UNIT – 2

CONVECTION

2.1. Convection Heat Transfer-Requirements

The heat transfer by convection requires a solid-fluid interface, a temperature difference between the solid surface and the surrounding fluid and a motion of the fluid. The process of heat transfer by convection would occur when there is a movement of macro-particles of the fluid in space from a region of higher temperature to lower temperature.

2.2. Convection Heat Transfer Mechanism

Let us imagine a heated solid surface, say a plane wall at a temperature T_w placed in an atmosphere at temperature T_∞ Fig. 2.1 Since all real fluids are viscous, the fluid particles adjacent to the solid surface will stick to the surface. The fluid particle at A, which is at a lower temperature, will receive heat energy from the plate by conduction. The internal energy of the particle would Increase and when the particle moves away from the solid surface (wall or plate) and collides with another fluid particle at B which is at the ambient temperature, it will transfer a part of its stored energy to B. And, the temperature of the fluid particle at B would increase. This way the heat energy is transferred from the heated plate to the surrounding fluid. Therefore the process of heat transfer by convection involves a combined action of heat conduction, energy storage and transfer of energy by mixing motion of fluid particles.

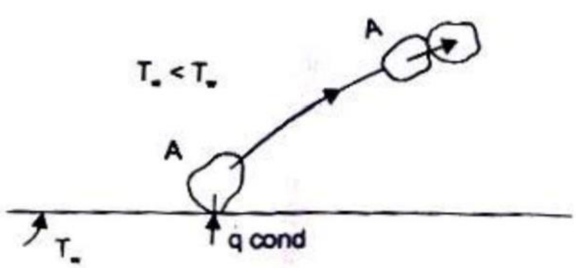


Fig. 2.1 Principle of heat transfer by convection

2.3. Free and Forced Convection

When the mixing motion of the fluid particles is the result of the density difference caused by a temperature gradient, the process of heat transfer is called natural or free convection.

When the mixing motion is created by an artificial means (by some external agent), the process of heat transfer is called forced convection Since the effectiveness of heat transfer by convection depends largely on the mixing motion of the fluid particles, it is essential to have a knowledge of the characteristics of fluid flow.

2.4. Basic Difference between Laminar and Turbulent Flow

In laminar or streamline flow, the fluid particles move in layers such that each fluid p article follows a smooth and continuous path. There is no macroscopic mixing of fluid particles between successive layers, and the order is maintained even when there is a turn around a comer or an obstacle is to be crossed. If a lime dependent fluctuating motion is observed indirections which are parallel and transverse to the main flow, i.e., there is a random macroscopic mixing of fluid particles across successive layers of fluid flow, the motion of the fluid is called' turbulent flow'. The path of a fluid particle would then be zigzag and irregular, but on a statistical basis, the overall motion of the macro particles would be regular and predictable.

2.5. Formation of a Boundary Layer

When a fluid flow, over a surface, irrespective of whether the flow is laminar or turbulent, the fluid particles adjacent to the solid surface will always stick to it and their velocity at the solid surface will be zero, because of the viscosity of the fluid. Due to the shearing action of one fluid layer over the adjacent layer moving at the faster rate, there would be a velocity gradient in a direction normal to the flow.

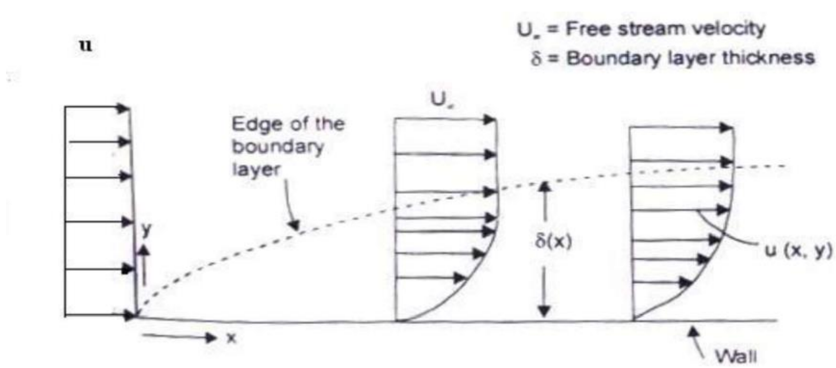


Fig 2.2: sketch of a boundary layer on a wall

Let us consider a two-dimensional flow of a real fluid about a solid (slender in cross-section) as shown in Fig. 2.2. Detailed investigations have revealed that the velocity of the fluid particles at the surface of the solid is zero. The transition from zero velocity at the surface of the solid to the free stream velocity at some distance away from the solid surface in the V-direction (normal to the direction of flow) takes place in a very thin layer called 'momentum or hydrodynamic boundary layer'. The flow field can thus be divided in two regions:

(i) A very thin layer in t he vicinity 0 f t he body w here a velocity gradient normal to the direction of flow exists, the velocity gradient du/dy being large. In this thin region, even a very small Viscosity of μ the fluid exerts a substantial Influence and the shearing stress $\tau = \mu$ du/dy may assume large values. The thickness of the boundary layer is very small and decreases with decreasing viscosity.

