3.2 SHAFT IN SERIES

3.2.1MODULUS OF RUPTURE:

The maximum shear stress calculated by the torsion formula by using the experimentally found maximum torque required to rupture a shaft.

Modulus of rupture in torsion $\tau_r = \frac{T_u \times R}{J}$

Where T_u= Ultimate torque

R = Radius of shaft

J = polar moment of inertia

FLANGED COUPLING:

A flanged coupling is used to connect two shaft are in coaxial. The following fig. shows the flange coupling. The flanges of the two shafts are jointed together by bolts and nuts and torque is then transferred from one shaft to another through the bolts. Connection between each shaft and coupling is provided by the key. The bolts are arranged along a circle called the pitch circle. The bolts are subjected to shear stress when torque is transmitted from one shaft to another.

Let τ = shear stress in the shaft

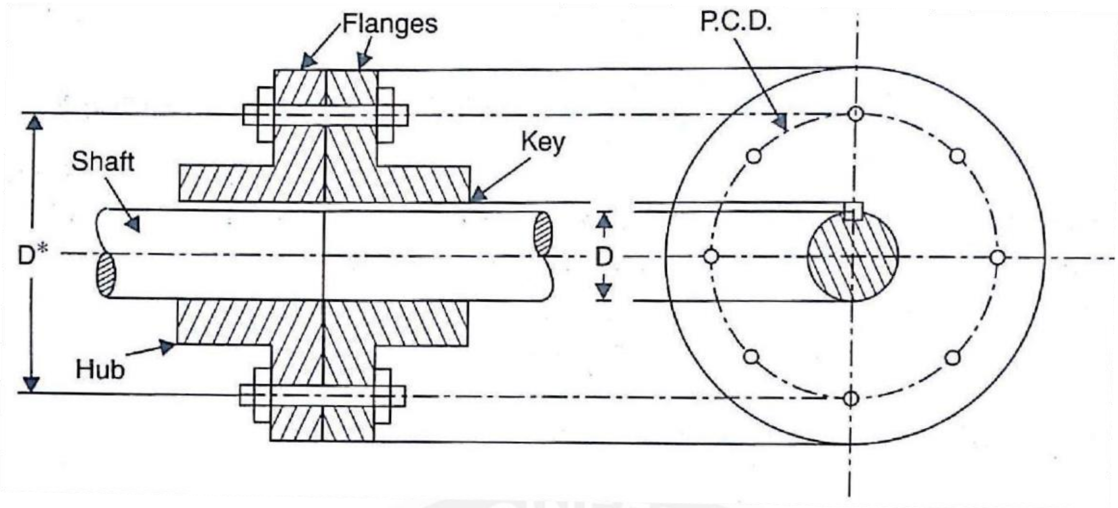
q = shear stress in the bolt

D = diameter of shaft d = diameter of bolt

D_p = diameter of pitch circle

n = number of bolt

Maximum load that can be resisted by one bolt



= Stress in bolt x Area of one bolt $= q \times \frac{\pi \times d^2}{4}$

Torque resisted by one bolt

= load resisted by one bolt x radius of pitch circle

$= q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2}$

∴Total torque resisted by n bolt

$= n \times q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2}$

But torque transmitted by the shaft,

$T = \frac{\pi}{16} \tau D^3$

Since the torque resisted by the bolts should be equal to the torque transmitted by the shafts, therefore equating (3.15) and (3.16), we get

$n \times q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2} = \frac{\pi}{16} \tau D^3$

Based on the above equation we have to calculate the unknown parameter.

STRENGTH OF A SHAFT OF VARYING SECTION

When a shaft is made up of different lengths and of different diameters, the torque transmitted by individual section should be calculated first. The strength of such a shaft is the minimum value of these torques. This varying cross section of shaft arranged in series is called ***series shaft.***

3.2.4. SHAFT IN SERIES

In this case, each shaft transmits the same torque but the angle of twist is the sum of the angle of twist of the two shaft connected in series. Torque transmitted, T = T_1= T_2

Angle of twist $\theta = \theta_1 + \theta_2$ (but $\theta_1 = \theta_2$)

