

ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY
Approved by AICTE & Affiliated to Anna University
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DEPARTMENT OF MECHANICAL ENGINEERING



NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

REGULATION 2021

**UNIT II: STATICS OF RIGID BODIES IN TWO
DIMENSIONAL**

Equilibrium of Rigidbodies – support Reactions

Beam:

A beam is horizontal structural member which carries a load transverse (perpendicular) to its axis and transfers the load through support reactions to supporting columns or walls

Frame:

A structure made up of up of several members riveted or welded together is known as frame.

Support Reactions of Beam:-

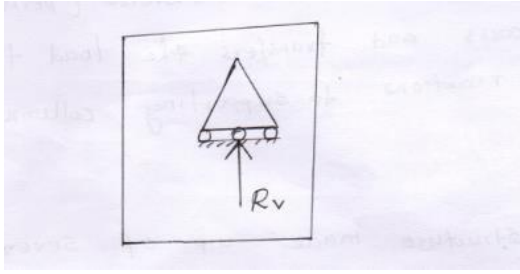
The force of resistance exerted by the support on the beam is called support reaction.

Types of support

1. Roller support
2. Hinged support
3. Fixed support

1. Roller support:

It consist of the rollers as the bottom. It has only one vertical reaction.

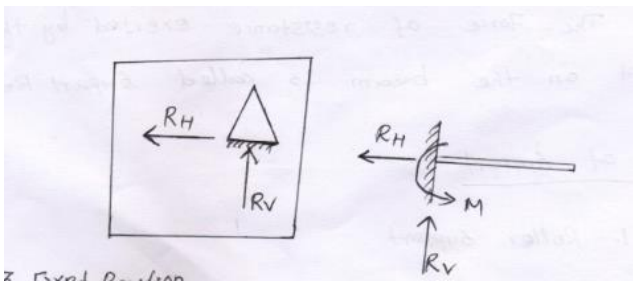


2. Hinged support:

It resists the horizontal and vertical moment. It has two Reaction

(i) Horizontal reaction

(ii) Vertical reaction



3. Fixed reaction:

It is the Stronged support. This support has following reaction

(i) Vertical reaction

(ii) Horizontal reaction

(iii) Rotational reaction (moment)

Types of load:

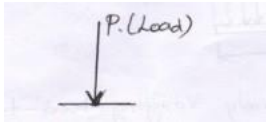
1. Point load

2. Uniformly distributed load (UDL)

3. Uniformly varying load (UVL)

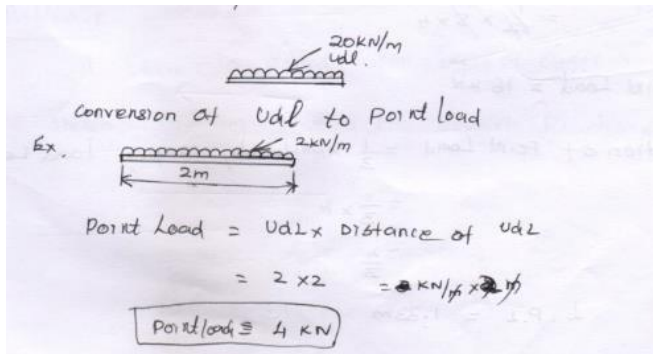
1. Point load:

Load which is acting at a particular point (i.e.) point load;



2. Uniformly distributed Load

The load which is spread over a beam in such a manner that each unit length at the beam carries same intensity of the load is called uniformly distributed load.



$$\text{Point load} = \text{udl} \times \text{distance of udl}$$

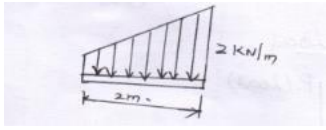
$$= 2 \times 2$$

$$\text{Point} = 4 \text{KN}$$

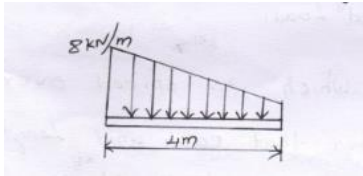
$$\text{Location of point} = \frac{\text{uniformly disbuted load length}}{2}$$

