4.2 Newton's Divided difference Interpolation

Let $y = f(x)$, take values $f(x_0), f(x_1), \ldots f(x_n)$ corresponding to the arguments $x_0, x_1, \ldots\ldots\ldots, x_n$.

Then the Newton's divided difference interpolation formula is

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \cdots \ldots\ldots\ldots. + (x - x_0)(x - x_1) \ldots\ldots (x - x_{n-1})f(x_0, x_1, \ldots\ldots\ldots, x_n)$$

1. Find $f(8)$ by Newton's divided difference formula for the data

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Solution:

Newton's divided difference interpolation formula is

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \cdots \ldots\ldots\ldots. + (x - x_0)(x - x_1) \ldots\ldots (x - x_{n-1})f(x_0, x_1, \ldots\ldots\ldots, x_n)$$

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3, x_4)$
4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$	0
5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$	0
7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-1} = 1$	0
10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$		
11	1210	$\frac{2028-1210}{13-11} = 409$			
13	2028				

$$f(8) = 48 + (8-4) * 52 + (8-4)(8-5) * 15 + (8-4)(8-5)(8-7) * 1$$
$$= 48 + 4 * 52 + 4 * 3 * 15 + 4 * 3 * 1 * 1$$

$$= 48 + 208 + 180 + 12$$
$$= 448$$

2. Using Newton's divided difference formula , find $f(x)$ and $f(6)$ from the following data

x	1	2	7	8
$f(x)$	1	5	5	4

Solution :

Newton's divided difference interpolation formula is

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \cdots \ldots \ldots \ldots + (x - x_0)(x - x_1) \ldots \ldots (x - x_{n-1})f(x_0, x_1, \ldots \ldots \ldots, x_n)$$

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$
1	1			
2	5	$\frac{5-1}{2-1} = 4$	$\frac{0-4}{7-1} = \frac{-2}{3}$	
7	5	$\frac{5-5}{7-2} = 0$	$\frac{-1-0}{8-2} = \frac{-1}{6}$	$\frac{\frac{-1}{6} + \frac{2}{3}}{8-1} = 0.07$
8	4	$\frac{4-5}{8-7} = -1$		

$$y = f(x) = 1 + (x-1) * 4 + (x-1)(x-2) * \frac{-2}{3} + (x-1)(x-2)(x-7) * 0.07$$
$$= 1 + 4x - 4 - (x^2 - 3x + 2) * \frac{-2}{3} + 0.07 * (x^3 - 3x^2 + 2x - 7x^2 - 21x - 14)$$
$$= 1 + 4x - 4 - 0.6x^2 + 1.8x - 1.2 + 0.07x^3 - 0.7x^2 - 1.33x - 0.98$$
$$= 0.07x^3 - 1.3x^2 + 7.62x - 5.32$$
$$f(6) = 0.07(6)^3 - 1.3x(6)^2 + 7.62(6) - 5.32$$
$$= 6.15$$

3. Given the following data, find $y'(5), y'(6)$and the maximum value of y

x	0	2	3	4	7	9
$y = f(x)$	4	26	58	112	466	922

Solution:

Newton's divided difference interpolation formula is

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \cdots \ldots \ldots \ldots . + (x - x_0)(x - x_1) \ldots \ldots (x - x_{n-1})f(x_0, x_1, \ldots \ldots \ldots, x_n)$$

x	$f(x)$	$f(x_0, x_1)$	$f(x_0, x_1, x_2)$	$f(x_0, x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3, x_4)$
0	4	$\frac{26-4}{2-0} = 11$	$\frac{32-11}{3-0} = 7$	$\frac{11-7}{4-0} = 1$	0
2	26	$\frac{58-26}{3-2} = 32$	$\frac{54-32}{4-2} = 11$	$\frac{16-11}{7-2} = 1$	0
3	58	$\frac{112-58}{4-3} = 54$	$\frac{118-54}{7-3} = 16$	$\frac{22-16}{9-3} = 1$	0
4	112	$\frac{466-112}{7-4} = 118$	$\frac{228-118}{9-4} = 22$		
7	466	$\frac{922-466}{9-7} = 228$			
139	922				

$$y = f(x) = 4 + (x - 0) * 11 + (x - 0)(x - 11) * 7 + (x - 0)(x - 2)(x - 3) * 1 + (x - 0)(x - 2)(x - 3)(x - 4) * 0$$

$$= 4 + 11x + x(x - 11) * 7 + x(x - 2)(x - 3)$$

$$= 4 + 11x + 7x^2 - 14x + x^3 - 3x^2 - 2x^2 + 6x$$

$$= x^3 + 2x^2 + 3x + 4$$

$$y'(x) = 3x^2 + 4x + 3$$

$$y'(5) = 3(5)^2 + 4(5) + 3 = 75 + 20 + 3 = 98$$

$$y'(6) = 3(6)^2 + 4(6) + 3 = 108 + 24 + 3 = 135$$