ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY
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DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

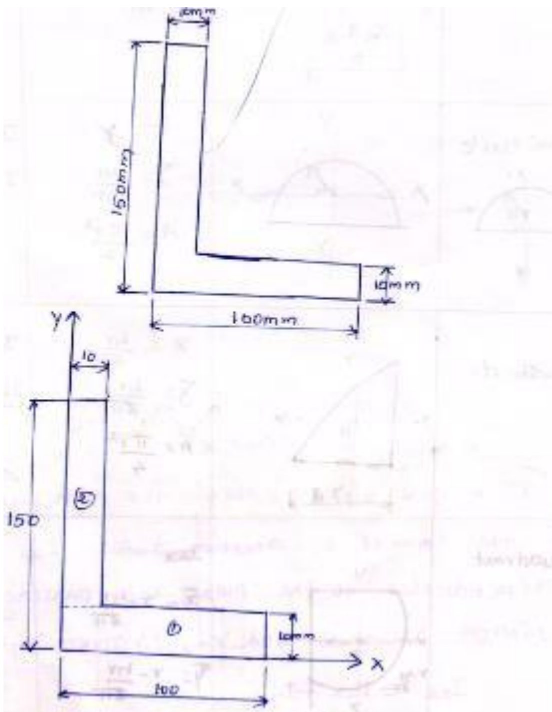
REGULATION 2021

UNIT III: DISTRIBUTED FORCES

Centroid:

Problem 1.

An area in the form of L section is shown in fig.



$I = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2$

$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2$

Section (1) Rectangle

$a_1 = 100 \times 10 = 1000mm^2$

$$x_1 = \frac{100}{2} = 50mm$$

$$y_1 = \frac{10}{2} = 5mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{100 \times 10^3}{12} = 8333.33mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{10 \times 100^3}{12} = 833.33 \times 10^3 mm^4$$

Section (2) rectangle

$$a_2 = 10 \times 140 = 1400mm^2$$

$$x_2 = \frac{10}{2} = 5mm$$

$$y_2 = 10 + \frac{140}{2} = 80mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{10 \times 140^3}{12} = 2.286 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{140 \times 10^3}{12} = 11.66 \times 10^3 mm^4$$

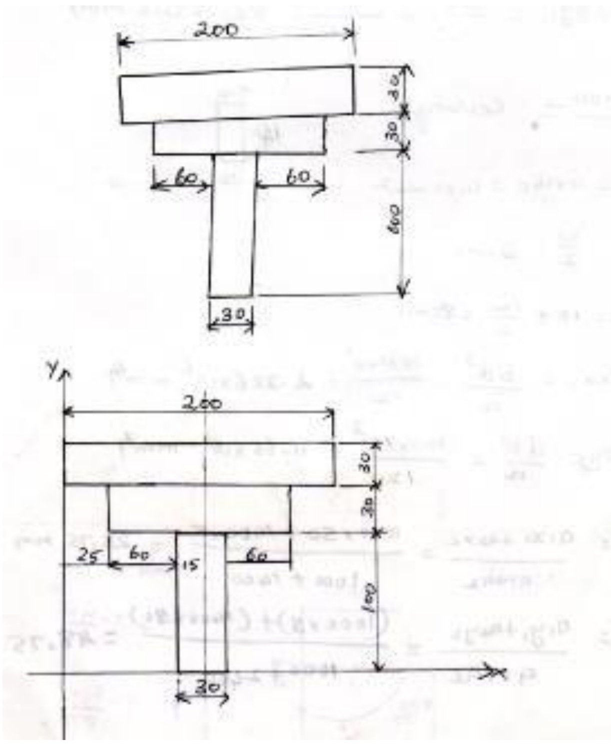
$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{1000 \times 50 + 1400 \times 5}{1000 + 1400} = 23.75mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{1000 \times 5 + 1400 \times 80}{1000 + 2400} = 48.75mm$$

$$I_{xx} = 8333.33 + 1000[48.75 - 5]^2 + 2.286 \times 10^6[1400(48.75 - 80)^2]$$
$$= 575 \times 10^6 mm^4$$

$$I_{yy} = 833.33 \times 10^3 + 1000[23.75 - 50]^2 + 11.66 \times 10^3[1400(23.75 - 5)^2]$$
$$= 2.02 \times 10^6 mm^4$$

2. Find the moment of inertia of the built up section shown in fig. about the axes passing through the centre of gravity parallel to the flange plate.