3. Uniformly varying load

A load which is varying from the particular point along particular length is called uniformly varying load



Conversion of uniformly varying load to point load



Point load = $1/2 \times$ *uniformly varying load* \times *length of uniformly load*

$$= 1/2 \times 8 \times 4$$

$$\text{Point load} = 16\text{KN}$$

Location of point load = $\frac{1}{3} \times$ *uniformly varying load length*

$$= \frac{1}{3} \times 4$$

$$= 4/3$$

$$\text{L.P.L} = 1.33\text{m}$$

Procedure for solving the support reaction problem

1. Sum of all the horizontal force is zero $\sum F_H = 0$

To find R_H (R_{HB})

2. Sum of all the vertical force is zero $\sum FV = 0$

To find $R_{VA} + R_{VB}$

3. Take moment of force about A ($\sum MA=0$ To $R_{FV}=0$ to find R_{VA})

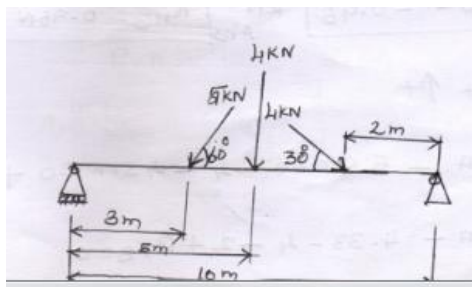
(OR)

4. Substitute R_{VB} in $\sum FV=0$ Eqn

To find R_{VA}

Problem-I

A beam is acted upon by a system of forces shown in fig. Find the support Reactions

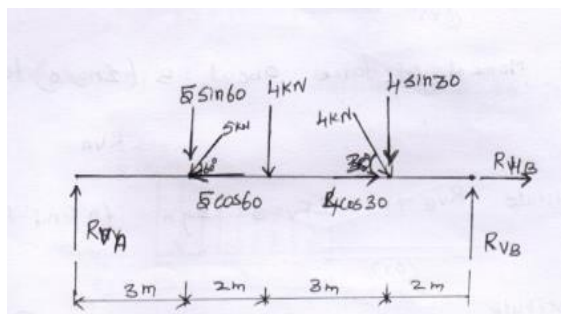


To find

Reaction at the support. R_V & R_{VB} , R_{HB}

Soln:

Free body diagram



$$\sum F_H = 0 \quad \begin{array}{c} + \\ \rightarrow \end{array} \quad \begin{array}{c} - \\ \leftarrow \end{array}$$

$$\sum F_H = -5 \cos 60 + 4 \cos 30 + R_{HB}$$

$$\sum F_H = 0.96 + R_{HB}$$

$$R_{HB} = -0.96 \text{KN}$$

$$R_{HB} = 0.96 \text{N}(\rightarrow)$$

$$\sum F_V = 0$$

$$\sum F_V = -R_{VA} - 5 \sin 60 - 4 - 4 \sin 30 + R_{VB} = 0$$

$$R_{VA} - 4.33 - 4 - 2 + R_{VB} = 0$$

$$R_{VA} + R_{VB} - 10.33 = 0$$

$$R_{VA} + R_{VB} = 10.33 \text{ -----} > (1)$$

Take moment of force about A

$$\sum M_A = 0$$

$$\sum M_A = (5 \sin 60 \times 3) + (4 \times 2) + (4 \sin 30 \times 8) + (R_{VB} \times 10) = 0$$

$$12.99 + 8 + 16 - 10 R_{VB} = 0$$

$$36.99 - 10 R_{VB} = 0$$

$$-10 R_{VB} = (-36.99)/(-10)$$

$$R_{VB} = \frac{-36.99}{-10}$$

$$\text{Ans: } R_{VB} = 3.69 \text{ N}$$

R_{VB} value sub in Eqn -----> (1)

