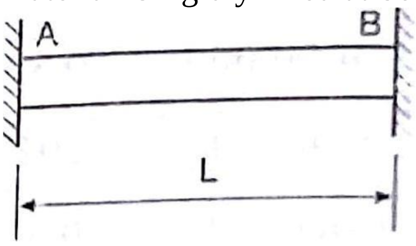
UNIT I

THERMAL STRESS AND STRAIN

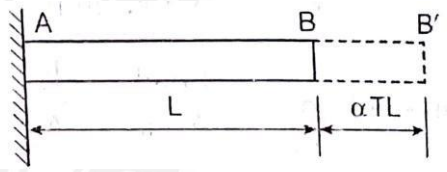
When a material is free to expand or contract due to change in temperature, no stress and strain will be developed in the material. But when the material is rigidly fixed at both the ends, the change in length is prevented. Due to change in temperature, stress will be developed in the material. Such stress is known as thermal stress and the corresponding strain is known as thermal strain. In other words, thermal stress is the stress developed in material due to change in temperature.



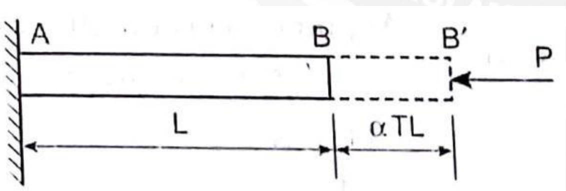
Determination of Thermal Stress and Strain

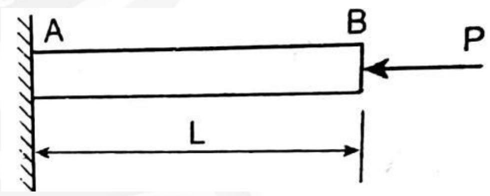
Consider a rod AB of length, fixed at both ends A and B as shown in figure. When the temperature of the rod is raised, it tends to expand by α TL. Where α is the coefficient of linear expansion.

When the fixity at the end B is removed, the rod is freely expanded by αTL. From that we came to know BB' = αTL.



Compressive load P is applied at B', the rod is decreased in its length from L + αTL to L as shown in below figure.





We Know that,

$$\text{Compressive strain or Thermal strain} = \frac{\text{Decrease in Length}}{\text{Original length}}$$

$$= \frac{\alpha TL}{L+\alpha TL}$$

$$= \frac{\alpha TL}{L}$$

$$= \alpha T$$

$$\text{Thermal Strain} = \alpha \text{T}$$

We Know that, Young's Modulus, $E = \frac{Stress}{Strain}$

$$Stress = Strain \times E$$

$$\text{Thermal Stress}, \sigma = \alpha TE$$

We Know that, $Stress = \frac{Load}{Area}$

Or $Load = Stress \times Area$

$$Load(P) = \alpha TE \times A$$

Suppose a rod of length L, when subjected to a rise of temperature is permitted to expand only be Δ, then

$$\text{Thermal Strain} = \frac{\text{Actual expansion allowed}}{\text{Original length}}$$

$$= \frac{\alpha TL - ð}{L}$$

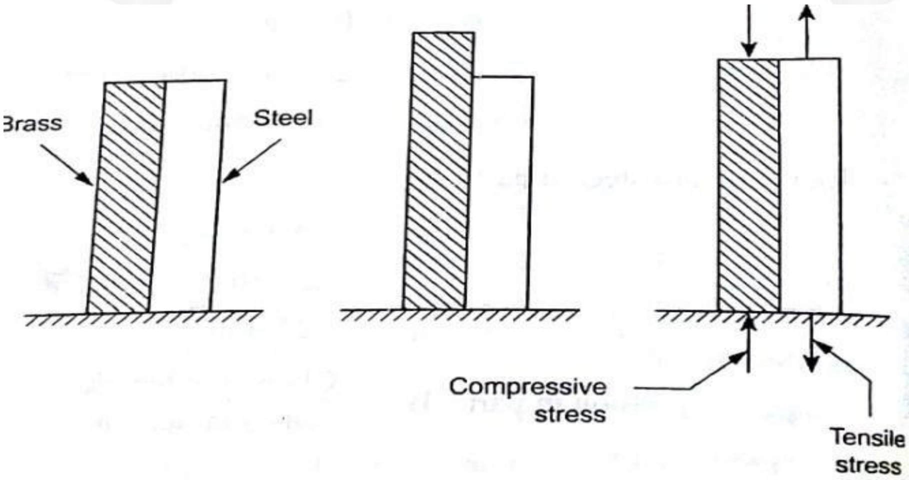
$$\text{Thermal Stress} = \text{Thermal Strain} \times E$$

