DC Response of RC Series Circuit

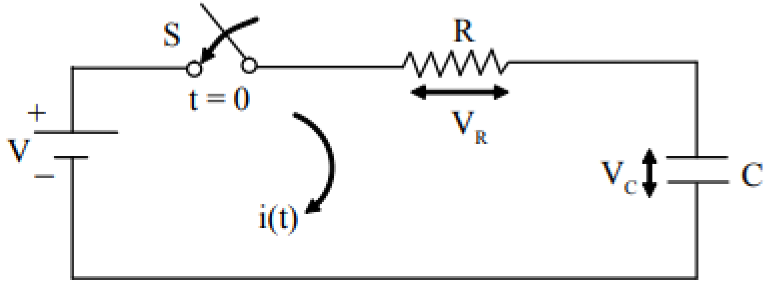

Fig. 3.6 RC Series Circuit

Consider the RC series circuit excited by a DC source as shown in above fig. 4.6. At t = 0, switch S is closed. Assume that at the time of switching, the voltage drop across the capacitor is zero.

By applying KVL to the circuit,

$$V = V_R + V_C$$

$$V = Ri(t) + \frac{1}{C}\int i(t)dt + V_0 \qquad \text{-------------(12)}$$

Applying Laplace transform on both sides to equation (12)

$$\frac{V}{S} = R\ I(S) + \frac{1}{C}\left[\frac{I(S)}{S}\right]$$

There is no loss, so assume $V_0 = \frac{Q_0}{C} = 0$

$$\therefore\ \frac{V}{S} = R\ I(S) + \frac{I(S)}{CS}$$

$$\frac{V}{S} = I(S)\left[R + \frac{1}{CS}\right]$$

$$\frac{V}{S} = I(S)\left[\frac{RCS+1}{CS}\right] \Rightarrow V = I(S)\left[\frac{RC\left(S + \frac{1}{RC}\right)}{C}\right]$$

$$\therefore\ I(S) = \frac{V}{R\left(S + \frac{1}{RC}\right)}$$

$$\text{Let } \frac{1}{RC} = a = \frac{1}{\tau}$$

$$\therefore\ I(S) = \frac{V}{R(s+a)} = \frac{V}{R} \cdot \frac{1}{(s+a)}$$

Applying Inverse Laplace transform to the above equation, we get

$$i(t) = \frac{V}{R} e^{-at}$$

$$\therefore i(t) = \frac{V}{R} e^{-t/\tau}$$

The initial value of the current is V/R Amperes and the final steady state value is ze Amperes. The current response of RC series circuit is shown in below fig. 4.7.

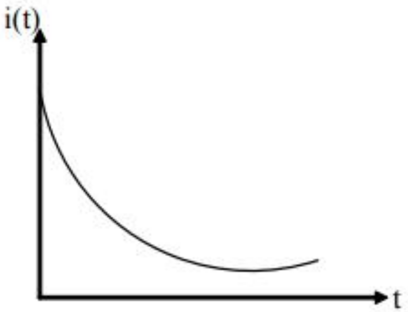

Fig. 3.7 Current response of RL series circuit

The voltage across the resistor, $V_R = iR = Ve^{-t/RC}$

The voltage across the capacitor, $V_C = \frac{1}{C}\int_0^t i(t)dt$

$$= \frac{V}{CR}\int_0^t e^{-t/RC} dt = -V\left[e^{-t/RC}\right]_0^t = V\left[1 - e^{-t/RC}\right]$$