$$R_{VA} + R_{VB} = 10.33$$

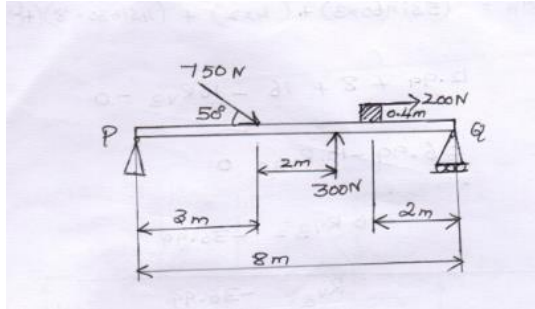
$$R_{VA} + 3.69 = 10.33$$

$$R_{VA} = 10.33 - 3.69$$

$$R_{VA} = 6.64 \text{N}$$

Problem 2

A beam is loaded as shown in fig find the magnitude direction and the location of the resultant of the system of forces.

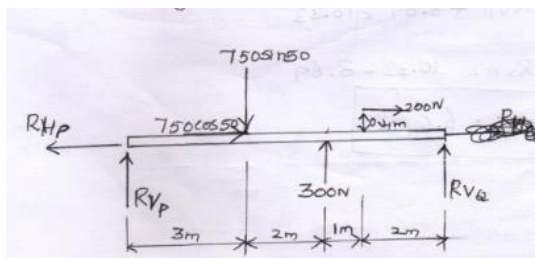


To find

1. Resultant force & direction
2. location of resultant force

Soln

Free body diagram



$$\sum F_V = 0 \quad \uparrow + \quad \downarrow -$$

$$-R_{HP} + 750 \cos 50 + 200 = 0$$

$$-R_{HP} + 482 + 200 = 0$$

$$-R_{HP} + 682 = 0$$

$$-R_{HP} = -682N$$



$$\sum F_V = 0 \quad + \quad -$$

$$R_{VP} - 750 \sin 50 + 300 + R_{VQ} = 0$$

$$R_{VP} + R_{VQ} - 574.53 + 300 = 0$$

$$R_{VP} + R_{VQ} = -300 + 574.53$$

$$R_{VP} + R_{VQ} = 274.53 \text{ N} \quad (1)$$

Take moment About 'p'

$$\sum M_P = 0$$

$$(750 \sin 50 \times 3) + (200 \times 0.4)(-R_{VQ} \times 8) = 0$$

$$1753.59 - 1500 + 80 - 8 R_{VQ} = 0$$

$$-8 R_{VQ} = -1753.59 + 1500 - 80$$

$$-8 R_{VQ} = 333.59$$

$$R_{VQ} = (-333.59)/(-8)$$

$$R_{VQ} = 41.9 \text{ N}$$

$$R_{VQ} = 41.69 \text{ N}$$

R_{VQ} value sub in Eqn(1)

$$R_{VP} + R_{VQ} = 274.53$$

$$R_{VP} + 41.69 = 274.53$$

$$R_{VP} = 274.53 + 41.69$$

$$R_{VP} = 232.83 \text{ N}$$

$$\text{Resultant } R = \sqrt{(\sum F_H)^2 + (\sum F_V)^2}$$

Resultant force consider $R = \sqrt{(\sum H_p)^2 + R V_p^2}$

Only hinged support $R = \sqrt{[682]^2 + [232.83]^2}$

$$R = 720N$$

Direction $\theta = \tan^{-1} \left[\frac{\sum V_p}{\sum V_H} \right]$

$$\theta = \tan^{-1} \left(\frac{232.83}{682} \right)$$

$$\theta = 18^\circ 50'$$

Location

By varignon's theorem

$$\sum M_p = R \times x$$

$$\sum M_p = 0 \quad +\downarrow \quad \uparrow-$$

$$\sum M_p = (750 \times \sin 50 \times 3) + (300 \times 5) + (200 \times 0.4) + (-R_{VQ} \times 8)$$

