SOLVED PROBLEMS ON IDEAL AND REAL GASES

Problem 5.1

A vessel of volume 0.3 m^3 contains 15 kg of air at 303 K. Determine the pressure exerted by the air using

1. Perfect gas equation

2. Van der Waals equation

3. Generalised compressibility chart.

Take critical temperature of air is 132.8 K and critical pressure of air is 37.7 bar.

Given data:

Volume, $V = 0.3\ m^3$

Mass, $m = 15\ kg$

Temperature, $T = 303\ K$

Critical temperature, $(T_c) = 132.8\ K$

Critical pressure, $(p_c) = 37.7\ bar = 37.7 \times 100 = 3770\ kN/m^2$

Solution:

1. Perfect gas equation:

$pV = mRT$

$$p = \frac{mRT}{V}$$

$$p = \frac{15 \times 0.287 \times 303}{0.3}$$

[∵ R for air is 0.287 kJ/kgK and 1 N/m^2 = 1 Pa]

$p =$ **4348.05 kPa** **Ans.**

2. Van der Waals equation:

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

$$a=\frac{27R^2(T_c)^2}{64p_c}=\frac{27\times(0.287)^2\times(132.8)^2}{64\times3770}=0.163$$

We know that

$$b=\frac{RT_c}{8p_c}=\frac{0.287\times132.8}{8\times3770}=1.26\times10^{-3}$$

Specific volume, $v=\frac{\text{Volume}}{\text{Mass}}=\frac{V}{m}=\frac{0.3}{15}=0.02\ m^3/kg$

Substituting a, b and v values in Van der Waals Equation

$$\left(p+\frac{0.163}{(0.02)^2}\right)(0.02-1.26\times10^{-3})=0.287\times303$$

$$p=\mathbf{4232.9\ kN/m^2}$$ **Ans.**

3. Generalised compressibility chart:

Reduced temperature and reduced specific volume can be calculated as follows:

$$T_r=\frac{T}{T_c}=\frac{303}{132.8}=2.28$$

$$v_r=\frac{v}{v_c}=\frac{v}{\frac{RT_c}{p_c}}=\frac{v\,p_c}{RT_c}=\frac{0.02\times3770}{0.287\times132.8}=1.98$$

The reduced temperature is 2.28 and reduced specific volume is 1.98. Both intersect at one point. Mark this point on compressibility chart. From chart, corresponding Z value can be read as 0.99.

We know that compressibility factor, $Z = \frac{pv}{RT}$

$$0.99 = \frac{p \times 0.02}{0.287 \times 303}$$

$$\therefore \quad p = 4304.57 \ kN/m^2 \qquad \textbf{Ans}$$

Problem 5.2

The gas neon has a molecular weight of 20.183 and its critical temperature, pressure and volume are 46 K, 2.5 MPa and 0.05 m3/kmol. Reading from a compressibility chart for a reduced pressure of 2 and a reduced temperature of 1.2, the compressibility factor Z is 0.75. What are the corresponding specific volume, pressure, temperature and reduced volume?

Given data:

Molecular weight of neon = 20.183

Critical temperature, $T_c = 46\ K$

Critical pressure, $p_c = 2.5\ MPa$

Critical volume, $v_c = 0.05\ m^3/kmol$

$T_r = 1.2$

$P_r = 2$

$Z = 0.75$

Solution:

We know that $p_r = \frac{p}{p_c} = 2$

$\therefore$ Pressure, $p = p_r \times p_c = 2 \times 2.5 = \mathbf{5\ MPa}$ **Ans.**

We know that $T_r = \frac{T}{T_c} = 1.2$

$\therefore$ Temperature, $T = T_r \times T_c = 1.2 \times 46 = \mathbf{55.2\ K}$ **Ans.**

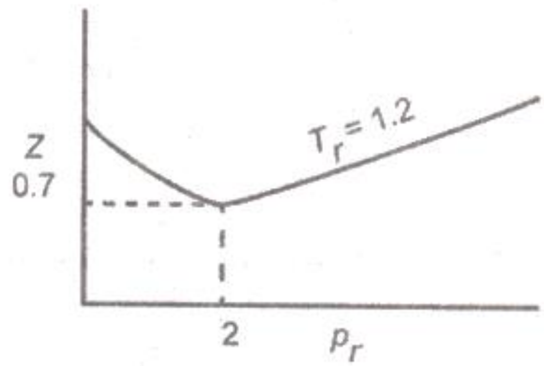

Figure 5.6

We know that

$$pv = ZRT$$

$$\therefore \quad v = \frac{ZRT}{p}$$

Also, gas constant, $R = \frac{\bar{R}}{M} = \frac{8.314}{20.183} = 0.412\ kJ/kgK$