From the torsion equation, $\theta = \frac{T \times l}{C \times J}$

Then angle of twist, $\theta = \frac{T_1 \times l_1}{C_1 \times J_1} + \frac{T_2 \times l_2}{C_2 \times J_2}$ $\theta = T \left(\frac{l_1}{C_1 \times J_1} + \frac{l_2}{C_2 \times J_2}\right)$ ($\therefore$ T = T_1 = T_2)

Where shafts are made of same material, then $C_1 = C_2$

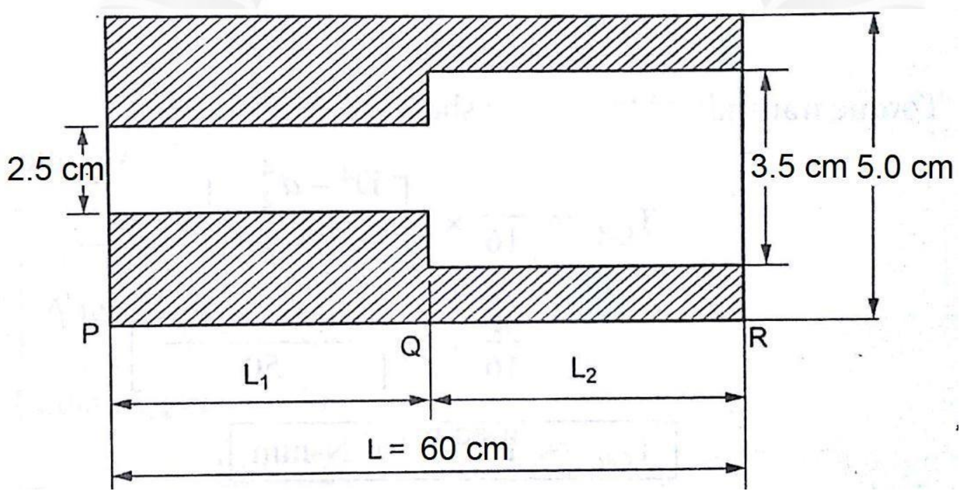
$\therefore$ $\theta = \frac{T}{C}\left(\frac{l_1}{J_1} + \frac{l_2}{J_2}\right)$

Here the driving toque is applied at one end and the resisting toque at the other end.

Problem 3.2.1: A shaft ABC of 60cm length and 5cm external diameter is bored for a part of its length AB to a 2.5cm diameter and for the remaining length BC to a 3.5 cm diameter bore. If the shear stress is not to exceed 70 N/mm^2, find the maximum power that the shaft can transmit at the speed of 150 rpm.

If the angle of twist in the length of 2.5cm diameter bore is equal to that in the 3.5cm diameter bore, find the length of the shaft that has been bored to 2.5cm and 3.5cm diameter.

Given data:



Let Shaft a

AB = shaft 1 and shaft BC = shaft 2

Total length of shaft L = 60 cm = 600mm

Outer Diameter of shaft 1 D_1 = 5cm = 50mm

Inner diameter of shaft 1 d_1 = 2.5cm = 25mm

Outer Diameter of shaft 2 D_2 = 5cm = 50mm

Inner diameter of shaft 1 d_2 = 3.5cm = 35mm

Shear stress r = 70 N/mm^2

Speed N = 150 rpm

Also given as $\theta_1 = \theta_2$

To find out:

Length of shaft 1 l_1 =? Power P =?

Length of shaft 2 l_2 =?

Solution:

Wkt, Condition for series shaft T = T_1 = T_2 θ = $\theta_1 + \theta_2$

Torque transmitted by the hollow shaft 1

$$T_1 = \frac{\pi}{16}\ \tau\ \left[\frac{{D_1}^4 - {d_1}^4}{D_1}\right] = \frac{\pi}{16} \times 70 \times \left[\frac{50^4 - 25^4}{50}\right]$$

$$= 1.61 \times 10^6 \text{ Nmm}$$

Torque transmitted by the hollow shaft 2

$$T_1 = \frac{\pi}{16}\ \tau\ \left[\frac{{D_2}^4 - {d_2}^4}{D_2}\right] = \frac{\pi}{16} \times 70 \times \left[\frac{50^4 - 35^4}{50}\right]$$

$$= 1.30 \times 10^6 \text{ Nmm}$$

From the above two torque value we have to find the maximum value that can be safely applied on the shaft is take the minimum value as 1.30×10^6 Nmm.