$I = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2$

$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$

Section (1) Rectangle

$a_1 = 30 \times 100 = 3000mm^2$

$x_1 = 25 + 60 + \frac{30}{2} = 100mm$

$y_1 = \frac{100}{2} = 50mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{30 \times 100^3}{12} = 2.5 \times 10^6 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{100 \times 30^3}{12} = 2.25 \times 10^5 mm^4$

Section (2) Rectangle

$a_2 = 150 \times 30 = 4500mm^2$

$x_2 = 25 + \frac{150}{2} = 100mm$

$y_2 = 100 + \frac{30}{2} = 115mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{150 \times 30^3}{12} = 3.37 \times 10^5 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 150^3}{12} = 8.43 \times 10^5 mm^4$

Section (3)

$a_3 = 300 \times 30 = 9000mm^2$

$x_2 = \frac{200}{2} = 100mm$

$y_2 = 100 + 30 + \frac{30}{2} = 145mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{300 \times 30^3}{12} = 6.75 \times 10^5 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 300^3}{12} = 67.5 \times 10^5 mm^4$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 100) + (4500 \times 100) + (9000 \times 100)}{3000 + 4500 + 9000}$$

$\bar{X} = 100mm$

$$\bar{Y} = \frac{a_1x_1 + a_2x_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 50) + (4500 \times 115) + (9000 \times 145)}{3000 + 4500 + 9000}$$

$\bar{Y} = 119.54mm$

$I_{xx} = 2.5 \times 10^6 + 3000[119.54 - 50]^2 + 3.37 \times 10^5 + 4500[119.54 - 115]^2$
$+ 6.75 \times 10^5 + 9000[119.54 - 145]^2$

$I_{xx} = 23.94 \times 10^6 mm^4$

$I_{yy} = 2.25 \times 10^5 + 3000[100 - 100]^2 + 8.43 \times 10^6 + 4500[100 - 100]^2$
$+ 6.75 \times 10^5 + 9000[100 - 100]^2$

$I_{yy} = 76.15 \times 10^6 mm^4$

Product of Inertia:

The moment of inertia of a plane fig. above a set of perpendicular axis is called product of inertia.

$$I_{xy} = \int_A xy \; da$$

$$= \sum axy$$

Problem: 1

Find the product of inertia and principal moment of inertia of the section about the centroidal axis

Product of inertia

$I_{xy} = I_{x_1y_1} + I_{x2y2} + I_{x3y3}$

$I = I_{xy} + a_1(x_1 - \bar{X})(y_1 - \bar{Y})$

$I_{x_{1y1}} = a_1 x_1^1 y_1^1$ $\quad$ $X_1^1 = x_1 - \bar{X}$ $\quad$ $Y_1^1 = y_1 - \bar{Y}$

$I_{x_{2y2}} = a_2 x_2^1 y_2^1$ $\quad$ $X_2^1 = x_2 - \bar{X}$ $\quad$ $Y_2^1 = y_2 - \bar{Y}$

$I_{x_{3y3}} = a_3 x_3^1 y_3^1$ $\quad$ $X_3^1 = x_3 - \bar{X}$ $\quad$ $Y_3^1 = y_3 - \bar{Y}$

Section (1)

$a_1 = 40 \times 8 = 320mm^2$

$x_1 = 25 + \frac{30}{2} = 52mm$

$y_1 = \frac{8}{2} = 4mm$

Section (2)

$a_3 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_2 = 8 + \frac{44}{2} = 30mm$

Section (3)

$a_1 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_3 = 8 + 44 + \frac{8}{2} = 56mm$

$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = \frac{(320 \times 52) + (352 \times 36) + (320 \times 20)}{320 + 352 + 320} = 36mm$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3} = \frac{(320 \times 4) + (352 \times 30) + (320 \times 56)}{320 + 352 + 320} = 30mm$$

