4.4 Leaf Springs

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \quad ...(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \quad ...\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibresare in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, asshown in Fig. 4.18. Hence for analysis, both stresses need not to be taken into account simultaneously.We shall consider the bending stress only.

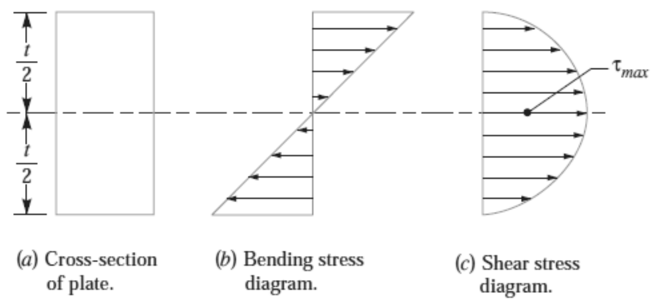


Fig. 4.18

If the spring is not of cantilever type but it is likea simply supported beam, with length 2*L* and load 2*W* in the centre, as shown in Fig. 4.19,

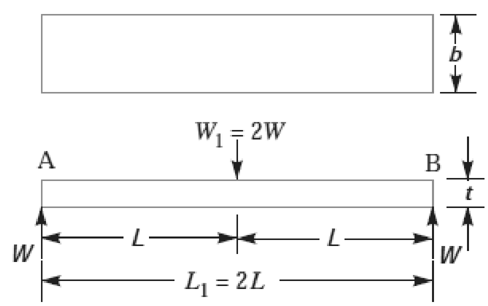


Fig. 4.19. Flat spring (simply supportedbeam type).

ThenMaximum bending moment in the centre

$$M = W.L$$

Section modulus, $Z = b.t^2/6$

$\therefore$ Bending stress, $\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$

$$= \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simplysupported beam loaded in the centre is given by

$$\delta = \frac{W_1 (L_1)^3}{48\, E.I} = \frac{(2W)\,(2L)^3}{48\, E.I} = \frac{W.L^3}{3\, E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

From above we see that a spring such as automobile spring (semi-elliptical spring) with length $2L$ and loaded in the centre by a load $2W$, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of n strips of width b and these are placed as shown in Fig. 4.20, then equations ***(i)*** and ***(ii)*** may be written as

$$\sigma = \frac{6\, W.L}{n.b.t^2} \quad \text{...}(iii)$$

and

$$\delta = \frac{4\, W.L^3}{n.E.b.t^3} = \frac{2\, \sigma.L^2}{3\, E.t} \quad \text{...}(iv)$$

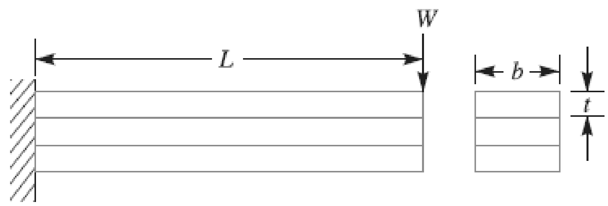


Fig. 4.20

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring is maximum at the support.

If a triangular plate is used as shown in Fig. 4.21 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 4.21 (*b*) to form a graduated or laminated leaf spring, then

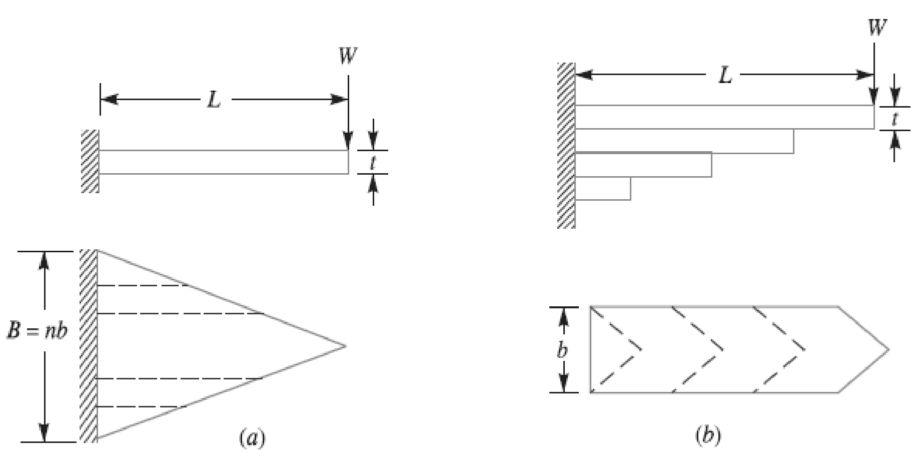


Fig. 4.21. Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(v)$$

and
$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad ...(vi)$$

Where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at theloaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessaryto have one or more leaves of uniform cross-section extending clear to the end. We see from equations**(*iv*)** and **(*vi*)** that for the same deflection, the stress in the uniform cross-section leaves (*i.e*. full lengthleaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects accordingto its own elastic curve. If the suffixes F and G are used to indicate the full length (or uniform crosssection) and graduated leaves, thenThe deflection in full length and graduated leaves is given by equation

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

4.1.14 Construction of Leaf Spring:

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 4.22. It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvatureor cambered so that they will tend to straighten under the load. The leaves are held together by meansof a band shrunk around them at the centre or by a bolt passing through the centre. Since the bandexerts a stiffening and strengthening effect, therefore the effective length of the spring for bendingwill be overall length of the spring *minus* width of band. In case of a centre bolt, two-third distancebetween centres of *U*-bolt should be subtracted from the overall length of the spring in order to findeffective length. The spring is clamped to the axle housing by means of *U*-bolts.

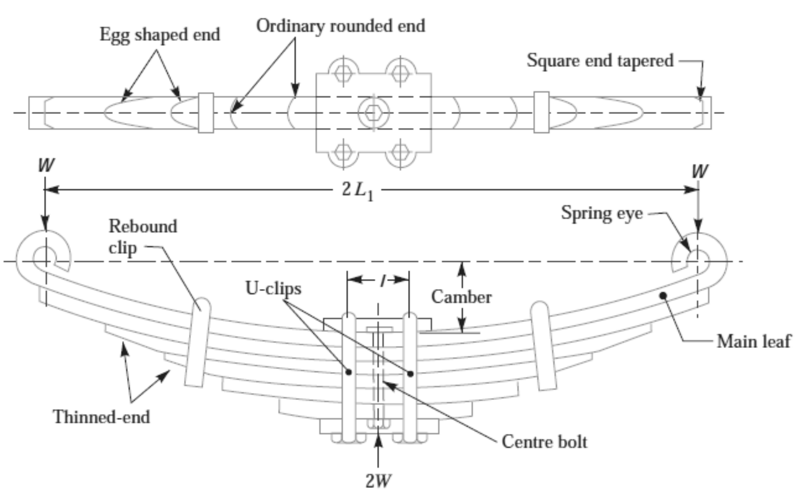


Fig. 4.22. Semi-elliptical leaf spring.

The longest leaf known as ***main leaf*** or ***master leaf***has its ends formed in the shape of an eye through whichthe bolts are passed to secure the spring to its supports.Usually the eyes, through which the spring is attached tothe hanger or shackle, are provided with bushings of someantifriction material such as bronze or rubber. The otherleaves of the spring are known as ***graduated leaves*.** Inorder to prevent digging in the adjacent leaves, the ends ofthe graduated leaves are trimmed in various forms as shownin Fig. 4.22. Since the master leaf has to with stand verticalbending loads as well as loads due to sideways of thevehicle and twisting, therefore due to the presence ofstresses caused by these loads, it is usual to provide twofull length leaves and the rest graduated leaves as shownin Fig. 4.22.Rebound clips are located at intermediate positionsin the length of the spring, so that the graduated leavesalso share the stresses induced in the full length leaveswhen the spring rebounds.

