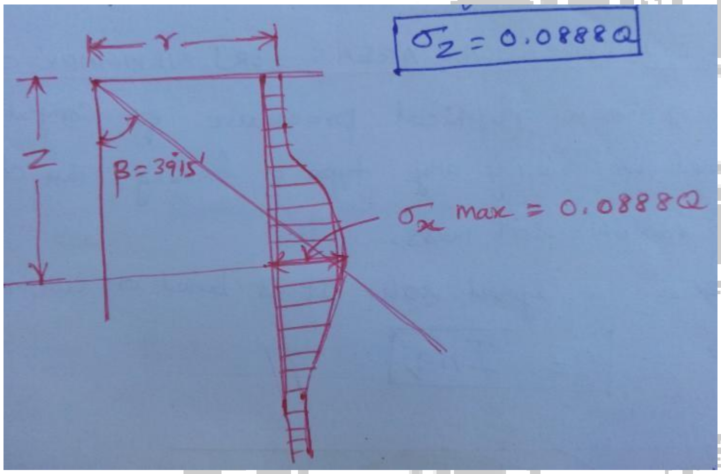
$$\sqrt{z} = 0.4775$$

In the horizontal plane the stress is maximumat the centre and reduces near the edges.

The maximum Vertical stress is 0.4775 $^{Q}/_{z^2}$

<u>Vertical pressure distribution of a vertical plane:</u>

$$\sigma_z = 0.0888Q$$



The vertical stress is <u>*maximum*</u> at the <u>*centre*</u> and <u>*minimum*</u> at the <u>*corners*</u>. The maximum stress is achieved at an angle of <u>39°15"</u> of <u>0.0888Q</u>

<u>Problem</u>

1)A concentrated load of 20KN acts at the ground surface. Compute the vertical stresses at 4m depth i) on the axis of load and ii) 2m away from the axis, also compute the horizontal sheet stress on the axis and 2m away from the axis.

Given: Load, $Q = 20K$; $Z = 4m$

a) $r = 0$; $z = 4m$; $Q = 20KN$

We know, Vertical stress

Vertical stress

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\sigma_z = \frac{3.20}{2\pi.4^2}\left[\frac{1}{\left(1+\frac{0^2}{4^2}\right)}\right]^{5/2}$$

$$\sigma_z = 0.597\ KN/m^2$$

Horizontal shear stress,

$$\tau_{rz} = \frac{3x20x0}{2\pi 4^3}\left[\frac{1}{\left(1+\frac{0^2}{4^2}\right)}\right]^{5/2}$$

$$\tau_{rz} = 0KN/m^2$$

b) $r = 2m;\ \ z = 4m;\ \ Q = 20KN;$

Vertical stress

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\sigma_z = \frac{3.20}{2\pi.4^2}\left[\frac{1}{\left(1+\frac{2^2}{4^2}\right)}\right]^{5/2}$$

$$\sigma_z = 0.342\ KN/m^2$$

Horizontal shear stress,

$$\tau_{rz} = \frac{3x20x2}{2\pi 4^3}\left[\frac{1}{\left(1+\frac{2^2}{4^2}\right)}\right]^{5/2}$$

$$\tau_{rz} = 0.171KN/m^2$$

2)Find the intensity of vertical pressure at a point 3m directly below 25 KN point load acting on a horizontal ground surface. What will be the vertical pressure at a point 2m horizontally away from the axis of loading and at same depth of 3m?use boussinesq`s equation.

Solution:

Casei)Z=3m ,r=0

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\sigma_z = \frac{3x25}{2\pi 3^2}\left[\frac{1}{\left(1+\frac{0^2}{3^2}\right)}\right]^{5/2}$$

=1.33KN/m^2

Case ii)z=3m,r=2m

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left(1+\frac{r^2}{z^2}\right)}\right]^{5/2}$$

$$\sigma_z = \frac{3x25}{2\pi x 3^2}\left[\frac{1}{\left(1+\frac{2^2}{3^2}\right)}\right]^{5/2}$$

=0.53 KN/m^2