UNIT I

Mechanics

1.1 Centre of mass of two particle system

Let us consider a system consisting of two particles of mass m_1 and m_2. P_1 and P_2 are the position at time t and r_1 and r_2 are the corresponding distances from the origin o. velocity and acceleration

$$v_1 = \frac{dr_1}{dt} \quad \text{and } a_1 = \frac{dv_1}{dt}$$

$$v_2 = \frac{dr_2}{dt} \quad \text{and } a_1 = \frac{dv_2}{dt}$$

particle at P_1 has two forces

(i) A force F_{12} due to the particle at P_1.

(ii) A force F_{1e}, external force.

Resultant force $F_1 = F_{12} + F_{1e}$ -------- (1)

net force F_2 acting on the particle P_1

$F_2 = F_{21} + F_{2e}$ -------- (2)

By Newton's law $F_1 = m_1 a_1$ -------- (3)

$F_2 = m_2 a_2$ -------- (4)

Eqn (3) + (4)

$F_1 + F_2 = m_1 a_1 + m_2 a_2$ -------- (5)

Sub. Eqn (1) and (2) in (5)

$F_{12} + F_{1e} + F_{21} + F_{2e} = m_1 a_1 + m_2 a_2$ -------- (6)

By Newton's third law

Force F_{12} exerted by particle at P_2 is equal and opposite to F_{21} exerted by particle at P_1

$$F_{12} = -F_{21}$$

Eqn (6) ⇒

$$F = F_{1e} + F_{2e} = m_1a_1 + m_2a_2 \qquad \text{--------} (7)$$

Total mass of the system

$$M = m_1 + m_2$$

Net force acting on the system produces and acceleration a_{CM}. celled acceleration of the centre of mass of the system

$$F = M\, a_{CM} = m_1a_1 + m_2a_2 \qquad \text{--------} (8)$$

R_{CM} = position vector of centre of mass

$$\therefore a_{CM} = \frac{d^2R_{CM}}{dt^2} \qquad \text{--------} (9)$$

Sub (8) in (9)

$$\frac{d^2R_{CM}}{dt^2} = \frac{1}{M}\left(m_1\frac{d^2r_1}{dt^2} + m_2\frac{d^2r_2}{dt^2}\right)$$

$$\frac{d^2R_{CM}}{dt^2} = \frac{1}{M}\frac{d^2}{dt^2}(m_1r_1 + m_2r_2)$$

$$R_{CM} = \frac{1}{M}(m_1r_1 + m_2r_2)$$

$$R_{CM} = \frac{m_1r_1 + m_2r_2}{m_1 + m_2} \qquad \text{--------} (10) \quad [\therefore M = m_1 + m_2]$$

This is the expression foe centre of mass of system consisting of two particle.

<u>Centre of mass of system consisting of n particle</u>

Let $m_1, m_2, m_3, \ldots..$ mass of the particle with position vectors $r_1, r_2, r_3, \ldots..$

$$M = m_1 + m_2 + m_3 + m_4, \ldots..$$

Centre of mass

$$R_{CM} = \frac{m_1r_1 + m_2r_2 + m_3r_3 \ldots\ldots\ldots\ldots\; m_nr_n}{m_1 + m_2 + m_3 \ldots\ldots\ldots\ldots\; m_n}$$

$$\frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i r_i}{\mathrm{M}}$$

X coordinate and y coordinate of centre of mass

$$x = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 \ldots\ldots\ldots\ldots\ m_n x_n}{m_1 + m_2 + m_3 \ldots\ldots\ldots\ldots\ m_n}$$ and

$$y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 \ldots\ldots\ldots\ldots\ m_n y_n}{m_1 + m_2 + m_3 \ldots\ldots\ldots\ldots\ m_n}$$