### 2.3 STRESS CONDITIONS IN SOIL:

## Total stress (or) Unit pressure ( $\sigma$ ):

Total load per unit area. This pressure may be due to

1) Self weight of soil (saturated weight, if soil is saturated).
2) Over-burden on the soil.

## Consists of two component:

1) Inter granular pressure (or) effective pressure (or) effective stress ( $\sigma^{\prime}$ ):

- It is the pressure transmitted from particle through their point of contact through their soil mass abovethe plane.
- It is effective in decreasing the voids ratio of the soil mass and in mobilizing its shear strength.


## 2) Neutral pressure (or) pore water pressure (u) :

$>$ It is the pressure transmitted through the pore fluid.
$>$ It is equal to water load per unit area above the plane.
> It does not have any influence (measurable) on the voids ratio or any other mechanical property of the soil such as shearing resistance.

$$
\sigma=\sigma^{\prime}+u
$$

Total vertical pressure $=$ Effective pressure + pore pressure
At any plane,
Pore pressure, $\mathrm{u}=$ piezometric head $\left(\mathrm{h}_{\mathrm{w}}\right) \mathrm{x}$ unit weight of water $\left(\gamma_{\mathrm{w}}\right)$

$$
u=h_{w} \gamma_{w}
$$

To find the value of effective pressure we shall consider different conditions of soil water system.

1) Submerged soil mass:
2) Soil mass with surcharge:
3) Partially saturated soil:
4) Saturated soil with capillary fringe:
5) Submerged soil mass:


Fig 2.5 Submerged soil mass
Fig2.5, shows saturated soil mass of depth z, submerged under water of height $\mathrm{z}_{1}$ above its top level. If a piezometric tube is inserted at level AA, water will rise in it upto level CC.

Now, total pressure at AA isgiven by,

$$
\sigma=Z \gamma_{s a t}+Z_{1} \gamma_{w}
$$

Pore pressure, $u=h_{w} . \gamma_{w}$

$$
\begin{aligned}
& \sigma^{\prime}=\sigma-\mathrm{u} \\
& \quad=\mathrm{Z} \gamma_{\mathrm{sat}}+\mathrm{Z}_{1} \gamma_{\mathrm{w}}-\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}} \\
& =\mathrm{Z} \gamma_{\mathrm{sat}}+\mathrm{Z}_{1} \gamma_{\mathrm{w}}-\left(\mathrm{Z}+\mathrm{Z}_{1}\right) \\
& \gamma_{\mathrm{w}} \sigma^{\prime}=\mathrm{Z}\left(\gamma_{\text {sat }}-\gamma_{\mathrm{w}}\right) \\
& \quad \sigma^{\prime}=Z \gamma^{\prime}
\end{aligned}
$$

@ B-B

$$
\begin{aligned}
& \sigma=\gamma_{\mathrm{W}} \cdot \mathrm{Z}_{2} \\
& \mathrm{U}=\gamma_{\mathrm{W}} \cdot \mathrm{~h}_{\mathrm{W}} \\
& \sigma^{\prime}=\sigma-\mathrm{U} \\
& =0
\end{aligned}
$$

## @ C-C

The total, effective and pore water pressure are zero

## 2) Soil mass with surcharge:



Fig 2.6 soil mass with surcharge
Let us now consider a moist soil mass of height $Z_{1}$ above the saturated mass of height Z. Soil mass supports a surcharge pressure of intensity ' $q$ ' per unit area.

At level AA, the pressure is,

$$
\begin{aligned}
& \sigma=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma_{\text {sat }} \\
& \begin{array}{l}
\mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}} \\
=\mathrm{Z} \gamma_{\mathrm{w}} \\
\sigma^{\prime}=\sigma-\mathrm{u} \\
\quad=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma_{\mathrm{sat}}-\mathrm{Z} \gamma_{\mathrm{w}} \\
\quad \sigma^{\prime}=\mathrm{q}+\mathrm{Z}_{1} \gamma+\mathrm{Z} \gamma^{\prime}
\end{array}
\end{aligned}
$$

At the plane BB ,

$$
\begin{array}{r}
\sigma=\mathrm{q}+\mathrm{Z}_{1} \gamma \\
\mathrm{u}=\mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}} \\
=0
\end{array}
$$

At the plane C-C

$$
\begin{gathered}
\sigma=\mathrm{q} \\
\mathrm{U}=0 \\
\sigma^{\prime}=\sigma-\mathrm{u} \\
=\mathrm{q}
\end{gathered}
$$

## 3) Partially saturated soil:

In a partially saturated soil, a part of void space is occupied by air. Hence, in addition to pore waterpressure $\left(\mathrm{u}_{\mathrm{w}}\right)$ pore air pressure $\left(\mathrm{u}_{\mathrm{a}}\right)$ will also to there.

