5.4 COULOMB'S WEDGE THEORY

Instead of considering the equilibrium of an element within the mass of the material, Coulomb (1776) considered of equilibrium of whole of the material supported by a retaining wall when the wall is on the point of moving slightly away from the filling. The wedge theory of earth pressure is based on the concept of a sliding wedge which is torn off from the rest of the backfill on movement of the wall. In the case of active earth pressure, the sliding wedge moves downwards on a slip surface relative to the intact backfill and in the case of passive earth pressure, the sliding wedge moves upwards and inwards. The pressure on the wall is, in fact, a force of reaction which it has to exert to keep the sliding wedge in equilibrium. Factors such as well friction, irregular soil surfaces and different soil strata can easily take into account in this method.

Following are the basic Assumptions of the wedge theory:

- The backfill is dry, cohesion less, homogenous, isotopic and elastically undeform able but breakable.
- The slip surface is plane which passes through the heel of the wall.
- The sliding wedge itself acts as a rigid body and the value of earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.
- The position and direction of the resultant earth pressure are known. The resultant pressure acts on the back of the wall at one-third the height of the wall from the base and is inclined at an angle δ (called the angle of wall friction) to the normal to the back. (The assumption means that the pressure distribution is hydrostatic, i.e., triangular). The back of wall is rough and a relative movement of the wall and the soil on the back takes place which develops frictional forces that influence the direction of the resultant pressure.

The forces acting on a wedge of soil are: its weight W, the reaction R along the plane of sliding and the active thrust Pa against the retaining wall. R will act at an angle $\emptyset$ to the normal of the plane of sliding. The pressure P is inclined at an angle of wall friction δ to the normal which is considered positive as marked in Fig. 2 Both R and P will be inclined in a direction so as to oppose the movement of the wedge. For the condition of the yield of the wall from the backfill the most dangerous or the critical slip surface is that for which the wall reaction is maximum, i.e., the wall must resist the maximum lateral pressure before it moves away from the fill.

Condition for maximum pressure from a sliding wedge. BD shows a plane inclined at an angle ϕ to the horizontal at which the soil is expected to stay in the absence of any lateral support. The line BD, therefore, is called the natural slope line, repose line or the $\emptyset$ – line. AD, inclined at β to the horizontal, is called the ground line or surcharge line. Plane BC, inclined at angle λ (to be determined) is the line or rupture plane or slip plane; the angle λ is called the critical slip angle. The reaction R inclined at an angle ϕ to the normal to the slip line; R is also inclined at an angle (λ-ϕ) to the vertical. The wall reaction P_a is inclined at an angle to the normal to the wall. The inclination of P_a to vertical is represented by angle $\psi = 90^\circ - \theta - \delta$ (= constant for given value of θ and δ). The value of P_a depends upon the slip angle λ. P_a is zero when λ = ϕ. As λ increases beyond ϕ, P also increases and after reaching a maximum value it again reduces to zero when λ equals 90 $+\theta$. Thus, the critical slip plane lies between the line and back of the wall.

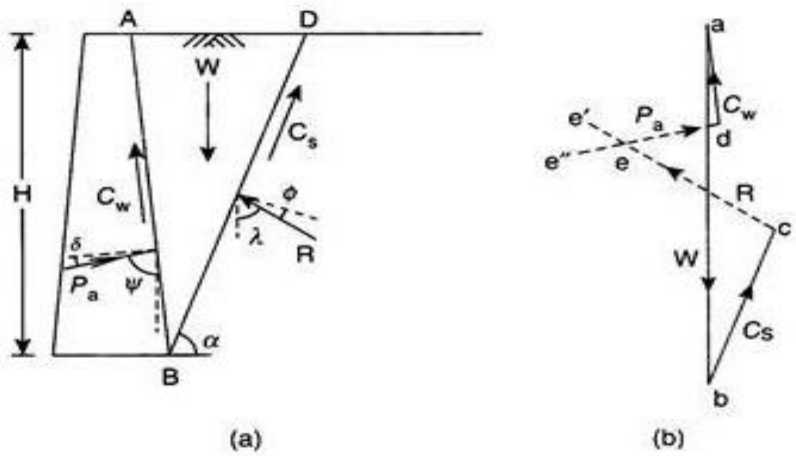

Fig1 Condition for maximum pressure from a sliding wedge

[Fig1https://www.soilmanagementindia.com/lateral-earth-pressure/coulombs-theory/coulombs-theory-for-earth-pressure-soil/13949]

In order to derive the condition for maximum active pressure P_a from the sliding wedge, draw line CE at an angle ψ to the ϕ –line. Let x and n be the perpendicular distance of points C and A from the ϕ -line, and m be the length of line BD. It will be seen triangle BCE and the force triangle similar.

$$\frac{P_a}{CE} = \frac{W}{BE}$$

$$P_a = W\frac{CE}{BE} - - - - - - (1)$$

From triangle CFE,

$$sin\varphi = \frac{x}{CE}$$

$$CE = \frac{x}{sin\varphi} - - - - (2)$$

Where A_1=cosec Φ

BE=BD-FD+FE

From triangle CFD,

$$\tan(\varphi-\beta)=\frac{x}{FD}$$

$$FD=xcot(\varphi-\beta)$$

From triangle CFE,

$$\tan(\varphi)=\frac{x}{FE}$$

FD=x cot Φ

Hence, BE=n-x[cot(Φ-β)-cot Φ]

or

BE=n-A_2X--------(3)

Where A_2=[cot(Φ-β)-cot Φ]

$$w=\gamma(\Delta ABC)=\gamma[\Delta ABD-\Delta BCD]$$

$$W=\frac{1}{2}\gamma(m-x)n----(4)$$

Substituting equation 2,3&4 in 1

$$P_a=\frac{1}{2}\gamma(m-x)n\frac{A_1x}{n-A_2x}=\left(\frac{1}{2}\gamma nA_1\right)\left(\frac{mx-x^2}{n-A_2x}\right)$$

In the above expression x is the only variable which depends upon the position of slip plane BC. For maxima $dP_a/dx = 0$

$$\frac{dP_a}{dx}=\left(\frac{1}{2}\gamma nA_1\right)\frac{(m-2x)(n-A_2x)-(-A_2)(mx-x^2)}{(n-A_2x)^2}=0$$

(m-2x) (n- A_2 x) = - A_2 (m x - x^2)

mn-A_2mx-2nx+2A2X^2=-A_2mx+A_2x^2

mn –2xn = - A_2 x^2

Rearranging,

mn-xn= xn - A_2 x^2=x(n- A_2 x)=x BE

We can Write,

$$\frac{mn}{2} - \frac{xn}{2} = X\frac{BE}{2}$$

$$\Delta ABD - \Delta BCD = \Delta BCE$$

$$\Delta ABC = \Delta BCE$$

Thus the criterion for maximum active pressure is that the slip plane is so chosen that

ΔABC and ΔBCE are equal in area.