$\left[\because \bar{R} = 8.314 kJ / kmol\ K \text{ and } M = 20.183 kg\right.$

$$\therefore \quad v = \frac{0.75 \times 0.412 \times 55.2}{5 \times 10^3} = \mathbf{0.00341\ m^3/kg}$$ **Ans.**

Volume ratio, $v_r = \frac{v}{v_c} = \frac{0.00341 \times 20.183}{0.05} = \mathbf{1.38}$ **Ans.**

Problem 5.3

Compute the specific volume of steam at 0.9 bar and 570 K using Van der Waals equation. Take critical temperature of steam as 647.3 K and critical pressure as 220.9 bar.

Given data:

Pressure, $p = 0.9\ bar = 0.9 \times 100\ kN/m^2 = 90\ kPa$

$[\because 1\ bar = 100\ kN/m^2 = 100\ kPa]$

Temperature, $T = 570\ K$

Critical temperature, $T_c = 647.3\ K$

Critical pressure, $p_c = 220.9\ bar = 220.9 \times 100 = 22090\ kPa$

Solution: We know that Van der Waals equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where $a = \dfrac{27R^2 (T_c)^2}{64\,p_c}$

where $R = \dfrac{\text{Universal gas constant}}{\text{Molecular weight of steam}}$

Molecular weight of steam (H_2O), $M = 2\times1 + 16 = 18\ kg/kmol$

$$\therefore\ R = \frac{8.314}{18} = 0.462\ kJ/kgK$$

$$a = \frac{27\times(0.462)^2\times(647.3)^2}{64\times22090} = 1.71$$

We know that $b = \dfrac{RT_c}{8\,p_c} = \dfrac{0.462\times647.3}{8\times22090} = 1.69\times10^{-3}$

Substituting a, b and pressure and temperature values in Van der Waals equation,

$$\left(90 + \frac{1.71}{v^2}\right)\times\left(v - 1.69\times10^{-3}\right) = 0.462 \times 570$$

$$\left(90 + \frac{1.71}{v^2}\right)\left(v - 1.69\times10^{-3}\right) = 263.34$$

$$\left(90v^2 + 1.71\right)\left(v - 1.69\times10^{-3}\right) = 263.34v^2$$

$$90v^3 - 0.1521v^2 + 1.71v - 0.0028899 = 263.34v^2$$

$$90v^3 - 263.4921v^2 + 1.71v - 0.0028899 = 0$$

By trial and error method, specific volume $v = \mathbf{0.0018\ m^3/kg}$ **Ans.**

Problem 5.4

A perfect gas of 0.2 kg has a pressure of 300 kPa, a temperature of 40° C and a volume of 0.06 m³. The gas undergoes an irreversible adiabatic process to a final pressure of 400kPa and final volume of 0.15 m³, work done on the gas is 50 kJ. Find C_p and C_v.

Given data:

$m = 0.2\ kg$

$p_1 = 300\ kPa$

$T_1 = 40°C = 40 + 273 = 313\ K$

$v_1 = 0.06\ m^3$

$p_2 = 400\ kPa$

$v_2 = 0.15\ m^3$

$W = -50\ kJ$ [Work done on the gas is negative value]

Solution:

We know that the perfect gas equation is written as

$$PV=MRT$$

$$p_1v_1 = mRT_1$$

$$R = \frac{p_1v_1}{mT_1} = \frac{300\times0.06}{0.2\times313} = 0.288 \ kJ/kgK$$

Similarly, $p_2v_2 = mRT_2$

$$T_2 = \frac{p_2v_2}{mR} = \frac{400\times0.15}{0.2\times0.288} = 1041.67 \ K$$

Heat transfer, $Q = W + \Delta U$

$$Q = W + m\,C_v\,(T_2 - T_1) \qquad [\because \ \Delta U = m\,C_v\,(T_2 - T$$

$$Q = -50 + 0.2\times C_v\,(1041.67 - 313)$$

For adiabatic process,

$$Q = 0$$

$$\therefore \quad 0 = -50 + 0.2\times C_v\,(1041.67 - 313)$$

C_v = **0.343** ***kJ/kgK*** **Ans.**

We know that $R = C_p - C_v$

$$0.288 = C_p - 0.343$$

C_p = **0.631** ***kJ/kgK*** **Ans.**