(ii) In the remaining region, no such large velocity gradients exist and the Influence of viscosity is unimportant. The flow can be considered frictionless and potential.

2.6. Thermal Boundary Layer

Since the heat transfer by convection involves the motion of fluid particles, we must superimpose the temperature field on the physical motion of fluid and the two fields are bound to interact. It is intuitively evident that the temperature distribution around a hot body in a fluid stream will often have the same character as the velocity distribution in the boundary layer flow. When a heated solid body IS placed in a fluid stream, the temperature of the fluid stream will also vary within a thin layer in the immediate neighborhood of the solid body. The variation in temperature of the fluid stream also takes place in a thin layer in the neighborhood of the body and is termed 'thermal boundary layer'. Fig. 2.3 shows the temperature profiles inside a thermal boundary layer.

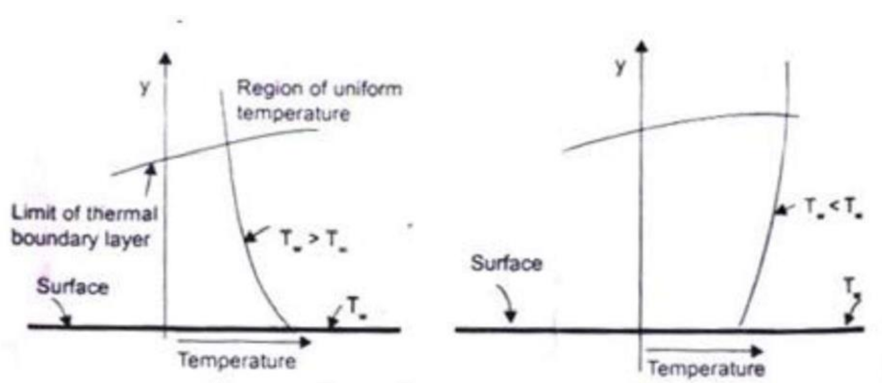


Fig2.3: The thermal boundary layer

2.7. Dimensionless Parameters and their Significance

The following dimensionless parameters are significant in evaluating the convection heat transfer coefficient:

(a) *The Nusselt Number (Nu)*-It is a dimensionless quantity defined as hL/ k, where h = convective heat transfer coefficient, L is the characteristic length and k is the thermal conductivity of the fluid The Nusselt number could be interpreted physically as the ratio of the temperature gradient in the fluid immediately in contact with the surface to a reference temperature gradient (T_s - T_∞) /L. The convective heat transfer coefficient can easily be obtained if the Nusselt number, the thermal conductivity of the fluid in that temperature range and the characteristic dimension of the object is known.

Let us consider a hot flat plate (temperature T_w) placed in a free stream (temperature $T_\infty < T_w$). The temperature distribution is shown ill Fig. 2.4. Newton's Law of Cooling says that the rate of heat transfer per unit area by convection is given by

$$\dot{Q}/\mathbf{A} = h(T_w - T_\infty)$$

$$\frac{\dot{Q}}{A} = h(T_w - T_\infty)$$

$$= k\frac{T_w - T_\infty}{\delta_t}$$

$$h = \frac{k}{\delta_t}$$

$$Nu = \frac{hL}{k} = \frac{L}{\delta_t}$$

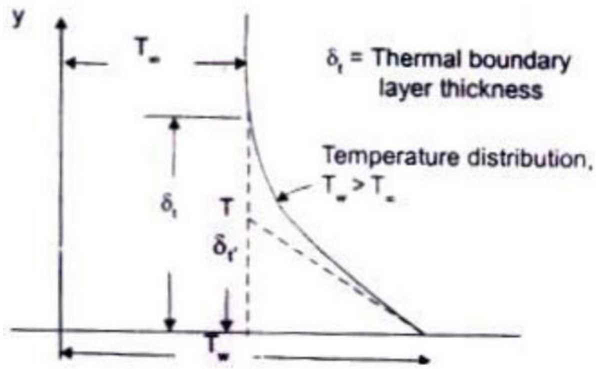


Fig. 2.4 Temperature distribution in a boundary layer: Nusselt modulus

The heat transfer by convection involves conduction and mixing motion of fluid particles. At the solid fluid interface (y = 0), the heat flows by conduction only, and is given by

$$\frac{\dot{Q}}{A} = -k\left(\frac{dT}{dy}\right)_{Y=0} \qquad \therefore h = \frac{\left(-k\,{}^{dT}\!/_{dy}\right)_{y=0}}{(T_w - T_\infty)}$$