$$\sigma = \frac{(\alpha TL - ð)}{L}$$

1.3.2 THERMAL STRESSES IN COMPOSITE BARS

A composite member is composed of two or more different materials which are joined together. The following figure shows the composite bars consisting of brass and steel which are subjected to temperature variation. Due to different coefficient of linear expansion, the two materials (brass and steel) expand or contract by different amount.

When the ends of bars are rigidly fixed, then the composites section as a whole will expand or contract. Since the linear expansion of brass is more than that of steel, the brass will expand more than steel. So, the actual expansion of the composite bars will be less than that of brass. Therefore, brass will be subjected to compressive stress, whereas steel will be subjected to tensile stress.



Let A_b be

the Area of cross section of brass bar,

σ_b be the stress in brass bar,

e_b be the Strain in brass bar,

α_b be the Coefficient of linear expansion for brass bar,

E_b be the Young's modulus for brass bar.

A_s be the Area of the cross section of steel bar,

σ_s be the Stress in Steel bar,

e_s be the strain in Steel bar,

α_s be the Coefficient of linear expansion for steel bar,

and E_s be the Young's modulus for steel bar

We Knoaw that,

$$\text{Stress in brass} = \frac{\textit{Load on the brass}}{\textit{Area of the brass}}$$

Load on the brass(P_b) = Stress in Brass (σ_b) × Area of the brass(A_b)

Similarly,

Load on the Steel (P_s) = Stress in Steel(σ_s) × Area of the Steel (A_s)

Under equilibrium condition, compression in the brass bar is equal to tension in the Steel bar

i.e., Load on the brass = Load on the steel

$$\sigma_b A_b = \sigma_s A_s \qquad \ldots(i)$$

We know that,

Actual expansion of steel = Actual expansion of brass …(ii)

Actual expansion of steel = Free expansion of Steel + Expansion due to Tensile Stress in steel

$$= \alpha_s TL + \frac{\sigma_s}{E_s} L \qquad \ldots (iii)$$

$$\left[since\ E = \frac{\sigma}{e}\ (or)\ e = \frac{\sigma}{E}\right]$$

Actual expansion of brass = Free expansion of brass - Contraction due to Compressive stress induced in brass

$$= \alpha_b TL - \frac{\sigma_b}{E_b} L \qquad \ldots (iv)$$

Substituting equation (iii) and (iv) in equation (ii) we get,

$$\alpha_s TL + \frac{\sigma_s}{E_s} L = \alpha_b TL - \frac{\sigma_b}{E_b} L$$

Cancelling the L terms on both sides we get,

$$a_s T + \frac{\sigma_s}{E_s} = a_b T - \frac{\sigma_b}{E_b}$$

Problem1.3.3 A steel rod of 30 mm diameter passes centrally through a copper tube of 60 mm external diameter and 50 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. Calculate the stress developed by copper and steel when the temperature of the assembly is raised by 60°C. Take E for steel = 2×10^5 N/mm^2,E for copper = 1×10^5N/mm^2, a for steel = 12×10^{-6}/°c, a for copper = 18×10^{-6}/°c.

Given Data:

Diameter of Steel rod D_s = 30mm

External diameter of copper, D_c = 60mm

Internal diameter of copper, d_c = 50mm

Rise in temperature, T = 60°c

Young's modulus for steel $E_s = 2\times10^5$ N/mm^2

Young's modulus for copper, $E_c = 1\times10^5$N/mm^2

Coefficient of linear expansion for Steel $\alpha_s = 12\times10^{-6}$,

Coefficient of linear expansion for copper $\alpha_c = 18\times10^{-6}$,

To find :

