5.1 Dual Nature of Radiation (Light) and Matter (Particles) – Matter Waves

The universe is made of Radiation (light) and matter (particles). The light exhibits the dual nature (i.e.) it can behave both as a wave (interference, diffraction phenomenon) and as a particle (Compton Effect, photo-electric effect etc.). Since the nature loves symmetry, in 1924 Louis de- Broglie suggested that an electron or any other material particle must exhibit wave like properties in addition to particle nature.

5.1.1 *The waves associated with a material particle are called as Matter waves or De-Braglie wave.*

De-Broglie Wavelength

From the theory of light, considering a photon as a particle the total energy of the photon is given by

$$E = mc^2 \quad \text{.......................................(1)}$$

Where, m - mass of the particle

c - velocity of light

Considering the photon as a wave, the total energy is given by

$$E = hv \quad \text{.................. (2)}$$

Where, h - Planck's constant

v - frequency of radiation

From equations (1) and (2) we can write

$$E = mc^2 = hv \quad \text{........ (3)}$$

We know momentum = mass x velocity

$$P = mc$$

Equation (3) becomes

$$hv = pc$$

$$P = \frac{hv}{c}$$

Since $\frac{c}{v} = \lambda$ we can write $p = \frac{h}{\lambda}$

The wave length of photon $\lambda = \frac{h}{mv}$-----------------(4)

De-Broglie suggested that equation (4) can be applied both for photons and material particles. If *m* is the mass of the particle and 'v' is the velocity of the particle, then

Momentum $p = mv$.

$$\boxed{\text{De-Broglie wavelength } \lambda = \frac{h}{mv}}$$

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OTHER FORMS OF DE-BROGLIE WAVELENGTH :

i) De-Broglie wavelength in terms of Energy:

We know kinetic energy $E = \frac{1}{2}mv^2$

Multiplying by 'm' on both sides we get

$$Em = \frac{1}{2} v^2 m^2$$

(Or)

$$m^2v^2 = 2Em$$

$$mv = \sqrt{2Em}$$

$$\boxed{\textbf{de-Broglie wave length } \lambda = \frac{h}{\sqrt{2Em}}}$$

ii) de-broglie wavelength in terms of voltage :

If a charged particle of charge 'e' is accelerated through a potential difference 'V'
Then the kinetic energy = $\frac{1}{2}mv^2$---------------(1)

Also, we know that energy = eV--------------(2)

Equating (1) and (2)

$$\frac{1}{2}mv^2 = eV$$

Multiplying by 'm' on both sides we get

$$m^2v^2 = 2meV$$

$$m\,v = \sqrt{2meV}$$

substituting in mv

$$\lambda = \frac{h}{mv}$$

$$\boxed{\text{de-Broglie wave length } \lambda = \frac{\boldsymbol{h}}{\sqrt{\boldsymbol{2meV}}}}$$

iii)De-Broglie wavelength in terms of Temperature

When a particle like neutron is in thermal equilibrium at temperature T, then they possess Maxwell distribution of velocities.

Therefore kinetic energy E $= \frac{1}{2}\mathrm{m}v_{rms}^2$ ----------------(1)

Where v_{rms} is the root mean square velocity of the particle

Also ,we know energy $= \frac{3}{2}K_B$ T------------------------(2)

K_B - Boltzmann constant.

Equating (1) and (2) we get

$$\frac{1}{2}\mathrm{m}v^2 = \frac{3}{2}K_B\,\mathrm{T}$$

$$m^2v^2 = 3\mathrm{m}K_B\,\mathrm{T}$$

$$\mathrm{mv} = \sqrt{3\mathrm{m}K_B\,\mathrm{T}}$$

$$\boxed{\text{De-Broglie wavelength } \lambda = \frac{h}{mv} = \frac{h}{\sqrt{3\mathrm{m}K_B\,\mathrm{T}}}}$$

PROPERTIES OF MATTER WAVE:

1. Matter wave are not an electromagnetic wave.
2. It motion due to the charge particles.
3. The wave and particle aspects cannot appear together.
4. Locating exact the position of the particle in the wave is uncertain.
5. Lighter particles will have high wavelength.
6. Particles moving with less velocity will have high wavelength.
7. Velocity of matter wave depends on the velocity of the particle.

The velocity of matter wave is greater than the velocity of light