4.1.15Equalised Stress in Spring Leaves(Nipping)

We have already discussed that the stress in the fulllength leaves is 50% greater than the stress in the graduatedleaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed.This condition may be obtained in the following two ways :

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way,the full length leaves will induce smaller bending stress due to small distance from the neutral axis tothe edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shownin Fig. 4.23, before the leaves are assembled to form a spring. By doing so, a gap or clearance will beleft between the leaves. This initial gap, as shown by *C* in Fig. 4.23, is called ***nip*.** When the central bolt,holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in

Fig. 4.23 and have an initial stress in a direction opposite to that of the normal load. The graduated

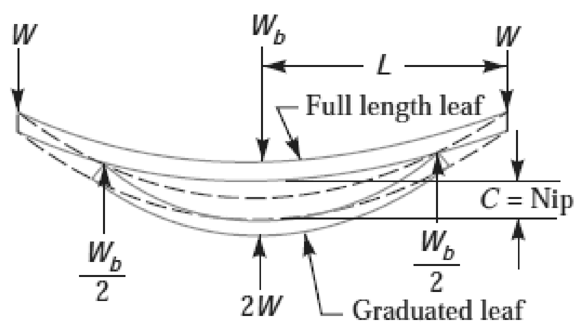


Fig. 4.23

leaves will have an initial stress in the same direction as that of the normal load. When the load isgradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full length leaves may have the lower stress. This is desirable inautomobile springs in which full length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip *C*.

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the fulllength leaves by an amount equal to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$\therefore \quad C = \delta_G - \delta_F = \frac{6\, W_G \cdot L^3}{n_G\, E.b.t^3} - \frac{4\, W_F.L^3}{n_F.E.b.t^3} \quad \text{...}(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\, W_G.L}{n_G.b.t^2} = \frac{6\, W_F.L}{n_F.b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} = \frac{W_F}{n_F}$$

$$\therefore \quad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

and

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of W_G and W_F in equation (*i*), we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \quad \text{...}(ii)$$

The load on the clip bolts (*Wb*) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \quad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\, L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\, W_b}{2\, n_G} = \frac{2\, n_G.W_b + 3\, n_F.W_b}{2\, n_F.n_G} = \frac{W_b\, (2\, n_G + 3\, n_F)}{2\, n_F.n_G}$$

$$\therefore \quad W_b = \frac{2\, n_F.n_G.W}{n\, (2\, n_G + 3\, n_F)} \quad \text{...}(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

$\therefore$ Final stress,

$$\sigma = \frac{6\, W_F.L}{n_F.b.t^2} - \frac{6\, L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\, L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\, L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\, (2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\, W.L}{b.t^2}\left[\frac{3n - n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\, W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n\,(2n_G + 3n_F)}\right] = \frac{6\, W.L}{n.b.t^2} \quad \text{...}(iv)$$

... (Substituting $n = n_F + n_G$)

4.1.16 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is theineffective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the Spring,

$$2L = 2L_1 - l \quad \text{...(When band is used)}$$

$$= 2L_1 - \frac{2}{3}l \quad \text{... (When } U\text{-bolts are used)}$$

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then thenumber of leaves to be cut will be n and when there are two full length leaves (including one masterleaf), then the number of leaves to be cut will be (n – 1). If a leaf spring has two full length leaves, thenthe length of leaves is obtained as follows :

$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The nth leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2L_1 + \pi(d+t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y(2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

4.1.17 Standard Sizes of Automobile Suspension Springs

Following are the standard sizes for the automobile suspension springs:

1. Standard nominal widths are : 32, 40*, 45, 50*, 55, 60*, 65, 70*, 75, 80, 90, 100 and 125 mm. (Dimensions marked* are the preferred widths)

2. Standard nominal thicknesses are : 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14 and 16 mm.

3. At the eye, the following bore diameters are recommended :

19, 20, 22, 23, 25, 27, 28, 30, 32, 35, 38, 50 and 55 mm.

4. Dimensions for the centre bolts, if employed, shall be as given in the following table.

Table 4.3. Dimensions for centre bolts.

Width of leaves in mm	*Dia. of centre bolt in mm*	*Dia. of head in mm*	*Length of bolt head in mm*
Upto and including 65	8 or 10	12 or 15	10 or 11
Above 65	12 or 16	17 or 20	11

5. Minimum clip sections and the corresponding sizes of rivets and bolts used with the clipsshall be as given in the following table (See Fig. 4.24).

Table 4.6. Dimensions of clip, rivet and bolts.

Spring width (B) in mm	*Clip section (b × t) in mm × mm*	*Dia. of rivet (d_1) in mm*	*Dia. of bolt (d_2) in mm*
Under 50	20 × 4	6	6
50, 55 and 60	25 × 5	8	8
65, 70, 75 and 80	25 × 6	10	8
90, 100 and 125	32 × 6	10	10

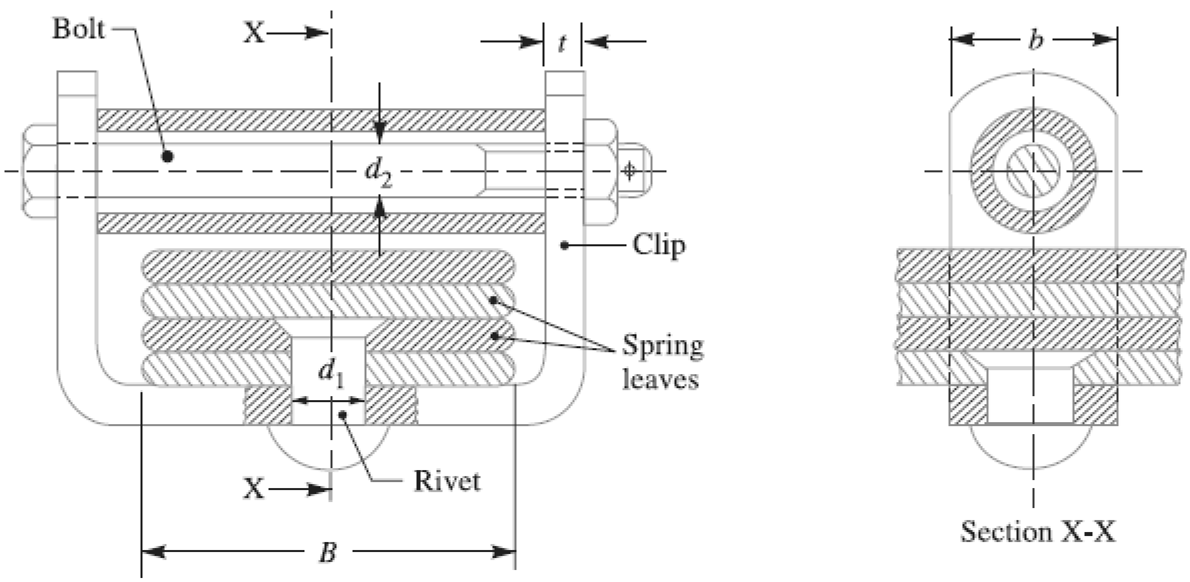


Fig. 4.24. Spring clip.

4.1.18 Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon.
The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

According to Indian standards, the recommended materials are :

1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.

2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (waterhardened) and 55 Si 2 Mn 90 (oil-hardened).

3. The physical properties of some of these materials are given in the following table. Allvalues are for oil quenched condition and for single heat only.

Table 4.4. Physical properties of materials commonly used for leaf springs.