Bishop (1959) based on his intuition gave the following expression for the effective stress.

$$
\sigma^{\prime}=\sigma-\mathrm{u}_{\mathrm{a}}+\mathrm{x}\left(\mathrm{u}_{\mathrm{a}}-\mathrm{u}_{\mathrm{w}}\right)
$$

where, $u_{\mathrm{a}}=$ pore air pressure
$\mathrm{u}_{\mathrm{w}}=$ pore water pressure
$x=$ factor of unit $c / s$ area
Occupied by water $\mathrm{x}=\frac{A_{w}}{A}$
$\mathrm{Aw}=$ Area of water $\mathrm{A}=$ Area of $\mathrm{c} / \mathrm{s}$ of soil
If ( $S>=90 \%$ ) near unity it is recommended to
take ' $x$ ' as unity (ie., 1). $\sigma$ ' $=\sigma-u_{w}$

$$
\sigma^{\prime}=\sigma-u
$$

4) Saturated soil with capillary fringe:


Fig 2.7 Saturated soil with capillary fringe
Fig 2.7 shows a saturated soil mass of height Z above this, there is a soil mass of height $Z_{1}$ saturated by capillary water. If we insert a piezometric tube at $A A$, water will rise to a height corresponding to the free water level BB Hence at the level AA,

$$
\begin{gathered}
\sigma=Z \gamma_{s a t}+Z_{1} \gamma_{s a t} \\
\mathrm{u}=\mathrm{Z} \gamma_{\mathrm{w}}
\end{gathered}
$$

$$
\begin{gathered}
\sigma^{\prime}=\sigma-u \\
=Z \gamma_{s a t}+Z_{1} \gamma_{s a t}-\mathrm{Z} \gamma_{\mathrm{w}} \\
\sigma^{\prime}=Z_{1} \gamma^{\prime}+Z_{1} \gamma_{s a t}
\end{gathered}
$$

Hence at the level BB,

$$
\sigma^{\prime}=Z_{1} \gamma^{\prime}+Z_{1} \gamma_{w}=Z_{1} \gamma_{s a t}
$$

At the level CC

$$
\begin{aligned}
& \quad \sigma^{1}=0 \\
& \quad \mathrm{U}=\mathrm{h}_{\mathrm{w}} \gamma_{\mathrm{w}}=\left(-\mathrm{z}_{1}\right) \mathrm{xh}_{\mathrm{w}} \\
& \sigma=\sigma^{1}-\mathrm{U} \\
& =0-\left(-\mathrm{h}_{\mathrm{w}} \mathrm{Z}_{1}\right) \\
& =\mathrm{h}_{\mathrm{w}} \mathrm{Z}_{1}
\end{aligned}
$$

## Problem

1) The water table in a deposit of sand 8 m thick, is at a depth of 3 m below the surface. Above the W.T. the sand is saturated with capillary water. The bulk density of sand is $19.62 \mathrm{kN} / \mathrm{m}^{3}$. Calculate the effective pressure at $1 \mathrm{~m}, 3 \mathrm{~m}$ and 8 m below the surface. Hence plot the variation of total, neutral pressure and effective pressure over the depth of 8 m .

## a) Stresses at $D, 8 \mathrm{~m}$ below ground:

If we insert a piezometric tube at $D$, water will be rise through a height $h_{w}=5 \mathrm{~m}$ in it.

