2.3 Permutation and Combination

The process of selecting things is called combination and that of arranging things is called permutation.

Examples of combinations and permutations:

(i) Formation of a team from a number of players.

(ii) Formation of a 3 member committee from 10members.

(iii) Arrangement of books on a shelf.

(iv) Formation of word with the given letters.

Permutation:

Each of the different arrangements which can be made by taking some or all of a number of things at a time is called a permutation.

The number of permutations of "n" things taken "r" at a time is denoted by nP_r

Examples:

$6P_2$ means the number of permutations of 6 things taken 2 at a time.

Formulae:

(i) $nP_r = \text{n}(n-1)(n-2)...(n-r+1)$

(ii) The number of permutations of "n" things taken all at a time is

$nP_n = \mathrm{n}(n-1)(n-2)...3 \cdot 2 \cdot 1$

$\Rightarrow nP_n = \mathrm{n}!$

Problems based on Permutations:

1. In how many ways can 6 persons occupy 3 vacant seats?

Solution:

Given $n = 6, r = 3$

Total number of ways = $nP_r = 6P_3$ ways

$= 6 \times 5 \times 4 = 120$ ways

2. How many permutations of the letters ABCDEFGH contain the string ABC.

Solution:

Given $n = 6$

No of arrangements = $nP_r = 6P_6 = 6!$ Ways

$= 720$ ways

3. In how many ways can letters of the word "INDIA" be arranged.

Solution:

The word INDIA contains 5 letters of which 2 are I's.

The number of word possible $= \frac{5!}{2!} = \frac{5\times4\times3\times2\times1}{2\times1}$

$$= 60 \text{ ways}$$

4. There are 6 books on Economics, 3 on Commerce and 2 on History. In how many ways can these be placed on a shelf if books on the same subject are to be together.

Solution:

6 Economics books can be arranged in $6P_6$ ways or 6! Ways.

3 Commerce books can be arranged in $3P_3$ ways or 3! Ways.

2 History books can be arranged in $2P_2$ ways or 2! Ways.

The three books can be arranged in $3P_3$ ways

The total number of required arrangements

= 6! × 3! × 2! × 3! Ways

$= 51840$ ways

5. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

Solution:

Number of ways of selecting 3 consonants from 7 $= {}^{7}C_{3}$

Number of ways of selecting 2 vowels from 4 $= {}^{4}C_{2}$

Number of ways of selecting 3 consonants from 7 and 2 vowels from 4

$= {}^{7}C_{3} \times {}^{4}C_{2}$

$$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1} = 210$$

6. Find the number of distinct permutations that can be formed from all the letters of each word (i) RADAR (ii) UNUSUAL

Solution:

The word contains 5 letters of which 2 are A's and 2 are R's.

The number of possible words $= \frac{5!}{2!2!} = 30$

(ii) The word contains 7 letters of which 3 U's are there

The number of possible words $= \frac{7!}{3!} = 40$

7. Find the value of n if $nP_2 = 20$

Solution:

We know that $nP_r = \frac{n!}{(n-r)!}$

$$nP_2 = \frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!}$$

$$\Rightarrow n(n-1) = 20$$

$$\Rightarrow n = 20\ (or)\ n - 1 = 20$$

$$\Rightarrow n = 21$$

Combinations:

Each of the different groups or selections which can be made by taking some or all of a number of things at a time is called a combination.

The number of combinations of "n" things taken "r" at a time is denoted by nC_r.

Formula:

$$nC_r = \frac{n!}{r!\ (n-r)!}$$

Problems based on Combinations:

1. In how many ways can 5 persons be selected from amongst 10 persons?

Solution:

The selection can be done in $10C_5$ ways.

$$= \frac{10\times9\times8\times7\times6}{1\times2\times3\times4\times5}$$

$$= 252 \text{ ways}$$

2. How many ways are there to select five players from 10 member tennis team to make a trip to match to another school.

Solution:

5 members can be selected from 10 members in $10C_5$ ways.

Now $10C_5 = \frac{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1}$

$= 252$ ways

3. Find the number of diagonals that can be drawn by joining the angular points of a heptagon.

Solution:

A heptagon has seven angular points and seven sides.

The join of two angular points is either a side or a diagonal.

The number of lines joining the angular points

$= 7C_2$

$= \frac{7\times6}{2\times1} = 21$

But the number of sides = 7

Hence the number of diagonals $= 21 - 7 = 14$

4. A team of 11 players is to be chosen from 15members. In how many ways can this be done if (i)one particular player is always included? (ii) Two such players have always to be included?

Solution:

(i) Let one player be fixed.

The remaining players are 14.

Out of these 14 players, we have to select 10 players in $14C_{10}$ ways.

$$14C_{10} = \frac{n!}{r!\,(n-r)!} = \frac{14!}{10!\,(14-10)!} = 1001 \text{ ways.}$$

(ii) Let 2 players be fixed.

The remaining players are 13.

Out of 13 players, we have to select 9 players in $13C_9$ ways.

$$13C_9 = \frac{n!}{r!\,(n-r)!} = \frac{13!}{9!\,(13-9)!} = 715 \text{ ways.}$$

5. If $\boldsymbol{nC_5 = 20nC_4}$, find the value of n.

Solution:

Given $nC_5 = 20nC_4$

$$\frac{n!}{5!\,(n-5)!} = \frac{20n!}{4!\,(n-4)!}$$

$\Rightarrow (n-4)!\,4! = 20 \times (n-5)!\,5!$

$\Rightarrow (n-4-1)!\,(n-4)4! = 20 \times (n-5)!\,5!$

$\Rightarrow (n-5)!\,(n-4)4! = 20 \times (n-5)!\,4! \times 5$

$\Rightarrow (n-4) = 100$

$\Rightarrow n = 100 + 4 = 104$

$\Rightarrow n = 104$

6. A question paper has 3 parts, Part A, Part B and Part C having 12, 4 and 4 questions respectively. A student has to answer 10 questions from Part A and 5 questions from Part B and Part C put together selecting atleast 2 from each one of these two parts. In how many ways the selection of questions can be done.

Solution:

12	4	4
Part A	Part B	Part C
10	2	3
10	3	2

The selection of questions can be done in

$$12C_{10} \times 4C_2 \times 4C_3 + 12C_{10} \times 4C_3 \times 4C_2$$

$$= 3168 \text{ ways}$$