$$\sum M_p = 1753.59 - 1500 - 800 - 333.52$$

$$\sum M_p = 0.07 N.m$$

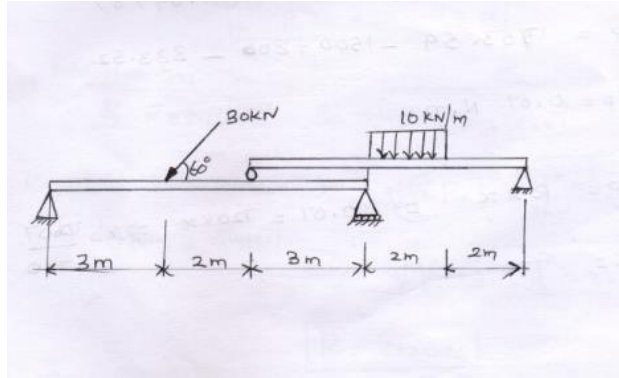
$$\sum M_p = R \times x \quad \Rightarrow 0.07 = 720 \times x \quad \Rightarrow x = 0.07/720$$

Problem: 3

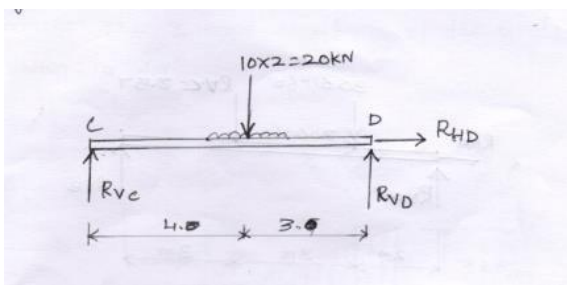
Two beams AB and CD are shown in fig A and D are hinged supports B and C are rollers supports.

(i) Sketch the free body diagram of the beam AB and determine the reaction at the support A and B.

(ii) Sketch the free body diagram of the beam CD and determine the reactions at the supports C and D.



Free diagram of beam CD



$$\sum F_H = 0 \rightarrow \leftarrow$$

$$R_{HD} = 0$$

$$R_{VC} - 20 + R_{VD} = 0$$

$$R_{VC} + R_{VD} = 20 \quad (2)$$

Take moment about c=0

$$\sum M_C = (20 \times 4) - (R_{VD} \times 7) = 0$$

$$80 - 7 R_{VD} = 0$$

$$R_{VD} = \frac{-80}{-7}$$

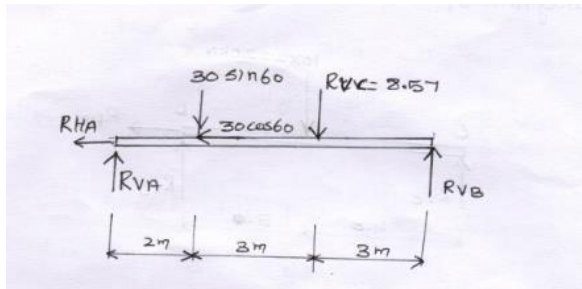
$$R_{VD} = 11.42 \text{ KN}$$

R_{VD} value sub in Eqn (1)

