

UNIT – 2

CONVECTION

2.1. Convection Heat Transfer-Requirements

The heat transfer by convection requires a solid-fluid interface, a temperature difference between the solid surface and the surrounding fluid and a motion of the fluid. The process of heat transfer by convection would occur when there is a movement of macro-particles of the fluid in space from a region of higher temperature to lower temperature.

2.2. Convection Heat Transfer Mechanism

Let us imagine a heated solid surface, say a plane wall at a temperature T_w placed in an atmosphere at temperature T_∞ . Fig. 2.1 Since all real fluids are viscous, the fluid particles adjacent to the solid surface will stick to the surface. The fluid particle at A, which is at a lower temperature, will receive heat energy from the plate by conduction. The internal energy of the particle would increase and when the particle moves away from the solid surface (wall or plate) and collides with another fluid particle at B which is at the ambient temperature, it will transfer a part of its stored energy to B. And, the temperature of the fluid particle at B would increase. This way the heat energy is transferred from the heated plate to the surrounding fluid. Therefore the process of heat transfer by convection involves a combined action of heat conduction, energy storage and transfer of energy by mixing motion of fluid particles.

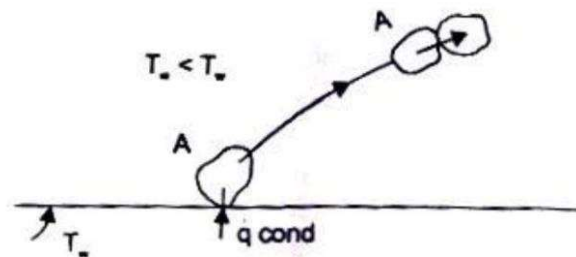


Fig. 2.1 Principle of heat transfer by convection

2.3. Free and Forced Convection

When the mixing motion of the fluid particles is the result of the density difference caused by a temperature gradient, the process of heat transfer is called natural or free convection.

When the mixing motion is created by an artificial means (by some external agent), the process of heat transfer is called forced convection. Since the effectiveness of heat transfer by convection depends largely on the mixing motion of the fluid particles, it is essential to have a knowledge of the characteristics of fluid flow.

2.4. Basic Difference between Laminar and Turbulent Flow

In laminar or streamline flow, the fluid particles move in layers such that each fluid particle follows a smooth and continuous path. There is no macroscopic mixing of fluid particles between successive layers, and the order is maintained even when there is a turn around a corner or an obstacle is to be crossed. If a time-dependent fluctuating motion is observed in directions which are parallel and transverse to the main flow, i.e., there is a random macroscopic mixing of fluid particles across successive layers of fluid flow, the motion of the fluid is called 'turbulent flow'. The path of a fluid particle would then be zigzag and irregular, but on a statistical basis, the overall motion of the macro particles would be regular and predictable.

2.5. Formation of a Boundary Layer

When a fluid flows over a surface, irrespective of whether the flow is laminar or turbulent, the fluid particles adjacent to the solid surface will always stick to it and their velocity at the solid surface will be zero, because of the viscosity of the fluid. Due to the shearing action of one fluid layer over the adjacent layer moving at the faster rate, there would be a velocity gradient in a direction normal to the flow.

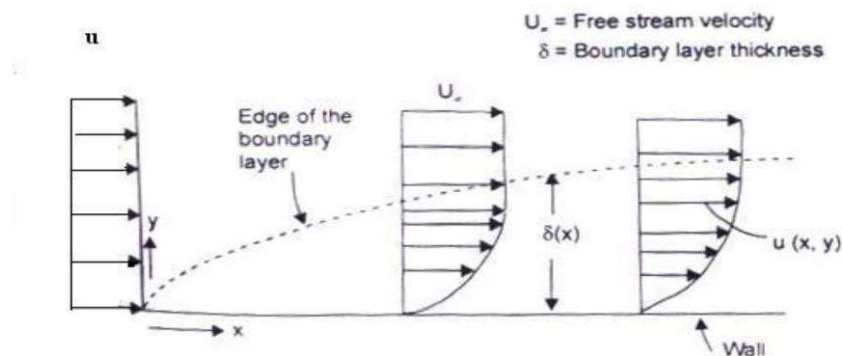


Fig 2.2: sketch of a boundary layer on a wall

Let us consider a two-dimensional flow of a real fluid about a solid (slender in cross-section) as shown in Fig. 2.2. Detailed investigations have revealed that the velocity of the fluid

particles at the surface of the solid is zero. The transition from zero velocity at the surface of the solid to the free stream velocity at some distance away from the solid surface in the V-direction (normal to the direction of flow) takes place in a very thin layer called 'momentum or hydrodynamic boundary layer'. The flow field can thus be divided in two regions:

(i) A very thin layer in the vicinity of the body where a velocity gradient normal to the direction of flow exists, the velocity gradient du/dy being large. In this thin region, even a very small Viscosity of the fluid exerts a substantial Influence and the shearing stress $\tau = \mu du/dy$ may assume large values. The thickness of the boundary layer is very small and decreases with decreasing viscosity.

(ii) In the remaining region, no such large velocity gradients exist and the Influence of viscosity is unimportant. The flow can be considered frictionless and potential.

2.6. Thermal Boundary Layer

Since the heat transfer by convection involves the motion of fluid particles, we must superimpose the temperature field on the physical motion of fluid and the two fields are bound to interact. It is intuitively evident that the temperature distribution around a hot body in a fluid stream will often have the same character as the velocity distribution in the boundary layer flow. When a heated solid body IS placed in a fluid stream, the temperature of the fluid stream will also vary within a thin layer in the immediate neighborhood of the solid body. The variation in temperature of the fluid stream also takes place in a thin layer in the neighborhood of the body and is termed 'thermal boundary layer'. Fig. 2.3 shows the temperature profiles inside a thermal boundary layer.

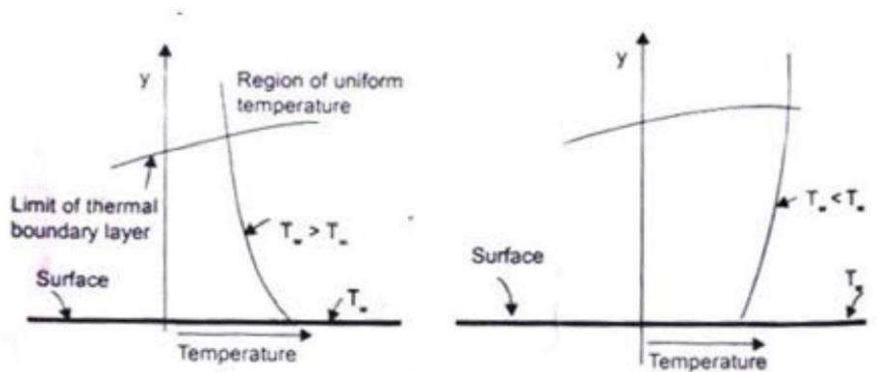


Fig2.3: The thermal boundary layer

2.7. Dimensionless Parameters and their Significance

The following dimensionless parameters are significant in evaluating the convection heat transfer coefficient:

(a) *The Nusselt Number (Nu)*-It is a dimensionless quantity defined as hL/k , where h = convective heat transfer coefficient, L is the characteristic length and k is the thermal conductivity of the fluid. The Nusselt number could be interpreted physically as the ratio of the temperature gradient in the fluid immediately in contact with the surface to a reference temperature gradient $(T_s - T_\infty)/L$. The convective heat transfer coefficient can easily be obtained if the Nusselt number, the thermal conductivity of the fluid in that temperature range and the characteristic dimension of the object is known.

Let us consider a hot flat plate (temperature T_w) placed in a free stream (temperature $T_\infty < T_w$). The temperature distribution is shown in Fig. 2.4. Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q}/A = h(T_w - T_\infty)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty)$$

$$= k \frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t}$$

$$Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

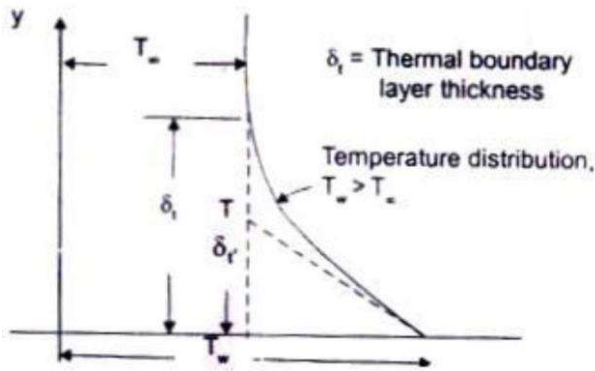


Fig. 2.4 Temperature distribution in a boundary layer: Nusselt modulus

The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface ($y = 0$), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k \left(\frac{dT}{dy} \right)_{y=0} \quad \therefore h = \frac{\left(-k \frac{dT}{dy} \right)_{y=0}}{(T_w - T_\infty)}$$

