

UNIFORM BENDING

DEFINATION:

If the beam is loaded uniformly on its both ends, bending of the beam forms an arc of a circle, The elevation is produced in the beam. This type of bending is known as Uniform Bending.

Theory of Uniform Bending:

Consider a beam AB on two knife edges C and D such that AC = BD = a.

The beam is loaded with equal weights W at each ends A and B.

The reaction on the knife edges are acting vertically upward.

$$\begin{aligned} \text{External Bending Moment} &= W(AF - CF) \\ &= W \times AC \\ &= Wa \text{ -----(1)} \end{aligned}$$

$$\text{Internal Bending Moment} = \frac{YI}{R} \text{ -----(2)}$$

Y – Youngs Modulus of the beam.

I- Geometrical Moment of Inertia of the Beam.

R- Radius of Curvature of the Beam.

In equilibrium position

External Bending Moment = External Bending Moment

$$Wa = \frac{YI}{R}$$

Let y is the elevation and l is the distance between the knife edges C & D. Then by

The property of Circles

$$CD = I$$

$$EF = Y$$

From the property of Circles

$$EF \times EG = CE \times ED \text{ ----- (4)}$$

$$EF (2R - EF) = CE^2$$

[Since CE = ED; EG = 2R - EF]

$$[EF = y \text{ and } CE = \frac{l}{2}]$$

$$y(2R - y) = \left(\frac{l}{2}\right)^2$$

$$2yR - y^2 = \left(\frac{l}{2}\right)^2$$

$$= \frac{l^2}{4}$$

$$2yR = \frac{l^2}{4}$$

$$y = \frac{l^2}{8R}$$

$$\frac{8y}{l^2} = \frac{1}{R}$$

$$\frac{1}{R} = \frac{8y}{l^2}$$

Substituting (5) in (3)

$$Wa = \frac{8y}{l^2} \frac{YI}{R}$$

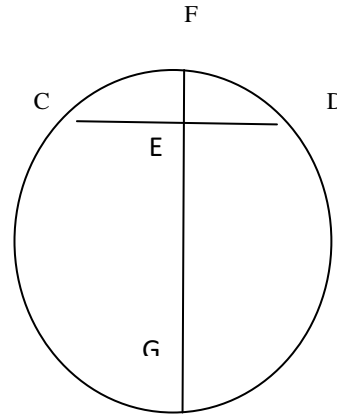
$$Y = \frac{Wal^2}{8Iy}$$

For Rectangular Cross Section

$$I = \frac{bd^3}{12}$$

Where b – breadth of beam

D – thickness of beam



$$W = Mg$$

$$\text{Then } Y = \frac{Mgal^2}{\frac{bd^3}{12}y}$$

$$Y = \frac{3Mgal^2}{ybd^3}$$

From which Young's Modulus of the beam is determined.