$I_{x_{1y1}} = a_1 x_1^1 y_1^1$ $\quad X_1^1 = x_1 - \bar{X} = 52 - 36 = 16$

$Y_1^1 = y_1 - \bar{Y} = 4 - 30 = -26$

$I_{x_{1y1}} = 320 \times 16 \times (-26)$

$I_{x_{1y1}} = -133120mm^4$

$I_{x_{2y2}} = a_2 x_2^1 y_2^1$ $\quad X_2^1 = x_2 - \bar{X} = 36 - 36 = 0$

$Y_2^1 = y_2 - \bar{Y} = 30 - 30 = 0$

$I_{x_{2y2}} = 0$

$I_{x_{3y3}} = a_3 x_3^1 y_3^1$ $\quad X_3^1 = x_3 - \bar{X} = 20 - 36 = -16$

$Y_3^1 = y_3 - \bar{Y} = 56 - 30 = 26mm$

$I_{x_{3y3}} = -133120mm^4$

Product of Inertia:

$I_{xy} = I_{X1Y1} + I_{X2Y2} + I_{X3Y3}$

$= -133120 + 0 + (-133120)$

$I_{xy} = -266240mm^4$

Principal Moment of Inertia:

Maximum Principal moment of inertia:

$$I_{Max} = \frac{I_{XX} + I_{YY}}{2} \pm \sqrt{\left(\frac{I_{XX} + I_{YY}}{2}\right)^2 + I^2 XY}$$

$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$

$I_{YY} = I_{YY1} + I_{YY2} + I_{YY3}$

$$I_{XX1} = \frac{bd^3}{12} + a_1(Y_1 - \bar{Y}_1)^2$$

$$I_{XX2} = \frac{bd^3}{12} + a_2(Y_1 - \bar{Y}_1)^2$$

$$I_{XX3} = \frac{bd^3}{12} + a_3(Y_1 - \bar{Y}_1)^2$$

$$I = I_{XX1} + a_1(Y_1 - \bar{Y}_1)^2 + I_{XX2} + a_2(\bar{Y} - Y_2)^2 + I_{XX3} + a_3(\bar{Y} - Y_3)^2$$

$$= 1706.66 + 320(30 - 4)^2 + 28394.66 + 352(30 - 30)^2 + 1706.66 + 320(30 - 56)^2$$

$$I_{XX} = 464.44 \times 10^3 mm^4$$

$$I_{yy} = I_{yy1} + a_1(X - X_1)^2 + I_{yy2} + a_2(X - X_2)^2 + I_{yy3} + a_3(X - X_3)^2$$

$$= 42666.66 + 320(36 - 52)^2 + 1877.33 + 352(36 - 36)^2 + 42666.66 + 320(36 - 20)^2$$

$$I_{yy} = 251.05 \times 10^3 mm^4$$

I_{Max}

$$I_{Max} = \frac{I + I_{yy}}{2} \sqrt{+ \left(\frac{I_{xx} + I}{2}\right)^2 + I^2 xy}$$

$$= \frac{464.44\times10^3+251.05\times10^3}{2} + \sqrt{\left(\frac{464.44\times10^3-251.05\times10^3}{2}\right)^2 + (-266240)^2}$$

$$I_{Max} = 357745 + \sqrt{1.138 \times 10^{10} + 7.083 \times 10^{10}}$$

$$I_{Max} = 357745 + 286.81 \times 10^3$$

$$I_{Max} = 644.55 \times 10^3 mm^4$$

$$I_{Min} = \frac{I + I_{yy}}{2} \sqrt{+ \left(\frac{I_{xx} + I}{2}\right)^2 + I^2 xy}$$

$$= \frac{464.44 \times 10^3 + 251.05 \times 10^3}{2} - \sqrt{\left(\frac{464.44 \times 10^3 + 251.05 \times 10^3}{2}\right) + (-266240)^2}$$

$$I_{Min} = 357745 - 268.81 \times 10^3$$

$$I_{Min} = 88.935 \times 10^3 mm^4$$

The position of Principal Axes is given by

$$tan2\theta = \left(\frac{I_{xy}}{\frac{I_x - I_{yy}}{2}}\right) = \left(\frac{266240}{\frac{464.44\times10^3 - 251.05\times10^3}{2}}\right)$$

$$tan2\theta = \left(\frac{-2I_{xy}}{I_{xx} - I_{yy}}\right)$$

$$tan2\theta = 2.49$$

$$2\theta = tan^{-1}(2.49)$$

$$2\theta = 68.16 \quad = \quad \frac{68.16}{2}$$

$$\theta = 34°4'$$