Since the magnitude of the temperature gradient in the fluid will remain the same, irrespective of the reference temperature, we can write $dT = d(T - T_w)$ and by introducing a characteristic length dimension L to indicate the geometry of the object from which the heat flows, we get

$$\begin{aligned} \frac{hL}{k} &= \frac{\left(\frac{dT}{dy} \right)_{y=0}}{(T_w - T_\infty)/L}, \text{ and in dimensionless form,} \\ &= \left(\frac{d(T_w - T)/(T_w - T_\infty)}{d(y/L)} \right)_{y=0} \end{aligned}$$

(b) *The Grashof Number (Gr)*-In natural or free convection heat transfer, the motion of fluid particles is created due to buoyancy effects. The driving force for fluid motion is the body force arising from the temperature gradient. If a body with a constant wall temperature T_w is exposed to a quiescent ambient fluid at T_∞ , the force per unit volume can be written as $\rho g \beta (T_w - T_\infty)$ where ρ = mass density of the fluid, β = volume coefficient of expansion and g is the acceleration due to gravity.

The ratio of inertia force \times Buoyancy force/(viscous force)² can be written as

$$\text{Gr} = \frac{(\rho V^2 L^2) \times \rho g \beta (T_w - T_\infty) L^3}{(\mu V L)^2}$$

$$= \frac{\rho^2 g \beta (T_w - T_\infty) L^3}{\mu^2} = g \beta L^3 (T_w - T_\infty) / \nu^2$$

The magnitude of Grashof number indicates whether the flow is laminar or turbulent. If the Grashof number is greater than 10^9 , the flow is turbulent and for Grashof number less than 10^8 , the flow is laminar. For $10^8 < \text{Gr} < 10^9$, It is the transition range.

(c) *The Prandtl Number (Pr)* - It is a dimensionless parameter defined as

$$\text{Pr} = \mu C_p / k = \nu / \alpha$$

Where μ is the dynamic viscosity of the fluid, ν = kinematic viscosity and α = thermal diffusivity.

This number assumes significance when both momentum and energy are propagated through the system. It is a physical parameter depending upon the properties of the medium. It is a measure of the relative magnitudes of momentum and thermal diffusion in the fluid: That is, for $\text{Pr} = 1$, the rate of diffusion of momentum and energy are equal which means that the calculated temperature and velocity fields will be similar, the thickness of the momentum and thermal boundary layers will be equal. For $\text{Pr} \ll 1$ (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa. The product of Grashof and Prandtl number is called Rayleigh number. Or, $\text{Ra} = \text{Gr} \times \text{Pr}$.

2.8. Evaluation of Convective Heat Transfer Coefficient

The convective heat transfer coefficient in free or natural convection can be evaluated by two methods:

- (a) Dimensional Analysis combined with experimental investigations
- (b) Analytical solution of momentum and energy equations in the boundary layer.

Dimensional Analysis and Its Limitations

Since the evaluation of convective heat transfer coefficient is quite complex, it is based on a combination of physical analysis and experimental studies. Experimental observations become necessary to study the influence of pertinent variables on the physical phenomena.

Dimensional analysis is a mathematical technique used in reducing the number of experiments to a minimum by determining an empirical relation connecting the relevant variables and in grouping the variables together in terms of dimensionless numbers. And, the method can only be applied after the pertinent variables controlling the phenomenon are identified and expressed in terms of the primary dimensions. (Table 1.1)

In natural convection heat transfer, the pertinent variables are: h , ρ , k , μ , C_p , L , (ΔT) , β and g . Buckingham's method provides a systematic technique for arranging the variables in dimensionless numbers. It states that the number of dimensionless groups, π 's, required to describe a phenomenon involving 'n' variables is equal to the number of variables minus the number of primary dimensions 'm' in the problem.