Hence torque transmitted T = 1.30×10^6 Nmm = 1.30×10^3Nm

Power transmitted $P = \frac{2\pi NT}{60} = \frac{2\times\pi\times150\times1.30\times10^3}{60} = 20.41\times 10^3$ W

= 20.41 kW

To find the length of shaft

Using the another condition $\theta_1 = \theta_2$

From the torsional equation $\theta_1 = \frac{T_1 \times l_1}{C_1 \times J_1}$ and $\theta_2 = \frac{T_2 \times l_2}{C_2 \times J_2}$

Based on the above condition

$$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2}$$

For same material and same torque transmission $C_1 = C_2$ and $T_1 = T_2$, then the above relation becomes

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} \qquad \gg l_1 = \frac{l_2}{J_2} \times J_1$$

Where

$$J_1 = \frac{\pi}{32} \times [D_1^4 - d_1^4] = J_1 = \frac{\pi}{32} \times [50^4 - 25^4] = 5.75 \times 10^5 \text{ mm}^4$$

$$J_2 = \frac{\pi}{32} \times [D_2^4 - d_2^4] = J_2 = \frac{\pi}{32} \times [50^4 - 35^4] = 4.66 \times 10^5 \text{mm}^4$$

$\gg$ $l_1 = \frac{l_2}{4.66 \times 10^5} \times 5.75 \times 10^5$

$\gg$ $l_1 = 1.23\ l_2$

But $l_2 = L - l_1$ $\gg l_2 = 600 - l_1$, then

$l_1 = 1.23\ (600 - l_1)$ $\gg$ $l_1 = 330.94$ mm

Then $l_2 = 600 - 330.94 = 296.06$ mm

Result:

Power transmitted P = **20.41 kW**

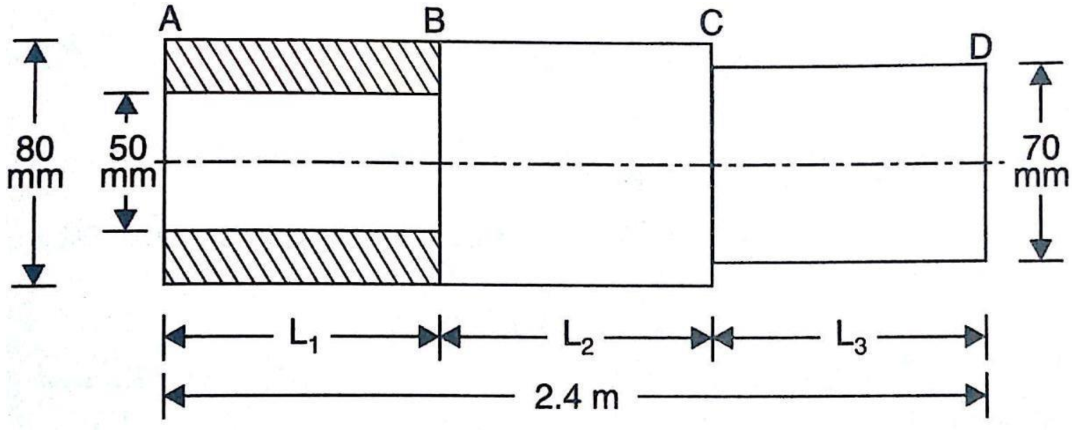
Length of shaft 1 l_1 = **330.94 mm**

Length of shaft 2 l_2 = **296.06 mm**

Problem 3.2.2: A steel shaft ABCD having a total length of 2.4 m consist of three lengths having different sections as follows;

AB is hollow having outside and inside diameter of 80mm and 50mm respectively and BC and CD are solid, BC having a diameter of 80mm and CD a diameter of 70mm. If the angle of twist is the same for each section, determine the length of each section and the total angle of twist if the maximum shear stress in the hollow potion is 50 N/mm^2. Take C = 8.2×10^4N/mm^2.