Since the magnitude of the temperature gradient in the fluid will remain the same, irrespective of the reference temperature, we can write dT = d(T - T_w) and by introducing a characteristic length dimension L to indicate the geometry of the object from which the heat flows, we get

$$\frac{hL}{k} = \frac{\left({}^{dT}\!/_{dy}\right)_{y=0}}{(T_w - T_\infty)/L}$$, and in dimensionless form,

$$= \left(\frac{d(T_w - T)/(T_w - T_\infty)}{d(y/L)}\right)_{y=0}$$

(b) *The Grashof Number (Gr)*-In natural or free convection heat transfer, die motion of fluid particles is created due to buoyancy effects. The driving force for fluid motion is the body force arising from the temperature gradient. If a body with a constant wall temperature T_w is exposed to a qui scent ambient fluid at T_∞, the force per unit volume can be written as $\rho g \beta (t_w - T_\infty)$ where ρ = mass density of the fluid, β = volume coefficient of expansion and g is the acceleration due to gravity.

The ratio of inertia force × Buoyancy force/(viscous force)^2 can be written as

$$Gr = \frac{\left(\rho V^2 L^2\right) \times \rho g \beta \left(T_w - T_\infty\right) L^3}{\left(\mu V L\right)^2}$$

$$= \frac{\rho^2 g \beta \left(T_w - T_\infty\right) L^3}{\mu^2} = g\beta L^3 \left(T_w - T_\infty\right) / \nu^2$$

The magnitude of Grashof number indicates whether the flow is laminar or turbulent. If the Grashof number is greater than 10^9, the flow is turbulent and for Grashof number less than 10^8, the flow is laminar. For $10^8 < Gr < 10^9$, It is the transition range.

(c) *The Prandtl Number (Pr)* - It is a dimensionless parameter defined as

$$Pr = \mu C_p / k = \nu / \alpha$$

Where μ is the dynamic viscosity of the fluid, v = kinematic viscosity and α = thermal diffusivity.

This number assumes significance when both momentum and energy are propagated through the system. It is a physical parameter depending upon the properties of the medium It is a measure of the relative magnitudes of momentum and thermal diffusion in the fluid: That is, for Pr = I, the r ate of diffusion of momentum and energy are equal which means that t he calculated temperature and velocity fields will be Similar, the thickness of the momentum and thermal boundary layers will be equal. For Pr <<I (in case of liquid metals), the thickness of the thermal boundary layer will be much more than the thickness of the momentum boundary layer and vice versa. The product of Grashof and Prandtl number is called Rayleigh number. Or, Ra = Gr × Pr.

2.8. Evaluation of Convective Heat Transfer Coefficient

The convective heat transfer coefficient in free or natural convection can be evaluated by two methods:

(a) Dimensional Analysis combined with experimental investigations

(b) Analytical solution of momentum and energy equations 10 the boundary layer.

Dimensional Analysis and Its Limitations

Since the evaluation of convective heat transfer coefficient is quite complex, it is based on a combination of physical analysis and experimental studies. Experimental observations become necessary to study the influence of pertinent variables on the physical phenomena.

Dimensional analysis is a mathematical technique used in reducing the number of experiments to a minimum by determining an empirical relation connecting the relevant variables and in grouping the variables together in terms of dimensionless numbers. And, the method can only be applied after the pertinent variables controlling t he phenomenon are Identified and expressed In terms of the primary dimensions. (Table 1.1)

In natural convection heat transfer, the pertinent variables are: h, ρ, k, μ, C_p, L, (ΔT), β and g. Buckingham π's method provides a systematic technique for arranging the variables in dimensionless numbers. It states that the number of dimensionless groups, π's, required to describe a phenomenon involving 'n' variables is equal to the number of variables minus the number of primary dimensions 'm' in the problem.

In SI system of units, the number of primary dimensions are 4 and the number of variables for free convection heat transfer phenomenon are 9 and therefore, we should expect (9 - 4) = 5 dimensionless numbers. Since the dimension of the coefficient of volume expansion, β, is θ^{-1}, one dimensionless number is obviously $\beta(\Delta T)$. The remaining variables are written in a functional form:

$$\phi\left(h, \rho, k, \mu, C_p, L, g\right) = 0.$$

Since the number of primary dimensions is 4, we arbitrarily choose 4 independent variables as primary variables such that all the four dimensions are represented. The selected primary variables are: ρ, g, k. L Thus the dimensionless group,

$$\pi_1 = \rho^a g^b k^c L^d h = \left(ML^{-3}\right)^a \left(LT^{-2}\right)^b . \left(MLT^{-3}\theta^{-1}\right) = M^0 L^0 T^0 \theta^0$$

Equating the powers of M, L, T, θ on both sides, we have

M : a + c + 1 = 0 } Upon solving them,
L : -3a + b + c + d = 0
T : -2b -3c -3 = 0
θ : -c - 1 = 0

Up on solving them,

c = 1, b = a = 0 and d = 1.

and π_1 = hL/k, the Nusselt number.