In SI system of units, the number of primary dimensions are 4 and the number of variables for free convection heat transfer phenomenon are 9 and therefore, we should expect $(9 - 4) = 5$ dimensionless numbers. Since the dimension of the coefficient of volume expansion, β , is θ^{-1} , one dimensionless number is obviously $\beta(\Delta T)$. The remaining variables are written in a functional form:

$$\phi(h, \rho, k, \mu, C_p, L, g) = 0.$$

Since the number of primary dimensions is 4, we arbitrarily choose 4 independent variables as primary variables such that all the four dimensions are represented. The selected primary variables are: g , k , L . Thus the dimensionless group,

$$\pi_1 = \rho^a g^b k^c L^d h = (ML^{-3})^a (LT^{-2})^b (MLT^{-3}\theta^{-1}) = M^0 L^0 T^0 \theta^0$$

Equating the powers of M, L, T, θ on both sides, we have

$$\left. \begin{array}{l} M : a + c + 1 = 0 \\ L : -3a + b + c + d = 0 \\ T : -2b - 3c - 3 = 0 \\ \theta : -c - 1 = 0 \end{array} \right\} \begin{array}{l} \text{Upon solving them,} \\ \\ \\ \text{Up on solving them,} \end{array}$$

$$c = 1, b = a = 0 \text{ and } d = 1.$$

and $\pi_1 = hL/k$, the Nusselt number.

The other dimensionless number

$\pi_2 = \rho^a g^b k^c L^d C_p = (ML^{-3})^a (LT^{-2})^b (MLT^{-3} \theta^{-1})^c (L)^d (MT^{-1} \theta^{-1}) = M^0 L^0 T^0 \theta^0$ Equating the powers of M, L, T and θ and upon solving, we get

$$\pi_3 = \mu^2 / \rho^2 g L^3$$

By combining π_2 and π_3 , we write $\pi_4 = [\pi_2 \times \pi_3]^{1/2}$

$$= \left[\rho^2 g L^3 C_p^2 / k^2 \times \mu^2 / g L^3 \right]^{1/2} = \frac{\mu C_p}{k} \text{ (the Prandtl number)}$$

By combining π_3 with $(\beta \Delta T)$, we have $\pi_5 = (\beta \Delta T) * \frac{1}{\pi_3}$

$$= \beta(\Delta T) \times \frac{\rho^2 g L^3}{\mu^2} = g \beta(\Delta T) L^3 / \nu^2 \text{ (the Grashof number)}$$

Therefore, the functional relationship is expressed as:

$$\phi(\text{Nu}, \text{Pr}, \text{Gr}) = 0; \text{ Or, } \text{Nu} = \phi_1(\text{Gr Pr}) = \text{Const} \times (\text{Gr} \times \text{Pr})^m \quad (2.1)$$

and values of the constant and 'm' are determined experimentally.

Table 2.1 gives the values of constants for use with Eq. (2.1) for isothermal surfaces.

Table 2.1 Constants for use with Eq. 2.1 for Isothermal Surfaces

<i>Geometry</i>	$G_{r_f} Pr_f$	<i>C</i>	<i>m</i>
Vertical planes and cylinders	$10^4 - 10^9$	0.59	1/4
	$10^9 - 10^{13}$	0.021	2/5
	$10^9 - 10^{13}$	0.10	1/3
Horizontal cylinders	$0 - 10^{-5}$	0.4	0
	$10^4 - 10^9$	0.53	1/4
	$10^9 - 10^{12}$	0.13	1/3
	$10^{10} - 10^{-2}$	0.675	0.058
	$10^{-2} - 10^2$	1.02	0.148
	$10^2 - 10^4$	0.85	0.188
	$10^4 - 10^7$	0.48	1/4
	$10^7 - 10^{12}$	0.125	1/3
	Upper surface of heated plates or lower surface of cooled plates	$8 \times 10^6 - 10^{11}$	0.15
- do -	$2 \times 10^4 - 8 \times 10^6$	0.54	1/4
Lower surface of heated plates or upper surface of cooled plates	$10^5 - 10^{11}$	0.27	1/4
Vertical cylinder height = diameter characteristic length = diameter	$10^4 - 10^6$	0.775	0.21
Irregular solids, characteristic length = distance the fluid particle travels in boundary layer	$10^4 - 10^9$	0.52	1/4