Given data:



Let Shaft a AB = shaft 1, shaft BC = shaft 2 and shaft CD = shaft 3

Total length of shaft L = 2.4m = 2400mm

Outer Diameter of shaft 1 D_1 = 80 mm

Inner diameter of shaft 1 d_1 = 50 mm

Outer Diameter of shaft 2 D_2 = 80mm

Outer Diameter of shaft 3 D_3 = 70mm

Also given as $\theta_1 = \theta_2 = \theta_3$

Shear stress τ = 50 N/mm^2

Modulus of rigidity C = 8.2 × 10^{4}N/mm^2

To find out:

Length of shaft 1 l_1 =? Length of shaft 3 l_3 =? Length of shaft 2 l_2 =? Total angle of twist θ =?

Solution:

Wkt, Condition for series shaft T = $T_1 = T_2$ $\theta_1 = \theta_2 = \theta_3$

Using the condition $\theta_1 = \theta_2 = \theta_3$

From the torsional equation $\theta_1 = \frac{T_1 \times l_1}{C_1 \times J_1}$;$\theta_2 = \frac{T_2 \times l_2}{C_2 \times J_2}$ and $\theta_2 = \frac{T_3 \times l_3}{C_3 \times J_3}$

Based on the above condition

$$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2} = \frac{T_3 \times l_3}{C_3 \times J_3}$$

For same material and same torque transmission $C_1 = C_2 = C_3$ and $T_1 = T_2 = T_3$ then the above relation becomes

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} = \frac{l_3}{J_3}$$

≫ $l_1 = \frac{l_2}{J_2} \times J_1$ and $l_2 = \frac{l_3}{J_3} \times J_2$

Where

$$J_1 = \frac{\pi}{32} \times [D_1^4 - d_1^4] = J_1 = \frac{\pi}{32} \times [80^4 - 50^4] = 34.09 \times 10^5 \text{ mm}^4$$

$$J_2 = \frac{\pi}{32} \times D_2^4 = J_2 = \frac{\pi}{32} \times 80^4 = 40.24 \times 10^5 \text{ mm}^4$$

$$J_3 = \frac{\pi}{32} \times D_3^4 = J_2 = \frac{\pi}{32} \times 70^4 = 23.58 \times 10^5 \text{mm}^4$$

≫ $$\frac{l_1}{34.09 \times 10^5} = \frac{l_2}{40.24 \times 10^5} = \frac{l_3}{23.58 \times 10^5}$$

≫ $l_1 = \frac{l_3}{23.58 \times 10^5} \times 34.09 \times 10^5$ and $l_2 = \frac{l_3}{23.58 \times 10^5} \times 40.24 \times 10^5$

≫ $l_1 = 1.44\, l_3$ and $l_2 = 1.71\, l_3$

But $L = l_1 + l_2 + l_3$

Substitute l_1 and l_2 value in the above equation

≫ $2400 = 1.44\, l_3 + 1.71\, l_3 + l_3$

≫ $4.15 l_3 = 2400$ ≫ $l_3 = 578.35$ mm

≫ $l_1 = 1.44 \times 578.35 = 832.75$ mm

≫ $l_2 = 1.71 \times 578.35 = 988.80$ mm

As the shear stress is given in shaft AB. The angle of twist of shaft AB can be obtained by using equation

$$\frac{\tau}{R} = \frac{C \times \theta}{l} \quad \text{where } R = \frac{D_1}{2} = \frac{80}{2} = 40\text{mm}$$

Then $$\theta_1 = \frac{\tau_1 \times l_1}{C_1 \times R_1} = \frac{50 \times 832.75}{8.2 \times 10^4 \times 40} = 0.01269 \text{ rad}$$

$$= 0.01269 \times \frac{180}{\pi} = 0.7273^{o}$$

∵ Total angle of twist of the whole shaft

$$\theta = \theta_1 + \theta_2 + \theta_3 = 3\theta_1 = 3 \times 0.7273 = 2.1819^o$$

Result:

Length of shaft 1 l_1 = **832.75 mm**

Length of shaft 2 l_2 = **988.80 mm**

Length of shaft 3 l_3 = **578.35 mm**

Total angle of twist θ = **2.1819 ^{o}**