The Stress in steel and copper

Solution

Area of steel rod $(A_s) = \frac{\pi D_s^2}{4}$

$= \frac{\pi\times30^2}{4}$

= 706.86 mm^2

Area of Copper tube $(A_C) = \frac{\pi}{4}[D_c^2 - d_c^2]$

$= \frac{\pi}{4}[60^2 - 50^2]$ = 63.94mm^2

Since the ends of the bars are rigidly fixed, the composite section as whole will expand or contract. In this problem, the coefficient of linear expansion of the copper is more than that of steel. So, the copper will expand more than steel, But the actual expansion of the composite bars will be less than that of copper. Therefore, copper will subjected to compressive stress, where as steel will be subjected to tensile stress

We Know that, Load on the Steel = Load on the Copper

Or, $\sigma_s A_s = \sigma_c A_c$ …(i)

$\sigma_s \times 706.86 = \sigma_s \times 863.9$

$\sigma_s = \frac{\sigma_s \times ....4}{706.86} = 1.22\sigma_c$ …(ii)

Also We know that,

Actual expansion of steel = Actual expansion of Copper

Or $a_s TL + \frac{\sigma_s}{E_s}L = a_c TL - \frac{\sigma_c}{E_c}L$

Cancelling the L terms on both sides we get,

$$a_s T + \frac{\sigma s}{E_s} = a_c T - \frac{\sigma_c}{E_c}$$

or, $(12 \times 10^{-6} \times 60) + \frac{1.22\sigma_c}{2\times10^5} = (18 \times 10^{-6} \times 60) - \frac{\sigma_c}{1\times10^5}$

$$\frac{1.22\sigma_c}{2\times10^5} + \frac{\sigma_c}{1\times10^5} = (18 \times 10^{-6} \times 60) - (12 \times 10^{-6} \times 60)$$

$$1.61 \times 10^{-5}\sigma_c = 3.6\times10^{-4}$$

$$\sigma_c = \frac{3.6 \times 10^{-4}}{1.61 \times 10^{-5}}$$

$$\boldsymbol{\sigma_c = 22.36\ N/mm^2}$$

Substituting the values of σc in equation ii we get,

$$\sigma_s = 1.22 \times 22.36$$

$$\boldsymbol{= 27.28\ N/mm^2}$$

Problem1.3.4 A gun metal rod 25 mm diameter screwed at the end passes through a steel tube 35 mm and 30mm external and internal diameters. The temperature of the whole assembly is raised to 125^{o}c and the nuts on the rod are then screwed lightly home on the ends of the tube. Calculate the stresses developed in gun metal and steel tube when the temperature of the assembly has fallen to 20^{o}c

Take E for gun metal = $1\times10^5 N/mm^2$,E for steel = $2.1\times10^5 N/mm^2$, a for gun metal = $20 \times 10^{-6}/^o c$, a for steel = $12 \times 10^{-6}/^o c$.

Given Data:

Diameter of gun metal rod (D_g) = 25 mm

External diameter of Steel, D_s = 35 mm

Internal diameter of Stee d_s= 30 mm

Fall in temperature, $T = 125^o c - 20^o c = 105^o c$

Young's modulus for gun metal,$E_g = 1 \times 10^5\ N/mm^2$

Young's modulus for Steel, $E_s = 2.1 \times 10^5 N/mm^2$

Coefficient of linear expansion for Gun metal $\alpha_g = 20 \times 10^{-6}$,

Coefficient of linear expansion for steel $\alpha_s = 12 \times 10^{-6}$.

To find :

The Stress in steel and gun metal

Solution:

Area of gun metal rod, $(A_g) = \frac{\pi}{4} D_g^2$

$= \frac{\pi \times 25^2}{4} = 490.87\ mm^2$

Area of steel tube, tube $(A_s) = \frac{\pi}{4}[D_s^2 - d_s^2]$

$= \frac{\pi}{4}[35^2 - 30^2] = 255.25 mm^2$

Coefficient of linear expansion of gun metal is more than that of steel. So, gun metal will be subjected to compressive stress whereas steel will be subjected to tensile stress.