Material	*Condition*	*Ultimate tensile strength (MPa)*	*Tensile yield strength (MPa)*	*Brinell hardness number*
50 Cr 1	Hardened	1680 – 2200	1540 – 1750	461 – 601
50 Cr 1 V 23	and	1900 – 2200	1680 – 1890	534 – 601
55 Si 2 Mn 90	tempered	1820 – 2060	1680 – 1920	534 – 601

Example 4.7. *Design a leaf spring for the following specifications :Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves= 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.Take Young's modulus, E = 200 kN/mm*^{2} *and allowable stress in spring material as 600 MPa*

Solution. Given :

Total load = 140 kN

No. of springs = 4; n = 10

$2L$ = 1000 mm or L = 500 mm

δ = 80 mm

$E = 200\ \text{kN/mm}^2 = 200 \times 103\ \text{N/mm}^2$

$\sigma = 600\ \text{MPa} = 600\ \text{N/mm}^2$

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35\ \text{kN}$$

$\therefore$ $W = 35 / 2 = 17.5\ \text{kN} = 17\,500\ \text{N}$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$\therefore$ $n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3$ ...(*i*)

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \qquad ...(ii)$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm Ans.}$$

Example 4.8. *A semi-elliptical laminated vehicle spring to carry a load of 6000 N is toconsist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. Thespring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts holdthe central portion of the spring so rigidly that they may be considered equivalent to a band havinga width equal to the distance between the bolts. Assume a design stress for spring material as 350MPa. Determine :*
1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5.
Radius to which leaves should be initially bent.
Sketch the semi-elliptical leaf-spring arrangement.
The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.

Solution.

Given :

$2W = 6000$ N
$W = 3000$ N
$n = 7$
$b = 65$ mm
$n_F = 2$
$2L_1 = 1.1$ m$= 1100$ mm or $L_1 = 550$ mm
$l = 80$ mm
$\sigma = 350$ MPa $= 350$ N/mm^2

1. *Thickness of leaves*

Let t = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress (σ_F),

$$350 = \frac{18\, W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\ (2 \times 5 + 3 \times 2)} = \frac{26\ 480}{t^2} \quad \text{... } (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\ 480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm } \textbf{Ans.}$$

2. *Deflection of spring*

We know that deflection of spring,

$$\delta = \frac{12\, W.L^3}{E.b.t^3\ (2n_G + 3n_F)} = \frac{12 \times 3000\ (510)^3}{210 \times 10^3 \times 65 \times 9^3\ (2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm } \textbf{Ans.} \quad \text{... (Taking } E = 210 \times 10^3 \text{ N/mm}^2\text{)}$$

3. *Diameter of eye*

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let d = Inner diameter of the eye or diameter of the pin,

l_1 = Length of the pin which is equal to the width of the eye or leaf (*i.e.* b) = 65 mm ...(Given)

p_b = Bearing pressure on the pin which may be taken as 8 N/mm^2.

We know that the load on pin (W),

$$3000 = d \times l_1 \times p_b$$

$$= d \times 65 \times 8 = 520\, d$$

$$\therefore \quad d = 3000 / 520$$

$$= 5.77 \text{ say } 6 \text{ mm}$$

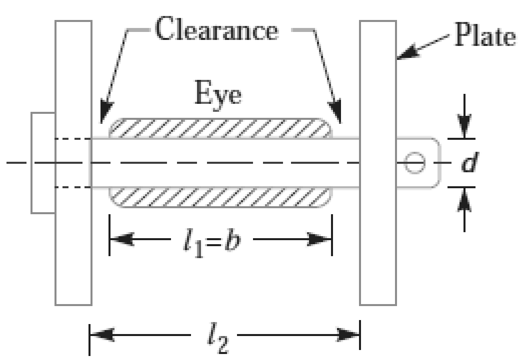


Fig. 4.25

Let us now consider the bending of the pin. Since thereis a clearance of about 2 mm between the shackle (or plate)and eye as shown in Fig. 4.25, therefore length of the pinunder bending,

$L_2 = l_1 + 2 \times 2 = 65 + 4 = 69$ mm

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\ 750 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\ d^3$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{51\ 750}{0.0982\ d^3} = \frac{527 \times 10^3}{d^3} \quad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2)$$

$\therefore$ $d^3 = 527 \times 10^3 / 80 = 6587$ or $d = 18.7$ say 20 mm **Ans.**

We shall take the inner diameter of eye or diameter of pin (d) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin (W),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\ \tau = 628.4\ \tau$$

$\therefore$ $\tau = 3000 / 628.4 = 4.77$ N/mm^2, which is safe.

4. *Length of leaves*

We know that ineffective length of the spring

$= l = 80$ mm ... ($\because$ *U*-bolts are considered equivalent to a band)

$$\therefore \text{Length of the smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \frac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \frac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \frac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \frac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \frac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$= 2L_1 + \pi (d + t)\ 2 = 1100 + \pi (20 + 9)2 = 1282.2$ mm **Ans.**

5. *Radius to which the leaves should be initially bent*

Let R = Radius to which the leaves should be initially bent, and

y = Camber of the spring.

We know that

$$y\,(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \text{ or } 2R - 30 = (550)^2/30 = 10\,083 \qquad \ldots (\because y = \delta)$$

$$\therefore \quad R = \frac{10\,083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

4.2 FLYWHEEL

4.2.1 Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply ofenergy is more than the requirement and releases it duringthe period when the requirement of energy is more thansupply.In case of steam engines, internal combustion engines,reciprocating compressors and pumps, the energy isdeveloped during one stroke and the engine is to run forthe whole cycle on the energy produced during this onestroke. For example, in I.C. engines, the energy is developedonly during power stroke which is much more than theengine load, and no energy is being developed duringsuction, compression and exhaust strokes in case of fourstroke engines and during compression in case of two strokeengines. The excess energy developed during power strokeis absorbed by the flywheel and releases it to the crankshaftduring other strokes in which no energy is developed, thusrotating the crankshaft at a uniform speed. A little consideration will show that when the flywheelabsorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel doesnot maintain a constant speed, it simply reduces the fluctuation of speed.In machines where the operation is intermittent like punching machines, shearing machines,riveting machines, crushers etc., the flywheel stores energy from the power source during the greaterportion of the operating cycle and gives it up during a small period of the cycle. Thus the energy fromthe power source to the machines is supplied practically at a constant rate throughout the operation.

4.2.2 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximumfluctuation of speed.*** The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed*.**

Let $N1$ = Maximum speed in r.p.m. during the cycle,

$N2$ = Minimum speed in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

$\therefore$ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2\,(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2\,(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \qquad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2\,(v_1 - v_2)}{v_1 + v_2} \qquad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It variesdepending upon the nature of service to which the flywheel is employed. Table 4.26 shows the permissiblevalues for coefficient of fluctuation of speed for some machines.

Table 4.5. Permissible values for coefficient of fluctuation of speed (*C*S).

S.No.	Type of machine or class of service	Coefficient of fluctuation of speed (C_S)
1.	Crushing machines	0.200
2.	Electrical machines	0.003
3.	Electrical machines (direct drive)	0.002
4.	Engines with belt transmission	0.030
5.	Gear wheel transmission	0.020
6.	Hammering machines	0.200
7.	Pumping machines	0.03 to 0.05
8.	Machine tools	0.030
9.	Paper making, textile and weaving machines	0.025
10.	Punching, shearing and power presses	0.10 to 0.15
11.	Spinning machinery	0.10 to 0.020
12.	Rolling mills and mining machines	0.025

4.2.3 Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one completecycle of operation. Consider a turning moment diagram for a single cylinder double acting steamengine as shown in Fig. 4.26. The vertical ordinate represents the turning moment and the horizontalordinate (abscissa) represents the crank angle.

