2.6 PRODUCING ELECTRO MAGNETIC WAVES:

We know a stationary charged particle produces an electric field and exerts force on nearby charged particles.

If a charged particle is moving, it can produce magnetic field and exerts force on nearby moving charged particle.

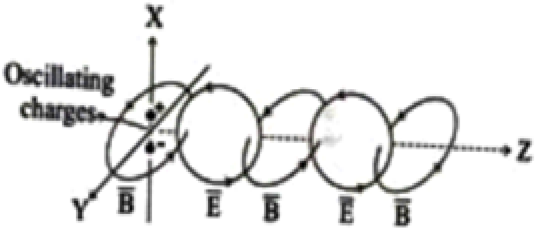

Fig.2.14

If a charged particle is oscillating w.r.t an equilibrium position, then it is known as accelerating charged particle.

An accelerating charged particle produce EM waves of its own frequency '*f*'.

The wavelength λ of EM wave is given by

$$\lambda = \frac{c}{f}$$

2.7 ENERGY AND MOMENTUM OF EM WAVES:

Total energy of EM waves is the sum of time average of energy content due to electric field and magnetic field.

Energy content due to electric field U_E:

Energy density due to electric field

$$U_E = \frac{1}{2}\epsilon_0 \vec{E}^2$$

Substitute $\vec{E} = \vec{E}_0 \sin(kz - \omega t)$

$$U_E = \frac{1}{2}\epsilon_0 \vec{E}_0^{\,2} \sin^2(kz - \omega t)$$

Time average of energy density

$$U_E = \frac{1}{T}\int_0^T \frac{1}{2}\epsilon_0 \vec{E}_0^{\,2} \sin^2(kz - \omega t)dt$$

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Since the value of $\frac{1}{T}\int_0^T \sin^2(kz - \omega t)dt = \frac{1}{2}$, the above equation becomes

$$U_E = \frac{1}{2}\epsilon_0 \vec{E}_0^{\,2} \frac{1}{2}$$

i.e $$U_E = \frac{1}{4}\epsilon_0 \vec{E}_0^{\,2}$$

The above equation represents the energy content in electromagnetic waves due to electric field.

Energy content due to magnetic field U_B:

Energy density due to magnetic field

$$U_B = \frac{1}{2\mu_0}\vec{B}^2$$

Substitute $\vec{B} = \vec{B}_0 \sin(kz - \omega t)$

$$U_B = \frac{1}{2\mu_0}\vec{B}_0^{\,2} \sin^2(kz - \omega t)$$

Time average of energy density

$$U_B = \frac{1}{T}\int_0^T \frac{1}{2\mu_0}\vec{B}_0^{\,2} \sin^2(kz - \omega t)dt$$

$$U_B = \frac{1}{2\mu_0}\vec{B}_0^{\,2} \frac{1}{T}\int_0^T \sin^2(kz - \omega t)dt$$

Since the value of $\frac{1}{T}\int_0^T \sin^2(kz - \omega t)dt = \frac{1}{2}$, the above equation becomes

$$U_B = \frac{1}{2\mu_0}\vec{B}_0^{\,2}\frac{1}{2}$$

i.e $$U_B = \frac{1}{4\mu_0}\vec{B}_0^{\,2}$$

but $$\vec{B}_o = \frac{\vec{E}_o}{c}$$

$$U_B = \frac{1}{4\mu_0}\left(\frac{\vec{E}_o}{c}\right)^2$$

$$U_B = \frac{1}{4\mu_0}\frac{\vec{E}_o^{\,2}}{c^2}$$

But we know

$$\frac{1}{c^2} = \mu_0\epsilon_0$$

So the above equation becomes

$$U_B = \frac{1}{4\mu_0}\vec{E}_o^{\,2}\mu_0\epsilon_0$$

$$U_B = \frac{1}{4}\epsilon_0\vec{E}_o^{\,2}$$

The above equation represents the energy content in electromagnetic waves due to magnetic field.

Total energy $$U = U_E + U_B$$

$$U = \frac{1}{4}\epsilon_0\vec{E}_o^{\,2} + \frac{1}{4}\epsilon_0\vec{E}_o^{\,2}$$

$$U = \frac{1}{2}\epsilon_0\vec{E}_o^{\,2}$$