The other dimensionless number

$\pi_2 = p^a g^b k^c L^d C_p = (ML^{-3})^a (LT^{-2})^b (MLT^{-3}\theta^{-1})^c (L)^d (MT^{-1}\theta^{-1}) = M^0 L^0 T^0 \theta$ Equating the powers of M,L,T and θ and upon solving, we get

$\pi_3 = \mu^2 / \rho^2 g L^3$

By combining π_2 and π_3, we write $\pi_4 = [\pi_2 \times \pi_3]^{1/2}$

$$= \left[\rho^2 g L^3 C_p^2 / k^2 \times \mu^2 / gL^3\right]^{1/2} = \frac{\mu C_p}{k} \text{ (the Prandtl number)}$$

By combining π_3 with $(\beta\Delta T)$, we have $\pi_5 = (\beta\Delta T) * \frac{1}{\pi_3}$

$$= \beta(\Delta T) \times \frac{\rho^2 g L^3}{\mu^2} = g\beta(\Delta T) L^3 / \nu^2 \text{ (the Grashof number)}$$

Therefore, the functional relationship is expressed as:

$$\phi(Nu, Pr, Gr) = 0; Or, Nu = \phi_1(Gr\,Pr) = Const \times (Gr \times Pr)^m \qquad (2.1)$$

and values of the constant and 'm' are determined experimentally.

Table 2.1 gives the values of constants for use with Eq. (2.1) for isothermal surfaces.

Table 2.1 Constants for use with Eq. 2.1 for Isothermal Surfaces

Geometry	$G_{r\,f}\,p_{r\,f}$	*C*	*m*
Vertical planes and cylinders	$10^4 - 10^9$	0.59	1/4
	$10^9 - 10^{13}$	0.021	2/5
	$10^9 - 10^{13}$	0.10	1/3
Horizontal cylinders	$0 - 10^{-5}$	0.4	0
	$10^4 - 10^9$	0.53	1/4
	$10^9 - 10^{12}$	0.13	1/3
	$10^{10} - 10^{-2}$	0.675	0.058
	$10^{-2} - 10^2$	1.02	0.148
	$10^2 - 10^4$	0.85	0.188
	$10^4 - 10^7$	0.48	1/4
	$10^7 - 10^{12}$	0.125	1/3
Upper surface of heated plates or lower surface of cooled plates	$8 \times 10^6 - 10^{11}$	0.15	1/3
- do -	$2 \times 10^4 - 8 \times 10^6$	0.54	1/4
Lower surface of heated plates or upper surface of cooled plates	$10^5 - 10^{11}$	0.27	1/4
Vertical cylinder height = diameter characteristic length = diameter	$10^4 - 10^6$	0.775	0.21
Irregular solids, characteristic length = distance the fluid particle travels in boundary layer	$10^4 - 10^9$	0.52	1/4

__Analytical Solution-Flow over a Heated Vertical Plate in Air__

Let us consider a heated vertical plate in air, shown in Fig. 2.5. The plate is maintained at uniform temperature T_w .The coordinates are chosen in such a way that x - is in the stream wise direction and y - is in the transverse direction. There will be a thin layer of fluid adjacent to the hot surface of the vertical plate within

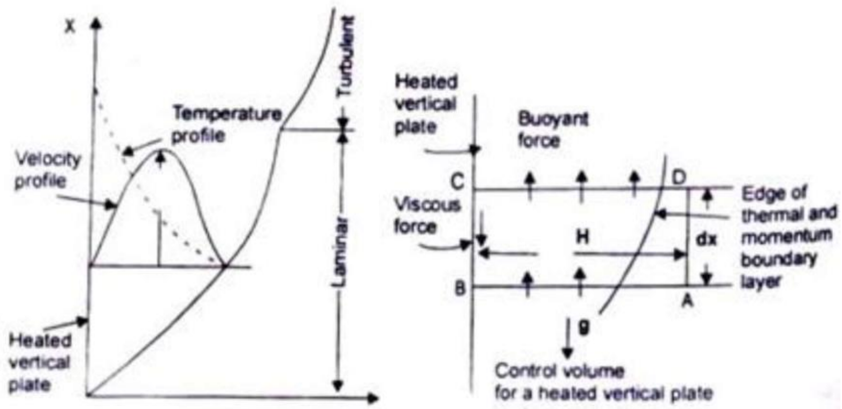


Fig. 2.5 Boundary layer on a heated vertical plate

Which the variations in velocity and temperature would remain confined. The relative thickness of the momentum and the thermal boundary layer strongly depends upon the Prandtl number. Since in natural convection heat transfer, the motion of the fluid particles is caused by the temperature difference between the temperatures of the wall and the ambient fluid, the thickness of the two boundary layers are expected to be equal. When the temperature of the vertical plate is less than the fluid temperature, the boundary layer will form from top to bottom but the mathematical analysis will remain the same.