We Know that, Load on the steel = Load on the gun metal

Or, $\sigma_s A_s = \sigma_g A_g$ …(i)

$\sigma_s \times 255.25 = \sigma_g \times 490.87$

$\sigma_s = \frac{\sigma_g \times 4}{255.25} = 1.92\sigma_g$ …(ii)

Also We know that,

Actual Contraction of steel = Actual Contraction of gun metal

Or $a_s TL + \frac{\sigma_s}{E_s} L = a_g TL - \frac{\sigma_g}{E_g} L$

Cancelling the L terms on both sides we get,

$$a_s T + \frac{\sigma_s}{E_s} = a_g T - \frac{\sigma_g}{E_g}$$

or, $(12 \times 10^{-6} \times 105) + \frac{1.92\sigma_g}{2.1 \times 10^5} = (20 \times 10^{-6} \times 105) - \frac{\sigma_g}{1 \times 10^5}$

$$\frac{1.92\sigma_g}{2.1 \times 10^5} + \frac{\sigma_g}{1 \times 10^5} = (20 \times 10^{-6} \times 105) - (12 \times 10^{-6} \times 105)$$

$19.1 \times 10^{-6} \sigma_g = 0.84 \times 10^{-3}$

$\sigma_g = \frac{0.84 \times 10^{-3}}{19.1 \times 10^{-6}}$ = **$43.97 N/mm^2$**.

Substituting the values of σ_g in equation (ii) we get,

$$\sigma_s = 1.92 \times 43.97 = \mathbf{84.42\ N/mm^2}.$$

Problem1.3.6 A composite bar is made with a copper flat of size 50mm × 30mm and steel flat of 50mm × 40mm of length 500mm each placed one over the other. Find the stress induced in the material, when the composite bar is subjected to an increase in temperature of 90°c. Take coefficient of thermal expansion of steel as $12\times10^{-6}/^{o}c$ and that of copper as $18\times10^{-6}/^{o}c$, Modulus of Elasticity of steel = 200Gpa and modulus of Elasticity of copper = 100Gpa.

Given Data:

Area of copper, $A_c = 50\times30 = 1500mm^2$

Area of steel, $A_s = 50\times40 = 2000mm^2$

Length of the flat (L) = 500mm

Rise in temperature, T = 90°c

Coefficient of linear expansion for Copper $\alpha_c = 18\times10^{-6}$,

Coefficient of linear expansion for Steel $\alpha_s = 12\times10^{-6}$.

Young's modulus for steel E_s = 200 Gpa $=200\times10^9Pa$

$=200\times10^9 N/m^2 = 200\times\frac{10^9}{10^6} = 200\times10^3 N/mm^2$

Young's modulus for copper E_c = 100 Gpa $=100\times10^9Pa$

$= 100\times10^9 N/m^2 = 100\times\frac{10^9}{10^6} = 100\times10^3 N/mm^2$

To find : The Stress induced in the material

Solution:

Coefficient of linear expansion of gun metal is more than that of steel. So, gun metal will be subjected to compressive stress whereas steel will be subjected to tensile stress.

Wkt, Load on the steel = Load on the copper

Or, $\sigma_s A_s = \sigma_c A_c$ …(i)

$\sigma_s \times 2000 = \sigma_c \times 1500$

$\sigma_s = \frac{\times 5}{2000} = 0.75\sigma_c$ …(ii)

Also We know that,

Actual Expansion of steel = Actual Expansion of copper

Or $\alpha_s TL + \frac{\sigma_s}{E_s}L = \alpha_c TL - \frac{\sigma_c}{E_c}L$

Cancelling the L terms on both sides we get,

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

or, $(12\times10^{-6}\times90) + \frac{0.75\sigma_c}{200\times10^3} = (18\times10^{-6}\times90) - \frac{\sigma_c}{100\times10^3}$

$$\frac{0.75\sigma_c}{200\times10^3} + \frac{\sigma_c}{100\times10^3} = (18\times10^{-6}\times90) - (12\times10^{-6}\times90)$$

$$1.375\times10^{-5}\sigma_c = 5.4\times10^{-4}$$

$$\sigma_c = \frac{5.4\times10^{-4}}{1.375\times10^{-5}} = \mathbf{39.27 N/mm^2}.$$

Substituting the values of σ_g in equation ii we get,

$$\sigma_s = 0.75\times39.27 = \mathbf{29.45\ N/mm^2}.$$