$$R_{VC} + 11.42 = 20 \Rightarrow R_{VC} = 20 - 11.42$$

$$R_{VC} = 8.57 \text{KN}$$

Free body diagram of beam AB



$$\Sigma F_H = 0 \quad \rightarrow \leftarrow$$

+ -

$$\Sigma F_H = R_{HA} - 30 \cos 60 = 0$$

$$R_{HA} - 15 = 0$$

$$R_{HA} = 15 \text{KN}$$

$$\Sigma F_V = 0 \quad \uparrow + \downarrow -$$

$$R_{VA} - 30 \sin 60 - 8.57 + R_{VB} = 0$$

$$R_{VA} + R_{VB} - 25.98 - 8.57 = 0$$

$$R_{VA} + R_{VB} = 34.55 \text{ (1)}$$

Take moment about A

$$\Sigma M_A = 0$$

$$\Sigma M_A = (30 \sin 60 \times 2) + (8.57 \times 5) + (R_{VB} \times 8) = 0$$

$$51.96 + 42.85 - 8 R_{VB} = 0$$

$$-8 R_{VB} = -94.81$$

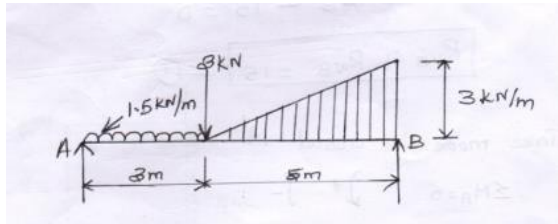
$$R_{VB} = 11.85 \text{KN}$$

$$R_{VA} + 11.85 = 34.85$$

$$R_{VA} = 22.69 \text{ KN}$$

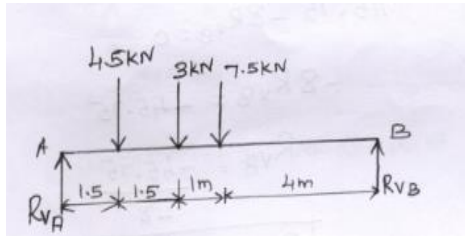
Problem:4

Determine the support reaction of the simply supported beam shown in fig



Soln:

Free body diagram



$$UDL \text{ to } PL = UDL \times \text{dis of VDL}$$

$$= 1.5 \times 3 = 4.5$$

$$\text{Length of UDL} = \frac{UDL}{2} = \frac{3}{2} = 1.5$$

$$UVL \text{ to } PL = \frac{1}{2} \times UVL \times \text{length of UVL}$$

$$= \frac{1}{2} \times 3 \times 5$$

$$PL = 7.5 \text{ KN}$$

$$\text{Length of PL} = \frac{1}{3} \times 3 = 1 \text{ m}$$

$$\Sigma F_H = 0 \rightarrow \leftarrow$$

$$\sum F_V = 0 \quad \downarrow + \quad \uparrow +$$

$$R_{VA} - 4.5 - 3 - 7.5 + R_{VB} = 0$$

$$+ R_{VA} - 15 = 0$$

$$R_{VA} + R_{VB} = 15 \quad \text{-----}(1)$$

Take moment about A

$$\sum M_A = 0 \quad \downarrow + \quad \uparrow -$$

$$(4.5 \times 1.5) + (3 \times 3) + (7.5 \times 4) + (R_{VB}) = 0$$

$$45.75 - 8 R_{VB} = 0$$

$$-8 R_{VB} = -45.75$$

$$R_{VB} = \frac{-45.75}{-8}$$

$$R_{VB} = 5.71 \text{KN}$$

$$R_{VB} + R_{VA} = 15 \quad \text{-----}(1)$$

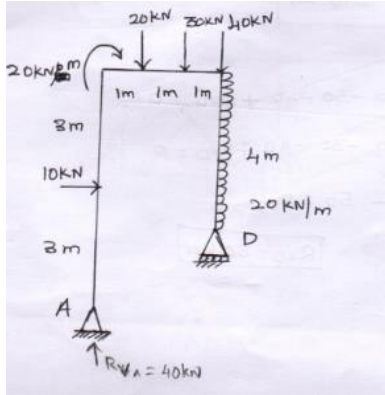
$$R_{VA} + 5.71 = 15$$

$$R_{VA} = 15 - 5.71$$

$$R_{VA} = 9.28 \text{KN}$$

Problem 5

Find the reactions for the frame shown in fig. the line of action of 40k passes through the point A.

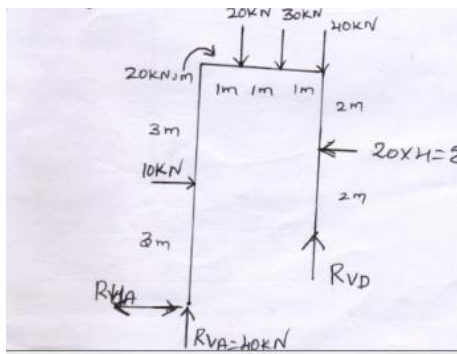


Given

Line of action at point A $R_{VA}=40\text{KN}$

Soln:

Free body diagram



$$\sum FH=0 \quad \rightarrow \leftarrow$$

$$-R_{HA} + 10 - 80 = 0$$

$$-R_{HA} - 70 = 0$$

$$-R_{HA} = 70$$

$$R_{HA} = -70\text{KN}$$

$$R_{HA} = -70\text{KN}$$

$$\sum FV=0$$

$$R_{VA} - 20 - 30 - 40 + R_{VD} = 0$$

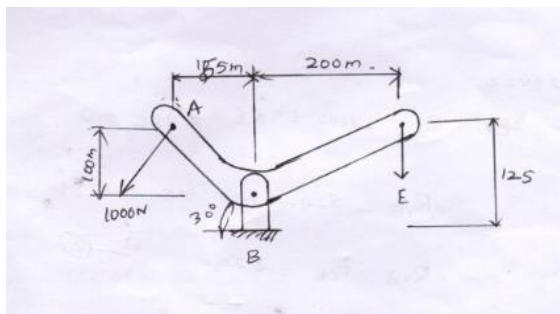
$$40 - 20 - 30 - 40 + R_{VD} = 0$$

$$-50 + R_{VD} = 0$$

$$R_{VD} = 50 \text{ KN}$$

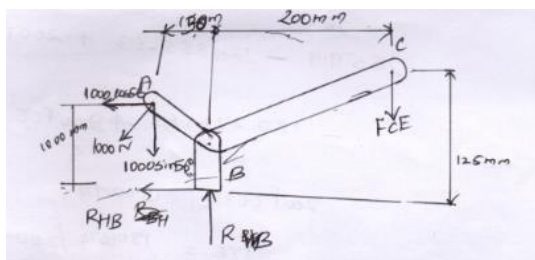
Problem:

Lever ABC of machine component is subjected to a force 1000n at point A as shown in fig. compute the reaction act B and the force at CE.



Soln:

Free body diagram



$$\theta = \tan^{-1}\left(\frac{100}{500}\right)$$

$$\theta = 33^{\circ} 41'$$

$$\sum F_H = 0$$

$$-R_{HB} - 1000 \cos 56 = 0$$

$$-R_{HB} = 1000 \cos 56$$

$$R_{HB} = -554.7 \text{ N} \quad (1)$$

$$\sum F_V = 0$$

$$R_{VB} - 1000 \sin 56 - F_{CE} = 0$$

$$R_{VB} = -829 - F_{CE} = 829 - - - - - (2)$$

$$M_B = [-1000 \cos 56 \times 100] + [-1000 \sin 56 \times 150] + [F_{CE}] = 0$$

$$-55919 - 124355.63 + 200 F_{CE} = 0$$

$$-180274.63200 F_{CE} = 0$$

$$200 F_{CE} = 180274$$

$$F_{CE} = 180274/200$$

$$F_{CE} = 901.373 \text{ N}$$

$$\text{Resultant } R_B = \sqrt{(R_{VB})^2 + (R_{HB})^2}$$

$$= \sqrt{(1730)^2 + (-554)^2}$$

$$R_B = 1817 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_{VB}}{R_{HB}}\right) = \tan^{-1}\left(\frac{1730}{554}\right)$$

$$\theta = 72^\circ 14'$$

To find

Reaction of support

Soln

Sum of horizontal forces $\sum F_H$

$$\sum F_H = 0$$

$$-RH_A - 10 \cos 60 = 0$$

$$-RH_A = -10 \cos 60 = -5$$

$$RH_A = 5N$$

Sum of vertical forces $\sum F_V$

$$\sum F_V = 0$$

$$R_{VA} = 10 \sin 60 - 8 = 0$$

$$R_{VA} = -8.66 - 8 = 0$$

$$R_{VA} - 16.66 = 0$$

$$R_{VA} = 16.66N$$

Moment About 'o'

$$M_o = 0$$

$$M_o = M = [10 \sin 60 \times 4] + 20 + [8 \times 8] = 0$$

$$M + 34.64 + 20 + 64 = 0$$

$$M + 118.64 = 0$$

$$M = 118.64KN.m$$

$$M = 118.64 m$$