The boundary layer will remain laminar upto a certain length of the plate ($Gr < 10^8$) and beyond which it will become turbulent ($Gr > 10^9$). In order to obtain the analytical solution, the integral approach, suggested by von-Karman is preferred.

We choose a control volume ABCD, having a height H, length dx and unit thickness normal to the plane of paper, as shown in Fig. 25. We have:

(b) Conservation of Mass:

Mass of fluid entering through face AB = $\dot{m}_{\mathbf{AB}} = \int_0^H \rho u dy$

Mass of fluid leaving face CD = $\dot{m}_{CD} = \int_0^H \rho u dy + \frac{d}{dx}\left[\int_0^H \rho u dy\right]dx$

$\therefore$ Mass of fluid entering the face DA = $\frac{d}{dx}\left[\int_0^H \rho u dy\right]dx$

(ii) Conservation of Momentum:

Momentum entering face AB = $\int_0^H \rho u^2 dy$

Momentum leaving face CD = $\int_0^H \rho u^2 dy + \frac{d}{dx}\left[\int_0^H \rho u^2 dy\right]dx$

$\therefore$ Net efflux of momentum in the + x-direction = $\frac{d}{dx}\left[\int_0^H \rho u^2 dy\right]dx$

The external forces acting on the control volume are:

(a) Viscous force = $\mu \left.\frac{du}{dy}\right|_{y=0} dx$ acting in the –ve x-direction

(b) Buoyant force approximated as $\left[\int_0^H \rho g \beta (T - T_\infty) dy\right]dx$

From Newton's law, the equation of motion can be written as:

$$\frac{d}{dx}\left[\int_0^\delta \rho u^2 dy\right] = -\mu \left.\frac{du}{dy}\right|_{y=0} + \int_0^\delta \rho g \beta (T - T_\infty) dy \qquad (2.2)$$

because the value of the integrand between δ and H would be zero.

(iii) Conservation of Energy:

$\dot{Q}_{AB}$, convection + $\dot{Q}_{AD}$, convection + $\dot{Q}_{BC}$, conduction = $\dot{Q}_{CD}$ convection

or, $\int_0^H \rho u C T dy + C T_\infty \left[\frac{d}{dx}\int_0^H \rho u dy\right]dx - k \left.\frac{dT}{dy}\right|_{y=0} dx$

$= \int_0^H \rho u C T dy + \frac{d}{dx}\left[\int_0^H \rho u T C dy\right]dx$

or $\frac{d}{dx}\int_0^{\delta}\rho u\left(T_{\infty}-T\right)dy \; \frac{k}{\rho C}\frac{dT}{dy}\bigg|_{y=0} = \alpha\frac{dT}{dy}\bigg|_{y=0}$ (2.3)

The boundary conditions are:

or,

(2.3)

Velocity profile	Temperature profile
u = 0 at y = 0	$T = T_w$ at y = 0
u = 0 at y = δ	$T = T_{\infty}$ at y = $\delta_1 \equiv \delta$
du/dy = 0 at y = δ	dT/dy $\equiv$ 0 at y = $\delta_1 \equiv \delta$

Since the equations (2.2) and (2.3) are coupled equations, it is essential that the functional form of both the velocity and temperature distribution are known in order to arrive at a solution.

The functional relationships for velocity and temperature profiles which satisfy the above boundary conditions are assumed of the form:

$$\frac{u}{u_*} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2 \qquad (2.4)$$

Where u_* is a fictitious velocity which is a function of x; and

$$\frac{\left(T-T_{\infty}\right)}{\left(T_w-T_{\infty}\right)} = \left(1-\frac{y}{\delta}\right)^2 \qquad (2.5)$$

After the Eqs. (5.4) and (5.5) are inserted in Eqs. (5.2) and (5.3) and the operations are performed (details of the solution are given in Chapman, A.J. Heat Transfer, Macmillan Company, New York), we get the expression for boundary layer thickness as:

$$\delta / x = 3.93 Pr^{-0.5}\left(0.952+Pr\right)^{0.25} Gr_x^{-0.25}$$

Where Gr, is the local Grashof number = $g\beta x^3\left(T_w - T_{\infty}\right)/\nu^2$

The heat transfer coefficient can be evaluated from:

$$\dot{q}_w = -k\left.\frac{dT}{dy}\right|_{y=0} = h\left(T_w - T_\infty\right)$$

Using Eq. (5.5) which gives the temperature distribution, we have

$h = 2k/\delta$ or, $hx/k = Nu_x = 2x/\delta$

The non-dimensional equation for the heat transfer coefficient is

$$Nu_x = 0.508\, Pr^{0.5}\,(0.952 + Pr)^{-0.25}\, Gr_x^{0.25} \qquad (2.7)$$

The average heat transfer coefficient, $\bar{h} = \frac{1}{L}\int_0^L h_x\, dx = 4/3 h_{x=L}$

$$Nu_L = 0.677\, Pr^{0.5}\,(0.952 + Pr)^{-0.25}\, Gr^{0.25} \qquad (2.8)$$

Limitations of Analytical Solution: Except for the analytical solution for flow over a flat plate, experimental measurements are required to evaluate the heat transfer coefficient. Since in free convection systems, the velocity at the surface of the wall and at the edge of the boundary layer is zero and its magnitude within the boundary layer is so small. It is very difficult to measure them. Therefore, velocity measurements require hydrogen-bubble technique or sensitive hot wire anemometers. The temperature field measurement is obtained by interferometer.