$\sigma^{\prime}=\sigma-\mathrm{u}=156.96-49.05=\underline{107.91} \mathrm{kN} / \mathrm{m}^{2}$
Alternatively, $\quad \sigma^{\prime}=5 \gamma^{\prime}+3 \gamma_{\text {sat }}$

$$
=5 \times 9.81+3 \times 19.62=\underline{107.91} \mathrm{kN} / \mathrm{m}^{2}
$$

## b) Stresses at C, 3m below G.L.:

$$
\begin{aligned}
& \sigma=3 \gamma_{\text {sat }}=3 \times 19.62 \\
& \quad=\underline{58.86} \mathrm{kN} / \mathrm{m}^{2} \\
& \mathrm{u}=0(\text { zero }) \\
& \sigma^{\prime}=58.86 \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

Alternatively, $\sigma^{\prime}=\mathrm{hx} \gamma_{\text {sat }}=3 \times 19.62$

$$
=\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}(\mathrm{or}) \quad \sigma^{\prime}=3
$$

$$
\mathrm{x} \gamma^{\prime}+\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}}
$$

$$
\begin{aligned}
& =3(19.62-9.81)+3 \times 9.81 \\
& =\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

c) Stresses at B, 1 m below G.L:
$\sigma=1 \gamma_{\text {sat }}=1 \times 19.62=\underline{19.62} \mathrm{kN} / \mathrm{m}^{2}$
$u=-2 \gamma_{w}=-2 \times 9.81=\underline{-19.62} \mathrm{kN} / \mathrm{m}^{2}$
(ie., pressure due to weight of water hanging below that level)

$$
\begin{aligned}
\sigma^{\prime} & =(\sigma-\mathrm{u}) \\
& =19.62-(-19.62) \\
& =\underline{39.24} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

d) Stresses at A, at Ground Level.:

$$
\begin{aligned}
\sigma & =0 \\
\mathrm{u} & =-\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}}=-3(9.81) \\
& =\underline{-29.43} \mathrm{kN} / \mathrm{m}^{2} \\
\sigma^{\prime} & =0-(-29.43) \\
\sigma^{\prime} & =\underline{29.43} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

2) The water table in a certain area is at a depth of 4 m below the ground surface, to a depth of 12 m , the soil consists of very fine sand having an average voids ratio of 0.7 . Above the water table the sand has an average degree of saturation of $50 \%$. Calculate the effective pressure on a horizontal plane at a depth 10 m below the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillary upto a height of 1 m above the W.T.? (Assume $\mathrm{G}=2.6$ ).


## Solution:

Height of sand layer above the w.t $=\mathrm{Z}_{1}=4 \mathrm{~m}$
Height of saturated layer $=12-4=8 \mathrm{~m}$
Depth of point x , where pressure is to be computed $=10 \mathrm{~m}$
Height of saturated layer above $x=Z_{2}=10-4=6 \mathrm{~m}$

$$
\begin{gathered}
\gamma_{d}=\frac{G \cdot \gamma_{w}}{1+e} \\
=\frac{2.65 x 9.81}{1+0.7} \\
=15.29 \mathrm{KN} / \mathrm{m}^{3}
\end{gathered}
$$

i) For sand above water table,

$$
\begin{aligned}
& \gamma_{1}=\frac{(G+e S) \gamma_{w}}{1+e} \\
& \gamma_{2}=\frac{(G+e S) \gamma_{w}}{1+e} \\
& e=\frac{w G}{S}
\end{aligned}
$$

$$
\begin{gathered}
w=\frac{e S}{G} \\
w=\frac{0.7 x 0.5}{2.65}=0.132 \\
\gamma_{1}=\gamma_{\mathrm{d}}(1+\mathrm{w})=15.29 \times(1+0.132)=\underline{17.31} \mathrm{kN} / \mathrm{m}^{3}
\end{gathered}
$$

ii) For saturated sand below water table, $w_{s a t}$

$$
w_{\text {sat }}=\frac{e S}{G}
$$

$$
w=\frac{0.7 x 1}{2.65}=0.264
$$

$\gamma_{2}=\gamma_{\mathrm{d}}\left(1+\mathrm{w}_{\mathrm{sat}}\right)=15.29 \times 1.264=19.33 \mathrm{kN} / \mathrm{m}^{3}$
$\gamma_{2}{ }^{\prime}=19.33-9.81=\underline{9.52} \mathrm{kN} / \mathrm{m}^{3}$
$\left(\gamma^{\prime}=\gamma_{\text {ref }}=\gamma_{\mathrm{sat}}-\gamma_{\mathrm{w}}\right)$
Effective pressure at $x$,