A little consideration will show that the turning moment is zero when the crank angle is zero. Itrises to a maximum value when crank angle reaches 90º and it is again zero when crank angle is 180º.This is shown by the curve *abc*in Fig. 4.26 and it represents the turning moment diagram for outstroke.The curve *cde*is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.Since the work done is the product of the turning moment and the angle turned, therefore thearea of the turning moment diagram represents the work done per revolution. In actual practice, theengine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. Theheight of the ordinate *aA*represents the mean height of the turning moment diagram. Since it isassumed that the work done by the turning moment per revolution is equal to the work done againstthe mean resisting torque, therefore the area of the rectangle *aA Fe* is proportional to the work doneagainst the mean resisting torque.

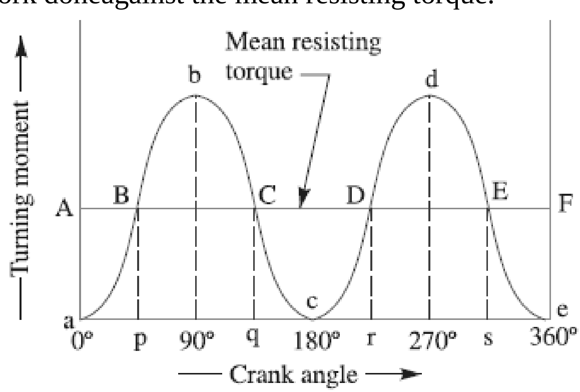


Fig. 4.26. Turning moment diagram for a single cylinder double acting steam engine.

We see in Fig. 4.26, that the mean resisting torque line *AF* cuts the turning moment diagram atpoints *B*, *C*, *D* and *E*. When the crank moves from '*a*' to '*p*' the work done by the engine is equal tothe area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the enginehas done less work (equal to the area *aAB*) than the requirement. This amount of energy is taken fromthe flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, thework done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is representedby the area *pBCq*. Therefore the engine has done more work than the requirement. This excess work(equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases whilethe crank moves from *p* to *q*.

Similarly when the crank moves from *q* to *r*, more work is taken from the engine than is developed.This loss of work is represented by the area *CcD*. To supply this loss, the flywheel gives up some ofits energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from*r* to *s*, excess energy is again developed given by the area *DdE*and the speed again increases. As thepiston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations ofenergy above and below the mean resisting torque line are called ***fluctuation of energy.*** The areas*BbC*, *CcD*, *DdE*etc. represent fluctuations of energy.

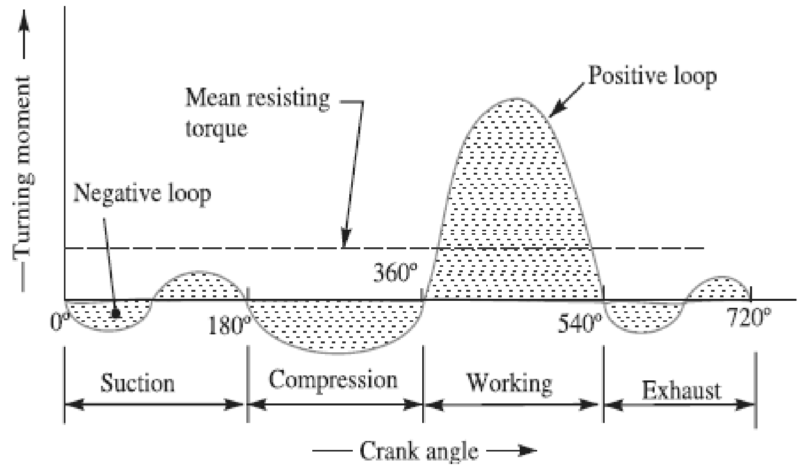


Fig. 4.27. Tunring moment diagram for a four stroke internal combustion engine.

A little consideration will show that the engine hasa maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crankmoves from *p* to *q* and from *r* to *s*. On the other hand,the engine has a minimum speed either at *p* or at *r*. Thereason is that the flywheel gives out some of its energywhen the crank moves from *a* to*p* and from *q* to *r*. Thedifference between the maximum and the minimumenergies is known as ***maximum fluctuation of energy.***A turning moment diagram for a four strokeinternal combustion engine is shown in Fig. 4.27. Weknow that in a four stroke internal combustion engine,there is one working stroke after the crank has turnedthrough 720º (or 4π radians). Since the pressure inside the engine cylinder is less than the atmosphericpressure during suction stroke, therefore a negative loop is formed as shown in Fig. 4.27. During thecompression stroke, the work is done on the gases, therefore a higher negative loop is obtained. In theworking stroke, the fuel burns and the gases expand, therefore a large positive loop is formed. Duringexhaust stroke, the work is done on the gases, therefore a negative loop is obtained.A turning moment diagram for a compound steam engine having three cylinders and the resultantturning moment diagram is shown in Fig. 4.28. The resultant turning moment diagram is the sum ofthe turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the highpressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressurecylinder. The cranks, in case of three cylinders are usually placed at 120º to each other.

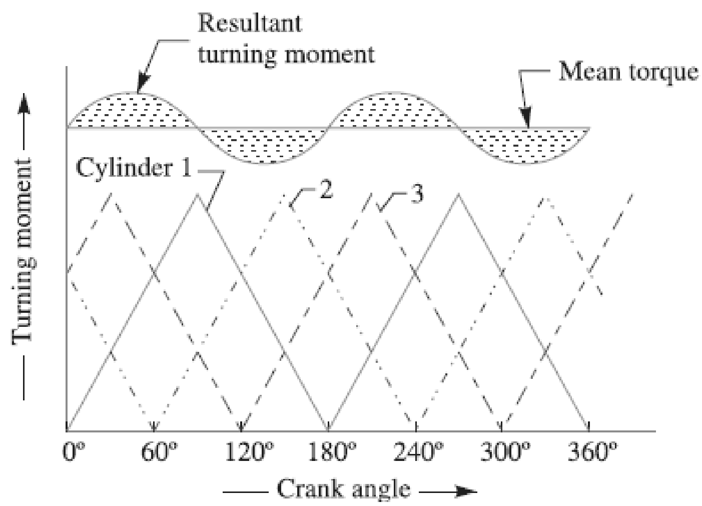
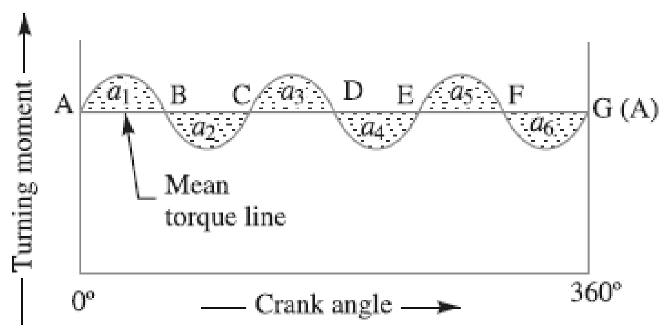


Fig. 4.28. Turning moment diagram for a compound steam engine.

4.2.4 Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 22.4.The horizontal line *AG* represents the mean torque line. Let *a*1, *a*3, *a*5 be the areas above the mean torque line and *a*2, *a*4 and *a*6 be the areas below the mean torque line. These areas represent somequantity of energy which is either added or subtracted from the energy of the moving parts of theengine.



Let the energy in the flywheel at $A = E$, then from Fig. 22.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A

Let us now suppose that the maximum of these energies is at B and minimum at E.

$\therefore$ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

4.2.5 Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It isusually denoted by *C*E. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

= 2π, in case of steam engines and two stroke internal combustion engines.