Expression for 'h' for a Heated Vertical Cylinder in Air

The characteristic length used in evaluating the Nusselt number and Grashof number for vertical surfaces is the height of the surface. If the boundary layer thickness is not to large compared with the diameter of the cylinder, the convective heat transfer coefficient can be evaluated by the equation used for vertical plane surfaces. That is, when $D/L \geq 25/\left(Gr_L\right)^{0.25}$

Example 2.1 A large vertical flat plate 3 m high and 2 m wide is maintained at 75°C and is exposed to atmosphere at 25°C. Calculate the rate of heat transfer.

Solution: The physical properties of air are evaluated at the mean temperature. i.e. T = (75 + 25)/2 = 50°C

From the data book, the values are:

$\rho = 1.088$ kg/m^3; $C_p = 1.00$ kJ/kg.K;

$\mu = 1.96 \times 10^{-5}$ Pa-s k = 0.028 W/mK.

Pr = $\mu C_p/k = 1.96 \times 10^{-5} \times 1.0 \times 10^3 /0.028 = 0.7$

$$\beta = \frac{1}{T} = \frac{1}{323}$$

$$Gr = \rho^2 g\beta(\Delta T)L^3/\mu^2$$

$$= \frac{(1.088)^2 \times 9.81 \times 1 \times (3)^3 \times 50}{323 \times (1.96 \times 10^{-5})^2}$$

$= 12.62 \times 10^{10}$

Gr.Pr $= 8.834 \times 10^{10}$

Since Gr.Pr lies between 10^9 and 10^{13}

We have from Table 2.1

$$Nu = \frac{hL}{k} = 0.1(Gr.Pr)^{1/3} = 441.64$$

$\therefore h = 441.64 \times 0.028/3 = 4.122$ W/m^{2}K

$$\dot{Q} = hA(\Delta T) = 4.122 \times 6 \times 50 = 1236.6W$$

We can also compute the boundary layer thickness at x = 3m

$$\delta = \frac{2x}{Nu_x} = \frac{2 \times 3}{441.64} = 1.4 \text{ cm}$$

Example 2.2 A vertical flat plate at 90°C. 0.6 m long and 0.3 m wide, rests in air at 30°C. Estimate the rate of heat transfer from the plate. If the plate is immersed in water at 30°C. Calculate the rate of heat transfer

Solution: (a) *Plate in Air*: Properties of air at mean temperature 60°C

Pr = 0.7, k = 0.02864 W/ mK, v = 19.036×10^{-6} m^2/s

Gr = $9.81 \times (90 - 30)(0.6)^3/ [333 (19.036 \times 10^{-6})^2]$

$= 1.054 \times 10^9$; $Gr \times Pr$ $1.054 \times 10^9 \times 0.7 = 7.37 \times 10^8 < 10^9$

From Table 5.1: for $Gr \times Pr < 10^9$, $Nu = 0.59\ (Gr.\ Pr)^{1/4}$

$\therefore$ $h = 0.02864 \times 0.59\ (7.37 \times 10^8)^{1/4}/0.6 = 4.64\ W/m^2K$

The boundary layer thickness, $\delta = 2\ k/h = 2 \times 0.02864/4.64 = 1.23$ cm

and $\dot{Q} = hA\ (\Delta T) = 4.64 \times (2 \times 0.6 \times 0.3) \times 60 = 100$ W.

Using Eq (2.8). $Nu = 0.677\ (0.7)^{0.5}\ (0.952 + 0.7)^{0.25}\ (1.054 \times 10^9)^{0.25}$,

Which gives $h = 4.297\ W/m^2K$ and heat transfer rate, $\dot{Q}$ 92.81 W

Churchill and Chu have demonstrated that the following relations fit very well with experimental data for all Prandtl numbers.

