Momentum of EM waves:

Momentum of EM waves can be found in terms of

- Energy
- Pointing vector

Momentum in terms of energy

We know

$$E = mc^2 \qquad ------(1)$$

Similarly the energy 'u' in terms of the effective mass of electromagnetic radiation can be written as

$$u = mc^2$$

$$m = \frac{u}{c^2} \qquad ------(2)$$

The momentum of the particle of mass 'm' and velocity 'v' is

$$\vec{p} = mv \qquad ------(3)$$

Substituting (2) in (3) we get

$$\vec{p} = \frac{u}{c^2}v \qquad ------(4)$$

If the EM wave is travelling along z-axis with velocity '*c*' it is represented as

$$c = c\hat{k}$$

Substituting for '*v*' in (4) we get

$$\vec{p} = \frac{u}{c^2}c\hat{k}$$

$$\vec{p} = \frac{u}{c^2} c\hat{k}$$

$$\vec{p} = \frac{u}{c}\hat{k} \qquad ------(5)$$

Magnitude of the momentum is

$$\vec{p} = \frac{u}{c}$$

This represents the momentum of EM waves in terms of energy 'u'.

Momentum in terms of pointing vector:

The pointing vector

$$\vec{S} = (\vec{E} \times \vec{H}) = uc\hat{k}$$

$$u\hat{k} = \frac{\vec{S}}{c} \qquad ------(6)$$

Substituting (6) in (5) we get

$$\vec{p} = \frac{1}{c}\frac{\vec{S}}{c}$$

$$\vec{p} = \frac{\vec{S}}{c^2}$$

since $c^2 = \frac{1}{\epsilon_0 \mu_0}$

We can write the above equation as

$$\vec{p} = \epsilon_0 \mu_0 \vec{S}$$

$$\vec{p} = \epsilon_0 \mu_0 (\vec{E} \times \vec{H})$$

The above equation represents the momentum of EM waves in terms of pointing vector.

Radiation pressure of electromagnetic waves:

When an EM waves strikes the surface, it exerts a force on the surface due to the change in momentum. The amount of force exerted per unit area on the surface due to the force is called radiation pressure.

We know the momentum

$$\vec{p} = \frac{u}{c}\hat{k}$$

The magnitude of momentum

$$\vec{p} = \frac{u}{c} \qquad ------(1)$$

Similarly we know the pointing vector

$$\vec{S} = uc\hat{k}$$

The magnitude of the pointing vector is

$$\vec{S} = uc \qquad ------(2)$$

According to pointing theorem, the electromagnetic energy passing normal to the surface per unit area and unit time is given by

$$\vec{S} = \frac{u}{At} \qquad ------(3)$$

Where 'A' is the area and 't' is the time.

Comparing equation (2) and (3) we get

$$uc = \frac{u}{At}$$

$$c = \frac{1}{At}$$

$$At = \frac{1}{c} \qquad ------(4)$$

Substituting (4) in (1) we get

$$\vec{p} = uAt \qquad ------(5)$$

According to Newton's law, the force 'F' acting on the surface is given by

$$F = \frac{p}{t} \qquad ------(6)$$

Substituting (5) in (6) we get,

$$F = \frac{uAt}{t}$$

$$F = uA \qquad ------(7)$$

We know the radiation pressure P_{rad} exerted on the surface is given by

$$P_{rad} = \frac{F}{A} \qquad ------(8)$$

Substituting (7) in (8) we get,

$$P_{rad} = \frac{uA}{A}$$

$$i.e \quad P_{rad} = u$$

Thus the radiation pressure of EM wave is equal to the energy of the striking EM wave.

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