$$
\begin{aligned}
\sigma= & \mathrm{Z}_{1} \gamma_{1}+\mathrm{Z}_{2} \gamma_{2}^{\prime}=(4 \times 17.31)+(6 \times 19.33) \\
\sigma= & \underline{185.222} \mathrm{kN} / \mathrm{m}^{2} \\
\mathrm{u}= & \mathrm{h}_{\mathrm{w}} \cdot \gamma_{\mathrm{w}}=6 \times 9.81 \\
& =\underline{58.86} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

$$
\sigma^{\prime}=\sigma-u=185.22-58.86
$$

$$
=\underline{126.36} \mathrm{kN} / \mathrm{m}^{2}
$$

Effective stress at x after capillary rise,

$$
\begin{aligned}
\sigma^{\prime} & =3 \gamma_{1}+(6+1) \gamma_{2}{ }^{\prime}+\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}} \\
& =(3 \times 17.31)+(7 \times 9.52)+(1 \times 9.81) \\
& =\underline{128.38} \mathrm{kN} / \mathrm{m}^{2}
\end{aligned}
$$

Increase in pressure $=128.38-126.36$

$$
=\underline{2.02} \mathrm{kN} / \mathrm{m}^{2}
$$

(or) $\quad \sigma=\mathrm{Z}_{1} \gamma_{1}+\mathrm{Z}_{2} \gamma_{2}{ }^{\prime}+\mathrm{h}_{\mathrm{c}} \cdot \gamma_{\mathrm{w}}$

$$
=3 \times 17.31+7 \times 19.33+1 \times 9.81=\underline{197.05} \mathrm{kN} / \mathrm{m}^{2}
$$

$$
\mathrm{U}=7 \times 9.81=\underline{68.67} \mathrm{kN} / \mathrm{m}^{2}
$$

$$
\sigma^{\prime}=\sigma-\mathrm{u}
$$

$$
=197.05-68.67
$$

$$
=128.38 \mathrm{kN} / \mathrm{m}^{2}
$$

3) At a construction site , a 3 m thick clay layer is followed by a 4 m thick gravel layer which is resting on impervious rock. A load of $25 \mathrm{kN} / \mathrm{m}^{2}$ is applied suddenly at the surface. The saturated unit weight of the soils is $19 \mathrm{kN} / \mathrm{m}^{3}$ and $20 \mathrm{kN} / \mathrm{m}^{3}$ for the clay and gravel layers, respectively. The water table is at the surface. Draw diagrams showing variation with depth of total, neutral and effective stress in the layers.


At A-A

$$
\sigma=25 \mathrm{KN} / \mathrm{m}^{2}
$$

$\mathrm{U}=25 \mathrm{KN} / \mathrm{m}^{2}$ (Since load is applied suddenly the entire load is taken by pore water)

$$
\begin{aligned}
& \sigma^{\prime}=\sigma-u \\
& =25-25 \\
& =0 \mathrm{KN} / \mathrm{m}^{2}
\end{aligned}
$$

## At B-B

$$
\sigma=25+3 \times 19=82 \mathrm{KN} / \mathrm{m}^{2}
$$

$$
\mathrm{U}=25+9.81 \times 3
$$

$=54.43 \mathrm{KN} / \mathrm{m}^{2}$ (Since load is applied suddenly the entire load is taken by pore water)

$$
\begin{aligned}
\sigma^{\prime} & =\sigma-\mathrm{u} \\
& =82-54.43 \\
& =27.57 \mathrm{KN} / \mathrm{m}^{2}
\end{aligned}
$$

At C-C
$\sigma=25+3 \times 19+4 \times 20=162 \mathrm{KN} / \mathrm{m}^{2}$
$\mathrm{U}=25+9.81 \times 3+9.81 \times 7=93.67 \mathrm{KN} / \mathrm{m}^{2}$ (Since load is applied suddenly the entire load is taken by pore water)

$$
\begin{gathered}
\sigma^{\prime}=\sigma-\mathrm{u} \\
=162-93.67=68.33 \mathrm{KN} / \mathrm{m}^{2}
\end{gathered}
$$