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Table 4.6. Coefficient of fluctuation of energy (*C*E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy* (C_E)
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinder, single acting, four stroke gas engine	0.066
5.	Six cylinder, single acting, four stroke gas engine	0.031

4.2.6 Energy Stored in a Flywheel

A flywheel is shown in Fig. 4.29. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

Let m = Mass of the flywheel inkg,
k = Radius of gyration of theflywheel in metres,
I = Mass moment of inertia ofthe flywheel about the axis of rotation in kg-m^2= $m.k^2$,
N_1 and N_2 = Maximum and minimum
speeds during the cycle inr.p.m.,
ω1 and ω2 = Maximum and minimumangular speeds during the cycle in rad / s,

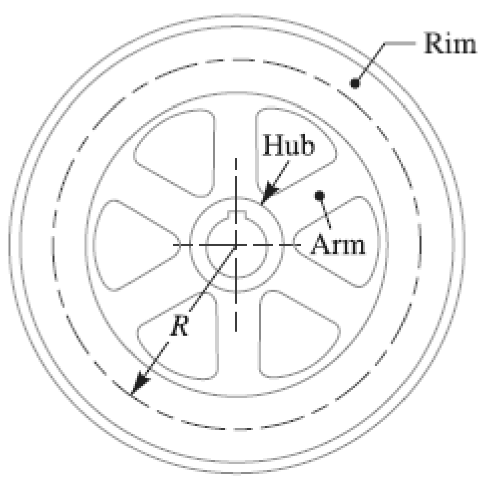


Fig. 4.29. Flywheel.

N = Mean speed during the cycle in r.p.m. = $\frac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad / s = $\frac{\omega_1 + \omega_2}{2}$

C_S = Coefficient of fluctuation of speed = $\frac{N_1 - N_2}{N}$ or $\frac{\omega_1 - \omega_2}{\omega}$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I (\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= I.\omega(\omega_1 - \omega_2) \quad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \ldots(i)$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) \quad \ldots\text{[Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \quad \ldots(\because I = m.k^2) \ldots(ii)$$

$$= 2E.C_S \quad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots(iii)$$

The radius of gyration (*k*) may be taken equal to the mean radius of the rim (*R*), because thethickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ inequation **(*ii*)**, we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \quad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.

Example 4.9. *The intercepted areas between the output torque curve and the mean resistanceline of a turning moment diagram for a multicylinder engine, taken in order from one end are asfollows:*

– 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm2.

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speedis 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The densityof the material of the flywheel may be taken as 7200 kg / m3. The rim is rectangular with the width2 times the thickness. Neglect effect of arms, etc.

Solution.
Given :
N = 900 r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s
$\omega_1 - \omega_2 = 2\% \ \omega$ or
$((\omega 1 - \omega 2)/2)) = CS = 2\% = 0.02$
D = 650 mm or R = 325 mm = 0.325 m
$\rho = 7200 \text{ kg / m}^3$

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.
First of all, let us find the maximum fluctuation of energy. The turning moment diagram for amulti-cylinder engine is shown in Fig. 4.30.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5°
= π / 40 rad, therefore 1 mm2 on the turning moment diagram.

= 70 × π / 40 = 5.5 N-m

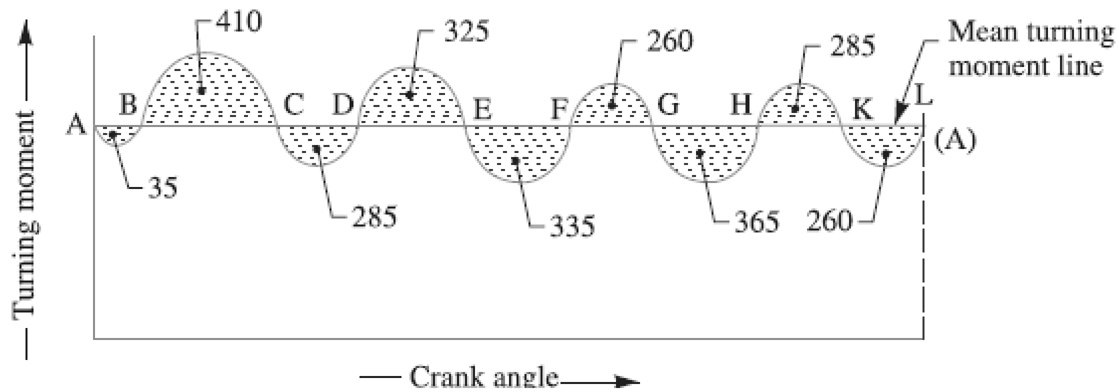


Fig. 4.30

Let the total energy at $A = E$. Therefore from Fig. 4.30, we find that

Energy at $B = E – 35$

Energy at $C = E – 35 + 410 = E + 375$

Energy at $D = E + 375 – 285 = E + 90$

Energy at $E = E + 90 + 325 = E + 415$

Energy at $F = E + 415 – 335 = E + 80$

Energy at $G = E + 80 + 260 = E + 340$

Energy at $H = E + 340 – 365 = E – 25$

Energy at $K = E – 25 + 285 = E + 260$

Energy at $L = E + 260 – 260 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

∴ Maximum energy = $E + 415$

and minimum energy = $E – 35$

We know that maximum fluctuation of energy,

$= (E + 415) – (E – 35) = 450 \text{ mm}^2$

= 450 × 5.5 = 2475 N-m

We also know that maximum fluctuation of energy (ΔE),

$$2475 = m.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$\therefore$ $m = 2475 / 18.77 = 132$ kg **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $2\,t$ ...(Given)

$\therefore$ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$\therefore$ $t^2 = 132 / 29\,409 = 0.0044$ or $t = 0.067$ m = 67 mm **Ans.**

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

4.2.7 Stresses in a Flywheel Rim

A flywheel, as shown in Fig. 4.31, consists of arim at which the major portion of the mass or weightof flywheel is concentrated, a boss or hub for fixingthe flywheel on to the shaft and a number of arms for supporting the rim on the hub.The following types of stresses are induced inthe rim of a flywheel:

1. Tensile stress due to centrifugal force,

2. Tensile bending stress caused by the restraintof the arms, and

3. The shrinkage stresses due to unequal rateof cooling of casting. These stresses may bevery high but there is no easy method of determining.This stress is taken care of by afactor of safety.

We shall now discuss the first two types ofstresses as follows:

1. ***Tensile stress due to the centrifugal force***

The tensile stress in the rim due to the centrifugal force, assuming that the rim is unstrained bythe arms, is determined in a similar way as a thin cylinder subjected to internal pressure.

Let

b = Width of rim,
t = Thickness of rim,
A = Cross-sectional area of rim = $b \times t$,
D = Mean diameter of flywheel
R = Mean radius of flywheel,
ρ = Density of flywheel material,
ω = Angular speed of flywheel,
v = Linear velocity of flywheel, and
σ_t = Tensile or hoop stress.

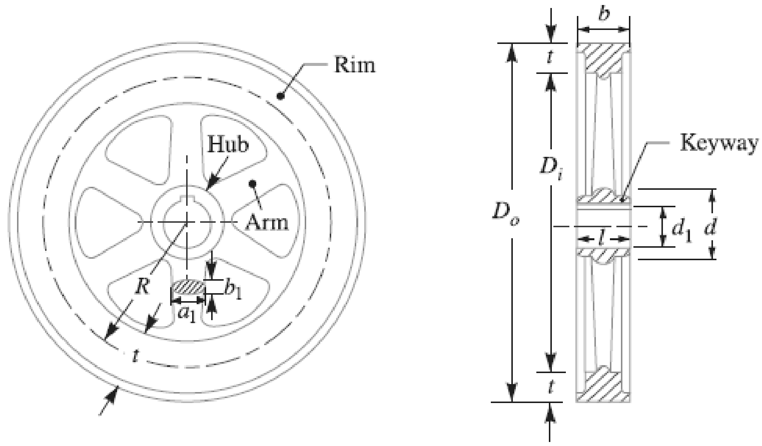


Fig. 4.31. Flywheel.