For $Ra_L < 10^9$, $Nu = 0.68 + (0.67\ Ra_L^{0.25})/\ [1 + (0.492/Pr)^{9/16}]^{4/9)}$ (5.9)

$Ra_L > 10^9$, $Nu = 0.825 + (0.387\ Ra_L^{1/6})/[1 + (0.492/Pr)^{9/16}]^{8/27}$ (5.10)

Using Eq (5.9): $Nu = 0.68 + [0.67(7.37 \times 10^8)^{0.25}]\ /\ [1 + (0.492/0.7)^{9/16}]^{4/9}$

$= 58.277$ and $h = 4.07\ W/m^2k$; $\dot{Q} = 87.9$ W

(b) Plate in Water: Properties of water from the Table

$Pr = 3.01$, $\rho^2\ g\ \beta\ C_p/\mu k = 6.48 \times 10^{10}$;

$Gr.Pr = \rho^2\ g\ \beta\ C_p\ L^3(\Delta T)/\ \mu k = 6.48 \times 10^{10} \times (0.6)^3 \times 60 = 8.4 \times 10^{11}$

Using Eq (5.10): $Nu = 0.825 + [0.387\ (8.4 \times 10^{11})^{1/6}]/\ [1 + (0.492/3.01)^{9/16})]^{8/27} = 89.48$ which gives $h = 97.533$ and $Q = 2.107$ kW.

2.9. Modified Grashof Number

When a surface is being heated by an external source like solar radiation incident on a wall, a surface heated by an electric heater or a wall near a furnace, there is a uniform heat flux distribution along the surface. The wall surface will not be an isothermal one. Extensive experiments have been performed by many research workers for free convection from vertical and inclined surfaces to water under constant heat flux conditions. Since the temperature difference (ΔT) is not known beforehand, the Grashof number is modified by multiplying it by

Nusselt number. That is,

$$Gr_x^* = Gr_x . Nu_x = (g\ \beta\ \Delta T / \nu^2) \times (hx/k) = g\ \beta\ x^4\ q/k\nu^2 \qquad (2.11)$$

Where q is the wall heat flux in Wm^2. ($q = h\ (\Delta T)$)

It has been observed that the boundary layer remains lam mar when the modified Rayleigh number, $Ra^* = Gr_x^*$ '. Pr is less than 3×10^{12} and fully turbulent flow appears for $Ra^* > 10^{14}$. The local heat transfer coefficient can be calculated from:

$$q \text{ constant and } 10^5 < Gr_x^* < 10^{11}: Nu_x = 0.60\ (Gr_x^* . Pr)^{0.2} \qquad (2.12)$$

$$q \text{ constant and } 2 \times 10^{13} < Gr_x^* < 10^{16}: Nu_x = 0.17\ (Gr_x^* . Pr)^{0.25} \qquad (2.13)$$

Although these results are based on experiments for water, they are applicable to air as well. The physical properties are to be evaluated at the local film temperature.

Example 2.3 Solar radiation of intensity 700W/m' is incident on a vertical wall, 3 m high and 3 m wide. Assuming that the wall does not transfer energy to the inside surface and all the incident energy is lost by free convection to the ambient air at 30oe, calculate the average temperature of the wall

Solution: Since the surface temperature of the wall is not known, we assume a value for $h = 7\ W/m^2 K$.

$\Delta T = \dot{q}\ / h = 700/7 = 100^oC$ and the film temperature $= (30 + 130)/2 = 80^oC$

The properties of air at 273 +80 = 353 are: $\beta = 1/353$, $Pr = 0.697$

$k = 0.03\ W/mK$, $\nu = 20.76 \times 10^{-6}\ m^2/s$.

Modified Grashof number, $Gr_x^* = 9.81. (1/353) \cdot (3)^4 \times 700/[0.03 \times (20.76 \times 10^{-6})^2] = 1.15 \times 10^{14}$

From Eq. (2.13), $h = (k/x)\ (0.17)\ (Gr_x^*\ Pr)^{0.25}$

$= (0.03/3)\ (0.17)\ (1.15 \times 10^{14} \times 0.697)^{1/4}$

$= 5.087\ W/m^2K$, a different value from the assumed value.

Second Trial: $\Delta T = \dot{q}\ / h = 700/5.087 = 137.66$ and film temperature

= 98.8°C

The properties of air at (273 + 98.8) °C are: β =1/372, k = 0.0318 W/mK

Pr = 0.693, v = 23.3 × 10^{-6} m^2/s

$Gr_x^* = 9.81. (1/372) \cdot (3)^4 \times 700 / [0.318(23.3 \times 10^{-6})^2] = 8.6 \times 10^{13}$

Using Eq (2.13), h = (k/x) (0.17) (Gr_x^* Pr$)^{1/4}$ = 5.015 W/m^2k, an acceptable value. In turbulent heat transfer by convection, Eq. (5.13) tells us that the local heat transfer coefficient h_x does not vary with x and therefore, the average and local heat transfer coefficients are the same.

2 Laminar Flow Forced Convection Heat Transfer

2.1Forced Convection Heat Transfer Principles

The mechanism of heat transfer by convection requires mixing of one portion of fluid with another portion due to gross movement of the mass of the fluid. The transfer of heat energy from one fluid particle or a molecule to another one is still by conduction but the energy is transported from one point in space to another by the displacement of fluid.