Consider a small element of the rim as shown shaded in Fig. 4.32. Let it subtends an angle ∂_θ at the centre of the flywheel.

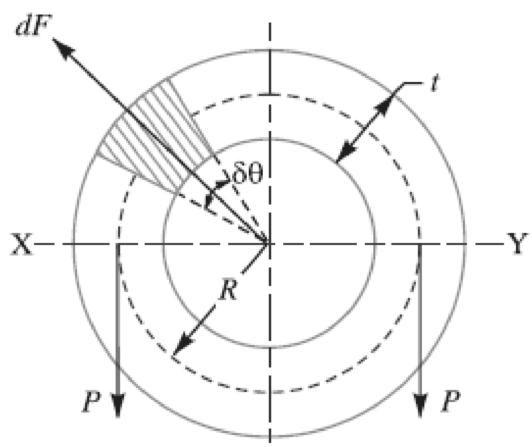


Fig. 4.32. Cross-section of a flywheel rim.

Volume of the small element

$$= A.R.\delta\theta$$

∴ Mass of the small element,

$$dm = \text{Volume} \times \text{Density}$$
$$= A.R.\delta\theta.\rho = \rho.A.R.\delta\theta$$

and centrifugal force on the element,

$$dF = dm.\omega^2.R = \rho.A.R.\delta\theta.\omega^2.R$$
$$= \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF

$$= dF.\sin\theta$$
$$= \rho.A.R^2.\omega^2.\delta\theta \sin\theta$$

∴ Total vertical bursting force across the rim diameter X-Y,

$$= \rho.A\,R^2.\omega^2 \int_0^{\pi} \sin\theta\, d\theta$$

$$= \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^{\pi} = 2\,\rho.A.R^2.\omega^2 \qquad ...(i)$$

This vertical force is resisted by a force of $2P$, such that

$$2P = 2\sigma_t \times A \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$2\rho A.R^2.\omega^2 = 2\,\sigma_t \times A$$

$$\therefore \quad \sigma_t = \rho.R^2.\omega^2 = \rho.v^2 \qquad ...(\because\ v = \omega.R) \qquad ...(iii)$$

when ρ is in kg / m^3 and v is in m / s, then σ_t will be in N / m^2 or Pa.

2. *Tensile bending stress caused by restraint of the arms*

The tensile bending stress in the rim due to the restraint of the arms is based on the assumptionthat each portion of the rim between a pair of arms behaves like a beam fixed at both ends anduniformly loaded, as shown in Fig. 4.33, such that length between fixed ends,

$$l = \frac{\pi\,D}{n} = \frac{2\,\pi\,R}{n}, \text{ where } n = \text{Number of arms.}$$

The uniformly distributed load (w) per metre length will be equal to the centrifugal force between a pair of arms.

$$\therefore \quad w = b.t.\rho.\omega^2.R \text{ N/m}$$

We know that maximum bending moment,

$$M = \frac{w\,.\,l^2}{12} = \frac{b\,.\,t\,\rho\,.\,\omega^2\,.\,R}{12}\left(\frac{2\,\pi\,R}{n}\right)^2$$

and section modulus, $\quad Z = \frac{1}{6}\,b \times t^2$

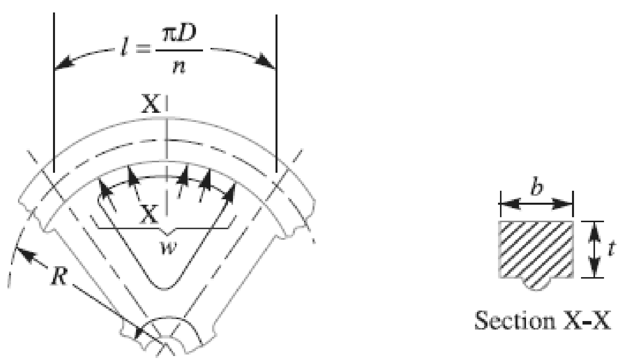


Fig. 4.33

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{b \,.\, t\, \rho \,.\, \omega^2 \,.\, R}{12}\left(\frac{2\,\pi\, R}{n}\right)^2 \times \frac{6}{b \times t^2}$$

$$= \frac{19.74\, \rho \,.\, \omega^2 \,.\, R^3}{n^2 \,.\, t} = \frac{19.74\, \rho \,.\, v^2 \,.\, R}{n^2 \,.\, t} \quad \text{...(iv)}$$

...(Substituting $\omega = v/R$)

Now total stress in the rim,

$$\sigma = \sigma_t + \sigma_b$$

If the arms of a flywheel do not stretch at all and are placed very close together, then centrifugal force will not set up stress in the rim. In other words, σ_t will be zero. On the other hand, if the arms are stretched enough to allow free expansion of the rim due to centrifugal action, there will be no restraint due to the arms, *i.e.* σ_b will be zero.

It has been shown by G. Lanza that the arms of a flywheel stretch about $\frac{3}{4}$ th of the amount necessary for free expansion. Therefore the total stress in the rim,

$$= \frac{3}{4}\,\sigma_t + \frac{1}{4}\,\sigma_b = \frac{3}{4}\,\rho . v^2 + \frac{1}{4} \times \frac{19.74\, \rho \,.\, v^2 \,.\, R}{n^2 \,.\, t} \quad \text{...(v)}$$

$$= \rho . v^2 \left(0.75 + \frac{4.935\, R}{n^2 \,.\, t}\right)$$

Example 4.10. *A multi-cylinder engine isto run at a constant load at a speed of 600 r.p.m.On drawing the crank effort diagram to a scaleof 1 m = 250 N-m and 1 mm = 3º, the areas in sqmm above and below the mean torque line are asfollows:*
+ 160, – 172, + 168, – 191, + 197, – 162 sq mm
The speed is to be kept within ± 1% of themean speed of the engine. Calculate the necessarymoment of inertia of the flywheel.
Determine suitable dimensions for cast ironflywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250kg / m^3*, and its working stress in tension is 6 MPa.Assume that the rim contributes 92% of theflywheel effect*.

Solution.
Given :
N = 600 r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s
$\rho = 7250$ kg / m^3
σ_t= 6 MPa = 6 × 106 N/m^2

Moment of inertia of the flywheel

Let

I = Moment of inertia of the flywheel.
First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 4.34.

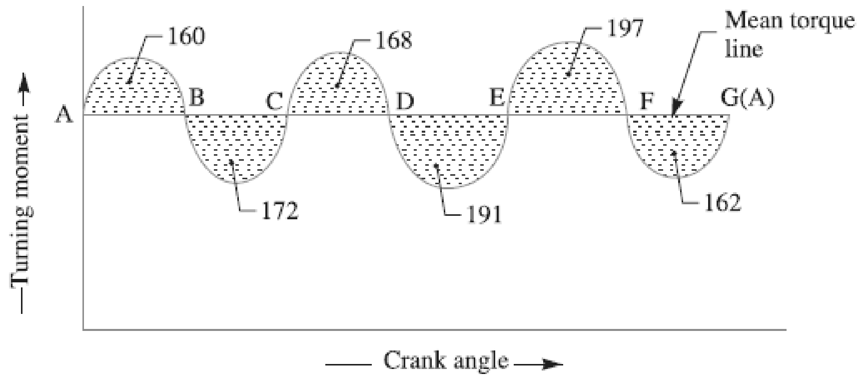


Fig. 4.34

Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is

1 mm = 3° = $\frac{\pi}{60}$ rad, therefore

1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 4.34, we find that

Energy at B = $E + 160$

Energy at C = $E + 160 - 172$ = $E - 12$

Energy at D = $E - 12 + 168$ = $E + 156$

Energy at E = $E + 156 - 191$ = $E - 35$

Energy at F = $E - 35 + 197$ = $E + 162$

Energy at G = $E + 162 - 162$ = E = Energy at A

From above, we find that the energy is maximum at F and minimum at E.

∴ Maximum energy $= E + 162$

and minimum energy $= E - 35$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m}$$

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 2\%\ \omega = 0.02\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I\,(62.84)^2\ 0.02 = 79\ I$$

∴ $I = 2581 / 79 = 32.7$ kg-m^2 **Ans.**

Dimensions of a flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres = 2 t ...(Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

$$6 \times 10^6 = \rho . v^2 = 7250 \times v^2$$

$$\therefore \quad v^2 = 6 \times 10^6 / 7250 = 827.6 \quad \text{or} \quad v = 28.76 \text{ m/s}$$

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi \, D \, . \, N}{60} = \frac{\pi \, D \times 600}{60} = 31.42 \, D$$

$$\therefore \quad D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm} \textbf{ Ans.}$$

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel (E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2 \, C_S = E \times 2 \times 0.02 = 0.04 \, E$$

$$\therefore \quad E = 2581 / 0.04 = 64\ 525 \text{ N-m}$$

and energy of the flywheel rim,

$$E_{rim} = 0.92 \, E = 0.92 \times 64\ 525 = 59\ 363 \text{ N-m}$$

Let m = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59\ 363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m \, (28.76)^2 = 413.6 \, m$$

$$\therefore \quad m = 59\ 363 / 413.6 = 143.5 \text{ kg}$$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi \, D \times \rho = 2 \, t \times t \times \pi \times 0.915 \times 7250 = 41\ 686 \, t^2$$

$$\therefore \quad t^2 = 143.5 / 41\ 686 = 0.003\ 44$$

or t = 0.0587 say 0.06 m = 60 mm **Ans.**

and $b = 2\,t = 2 \times 60 = 120$ mm **Ans.**

4.3 Connecting Rod and Crank Shaft

4.3.1 Introduction

The connecting rod is the intermediate member between the piston and the crankshaft. Its primaryfunction is to transmit the push and pull from the piston pin to the crankpin and thus convert thereciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 4.35. It consists of a long shank, a small end and abig end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-section.Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines.

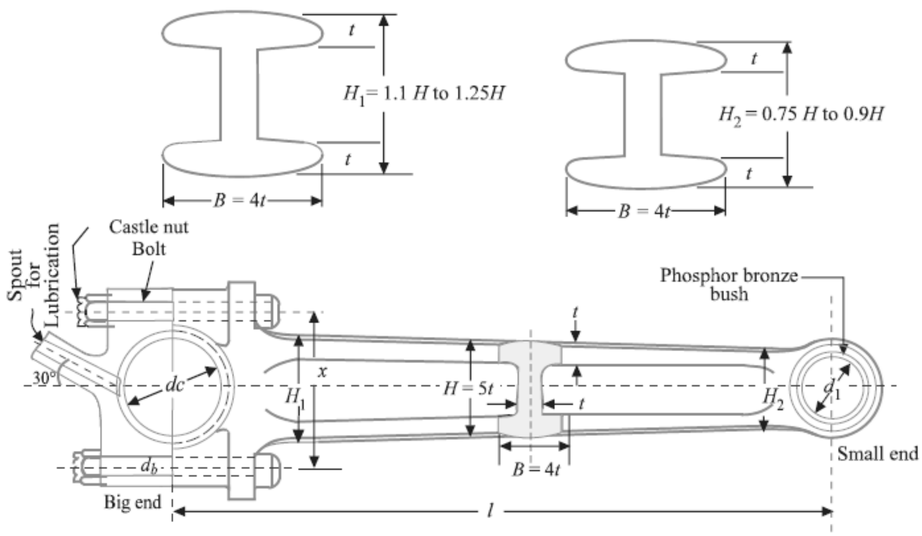


Fig. 4.35. Connecting rod.

The length of the connecting rod (*l*) depends upon the ratio of *l* / *r*, where *r* is the radius ofcrank. It may be noted that the smaller length will decrease the ratio *l* / *r*. This increases the angularityof the connecting rod which increases the side thrust of the piston against the cylinder liner which inturn increases the wear of the liner. The larger length of the connecting rod will increase the ratio*l* / *r*. This decreases the angularity of the connecting rod and thus decreases the side thrust and theresulting wear of the cylinder. But the larger length of the connecting rod increases the overall heightof the engine. Hence, a compromise is made and the ratio *l* / *r* is generally kept as 4 to 5.The small end of the connecting rod is usually made in the form of an eye and is provided witha bush of phosphor bronze. It is connected to the piston by means of a piston pin.The big end of the connecting rod is usually made split (in two halves) so that it can bemounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two capbolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by insertingthin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of theconnecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

The connecting rods are usually manufactured by drop forging process and it should have adequatestrength, stiffness and minimum weight. The material mostly used for connecting rods varies frommild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chromemolybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength ofabout 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimatetensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloysteels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods ofaeroengines and automobile engines.The bearings at the two ends of the connecting rod are either splash lubricated or pressurelubricated. The big end bearing is usually splash lubricated while the small end bearing is pressurelubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spoutand set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holesprovided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearingthrough the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed tosmall end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the smallend bearing is lubricated by the oil scrapped from the walls of the cyinder liner by the oil scraper rings.

4.3.2 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows :

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,

2. Force due to inertia of the connecting rod or inertia bending forces,

3. Force due to friction of the piston rings and of the piston, and

4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

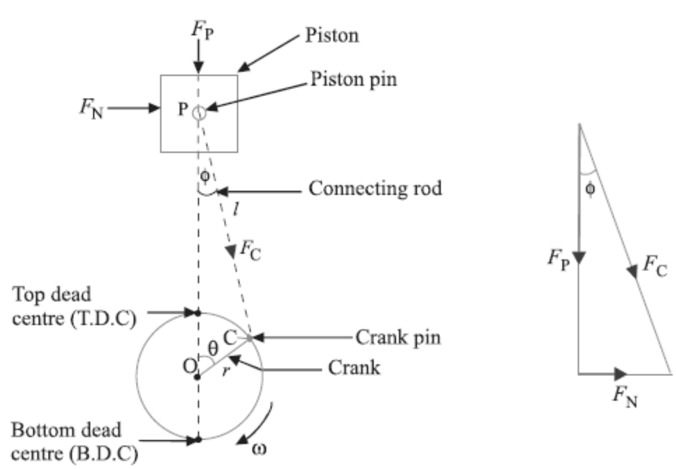
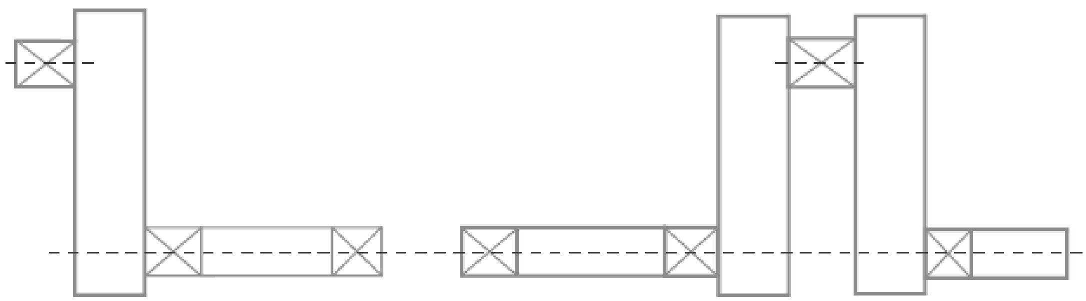


Fig. 4.36. Forces on the connecting rod.

4.3.3 Crankshaft

A crankshaft (*i.e.* a shaft with a crank) is used to convert reciprocating motion of the piston intorotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the mainbearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms orwebs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, dependingupon the position of crank, may be divided into the following two types :
1. Side crankshaft or overhung crankshaft, as shown in Fig. 4.37 (*a*), and
2. Centre crankshaft, as shown in Fig. 4.37 (*b*).



(*a*) Side Crankshaft.

(*a*) Centre Crankshaft.

Fig. 4.37. Types of crankshafts.

The crankshaft, depending upon the number of cranks in the shaft, may also be classfied assingle throw or multi-throw crankshafts. A crankhaft with only one side crank or centre crank iscalled a ***single throw crankshaft*** whereas the crankshaft with two side cranks, one on each end orwith two or more centre cranks is known as ***multi-throw crankshaft.***The side crankshafts are used for medium and large size horizontal engines.

4.3.4 Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft shouldbe tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel orspecial cast iron. In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8,55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 aregenerally used for the making of crankshaft. In aero engines, nickel chromium steel such as35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.The crankshafts are made by drop forging or casting process but the former method is morecommon. The surface of the crankpin is hardened by case carburizing, nitriding or inductionhardening.

4.3.5 Bearing Pressures and Stresses in Crankshaft

The bearing pressures are very important in the design of crankshafts. The *maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure.
The following two types of stresses are induced in the crankshaft.

1. Bending stress ; and **2.** Shear stress due to torsional moment on the shaft.
2. Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed
3. torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be
4. based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine

5. destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined
6. accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

EXERCISES

1. A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 600 N to 1600 N. The spring index is 6 and the design factor of safety is 1.43. If the yield shearstress is 700 MPa and the endurance stress is 350 MPa, find the size of the spring wire and meandiameter of the spring coil. **[Ans. 10 mm ; 60 mm]**

2. A helical spring *B* is placed inside the coils of a second helical spring *A*, having the same number of coils and free length. The springs are made of the same material. The composite spring is compressed by an axial load of 2300 N which is shared between them. The mean diameters of the spring *A* and *B* are 100 mm and 70 mm respectively and wire diameters are 13 mm and 8 mm respectively. Find the load taken and the maximum stress in each spring.

[Ans. *W*A = 1670 N ;*W*B = 630 N ; σA = 230 MPa ; σB = 256 MPa]

3. Design a concentric spring for an air craft engine valve to exert a maximum force of 5000 N under a deflection of 40 mm. Both the springs have same free length, solid length and are subjected to equal maximum shear stress of 850 MPa. The spring index for both the springs is 6.
[Ans. d_1 = 8 mm ;d_2 = 6 mm ; n = 4]

4. The free end of a torsional spring deflects through 90° when subjected to a torque of 4 N-m. The spring indexis 6. Determine the coil wire diameter and number of turns with the following data :
Modulus of rigidity = 80 GPa ; Modulus of elasticity = 200 GPa; Allowable stress = 500 MPa.
[Ans. 5 mm ; 26]

5. A flat spiral steel spring is to give a maximum torque of 1500 N-mm for a maximum stress of 1000 MPa.Find the thickness and length of the spring to give three complete turns of motion, when the stress decreases from 1000 to zero. The width of the spring strip is 12 mm. The Young's modulus for the material of the strip is 200 kN/mm^2. **[Ans. 1.225 mm ; 4.6 m]**

6. A semi-elliptical spring has ten leaves in all, with the two full length leaves extending 625 mm. It is 62.5 mm wide and 6.25 mm thick. Design a helical spring with mean diameter of coil 100 mm which will have approximately the same induced stress and deflection for any load. The Young's modulus for the material of the semi-elliptical spring may be taken as 200 kN/mm^2 and modulus of rigidity for the material of helical spring is 80 kN/mm^2.

7. A carriage spring 800 mm long is required to carry a proof load of 5000 N at the centre. The spring is made of plates 80 mm wide and 7.5 mm thick. If the maximum permissible stress for the material of the plates is not to exceed 190 MPa, determine :
1. The number of plates required, 2. The deflection of the spring, and 3. The radius to which the plates must be initially bent.
The modulus of elasticity may be taken as 205 kN/mm^2. **[Ans. 6 ; 23 mm ; 3.5 m]**

8. A semi-elliptical laminated spring 900 mm long and 55 mm wide is held together at the centre by a band 50 mm wide. If the thickness of each leaf is 5 mm, find the number of leaves required to carry a load of 4500 N. Assume a maximum working stress of 490 MPa.If the two of these leaves extend the full length of the spring, find the deflection of the spring. The

Young's modulus for the spring material may be taken as 210 kN/mm^2.
[Ans. 9 ; 71.8 mm]

9. A semi-elliptical laminated spring is made of 50 mm wide and 3 mm thick plates. The length between the supports is 650 mm and the width of the band is 60 mm. The spring has two full length leaves and five graduated leaves. If the spring carries a central load of 1600 N, find :
1. Maximum stress in full length and graduated leaves for an initial condition of no stress in the leaves.
2. The maximum stress if the initial stress is provided to cause equal stress when loaded.
3. The deflection in parts (1) and (2). **[Ans. 590 MPa ; 390 MPa ; 450 MPa ; 54 mm]**

10. A machine has to carry out punching operation at the rate of 10 holes/min. It does 6 N-m of work per sq mm of the sheared area in cutting 25 mm diameter holes in 20 mm thick plates. A flywheel is fitted to the machine shaft which is driven by a constant torque. The fluctuation of speed is between 180 and 200 r.p.m. Actual punching takes 1.5 seconds. Frictional losses are equivalent to 1/6 of the workdone during punching. Find:
(*a*) Power required to drive the punching machine, and
(*b*) Mass of the flywheel, if radius of gyration of the wheel is 450 mm.

11. A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 r.p.m. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If thefluctuation of speed is not to exceed 1 per cent and the maximum centrifugal stress in the flywheel isto be 5.5 MPa, estimate the mean diameter and the cross-sectional area of the rim. The material of therim has a density of 7200 kg / m^3. **[Ans. 1.464 m ; 0.09 m^2]**

12. Design a cast iron flywheel for a four stroke cycle engine to develop 110 kW at 150 r.p.m. The work done in the power stroke is 1.3 times the average work done during the whole cycle. Take the meandiameter of the flywheel as 3 metres. The total fluctuation of speed is limited to 5 per cent of the meanspeed. The material density is 7250 kg / m^3. The permissible shear stress for the shaft material is 40 MPa and flexural stress for the arms of the flywheel is 20 MPa.

13. A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm^2 of sheared area. Determine the momentof inertia of the flywheel if the punching takes one-tenth of a second and the r.p.m. of the flywheelvaries from 160 to 140.

14. A punch press is fitted with a flywheel capable of furnishing 3000 N-m of energy during quarter of a revolution near the bottom dead centre while blanking a hole on sheet metal. The maximum speed ofthe flywheel during the operation is 200 r.p.m. and the speed decreases by 10% during the cuttingstroke. The mean radius of the rim is 900 mm. Calculate the approximate mass of the flywheel rimassuming that it contributes 90% of the energy requirements.