When the motion of fluid is created by the imposition of external forces in the form of pressure differences, the process of heat transfer is called 'forced convection'. And, the motion of fluid particles may be either laminar or turbulent and that depends upon the relative magnitude of inertia and viscous forces, determined by the dimensionless parameter Reynolds number. In free convection, the velocity of fluid particle is very small in comparison with the velocity of fluid particles in forced convection, whether laminar or turbulent. In forced convection heat transfer, $Gr/Re^2 << 1$, in free convection heat transfer, $GrRe^2 >> 1$ and we have combined free and forced convection when $Gr/Re^2 \approx 1$.

2.2. Methods for Determining Heat Transfer Coefficient

The convective heat transfer coefficient in forced flow can be evaluated by: (a) Dimensional Analysis combined with experiments;

(b) Reynolds Analogy – an analogy between heat and momentum transfer; (c) Analytical Methods – exact and approximate analyses of boundary layer equations.

2.3. Method of Dimensional Analysis

As pointed out in Chapter 5, dimensional analysis does not yield equations which can be solved. It simply combines the pertinent variables into non-dimensional numbers which facilitate the interpretation and extend the range of application of experimental data. The relevant variables for forced convection heat transfer phenomenon whether laminar or turbulent, are

(b) The properties of the fluid – density p, specific heat capacity C_p, dynamic or absolute viscosity μ, thermal conductivity k.

(ii) The properties of flow – flow velocity Y, and the characteristic dimension of the system L.

As such, the convective heat transfer coefficient, h, is written as h = f (ρ, V, L, μ, Cp, k) = 0 (5.14)

Since there are seven variables and four primary dimensions, we would expect three dimensionless numbers. As before, we choose four independent or core variables as ρ, V, L, k, and calculate the dimensionless numbers by applying Buckingham π's method:

$$\pi_1 = \rho^a V^b L^c K^d h = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(MT^{-3}\theta^{-1}\right)$$

$$= M^o L^o T^o \theta^o$$

Equating the powers of M, L, T and θ on both sides, we get

M : a + d + 1 = O

L : - 3a + b + c + d = 0

T : - b – 3d – 3 = 0 By solving them, we have

θ : - d – 1= 0. D = -1, a = 0, b = 0, c = 1.

Therefore, π_1 = hL/k is the Nusselt number.

$$\pi_2 = \rho^a V^b L^c K^d \mu = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(ML^{-1}T^{-1}\right)$$

$$= M^o L^o T^o \theta^o$$

Equating the powers of M, L, T and on both sides, we get

M : a + d +1 = 0

L : - 3a + b + c + d = 1 = 0

T : - b – 3d – 1 = 0

θ : - d = 0.

By solving them, d = 0, b = - 1, a = - 1, c = - 1

and $\pi_2 = \mu / \rho VL; or, \pi_3 = \frac{1}{\pi_2} = \frac{\rho VL}{\mu}$

(Reynolds number is a flow parameter of greatest significance. It is the ratio of inertia forces to viscous forces and is of prime importance to ascertain the conditions under which a flow is laminar or turbulent. It also compares one flow with another provided the corresponding length and velocities are comparable in two flows. There would be a similarity in flow between two flows when the Reynolds numbers are equal and the geometrical similarities are taken into consideration.)

$$\pi_4 = \rho^a V^b L^c k^d C_p = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(L^2T^{-2}\theta^{-1}\right)$$

$M^o L^o T^o \theta^o$

Equating the powers of M, L, T, on both Sides, we get

M : a + d = 0; L : - 3a + b + c + d + 2 = 0

T : - b – 3d – 2 = 0; θ : - d – 1 = 0

By solving them,

d =- 1,a = l, b = l, c = l,

$$\pi_4 = \frac{\rho VL}{k} C_p; \quad \pi_5 = \pi_4 \times \pi_2$$

$$= \frac{\rho VL}{k} C_p \times \frac{\mu}{\rho VL} = \frac{\mu C_p}{k}$$

$\therefore$ π_5 is Prandtl number.

Therefore, the functional relationship is expressed as:

$$Nu = f(Re, Pr); \text{ or } Nu = C\,Re^m\,Pr^n \qquad (5.15)$$

Where the values of c, m and n are determined experimentally.

2.4. Principles of Reynolds Analogy

Reynolds was the first person to observe that there exists a similarity between the exchange of momentum and the exchange of heat energy in laminar motion and for that reason it has been termed 'Reynolds analogy'. Let us consi der the motion of a fluid where the fluid is flowing over a plane wall. The X-coordinate is measured parallel to the surface and the V-coordinate is measured normal to it. Since all fluids are real and viscous, there would be a thin layer, called momentum boundary layer, in the vicinity of the wall where a velocity gradient normal to the direction of flow exists. When the temperature of the surface of the wall is different than the temperature of the fluid stream, there would also be a thin layer, called thermal boundary layer, where there is a variation in temperature normal to the direction of flow. Fig. 2.6 depicts the velocity distribution and temperature profile for the laminar motion of the fluid flowing past a plane wall.

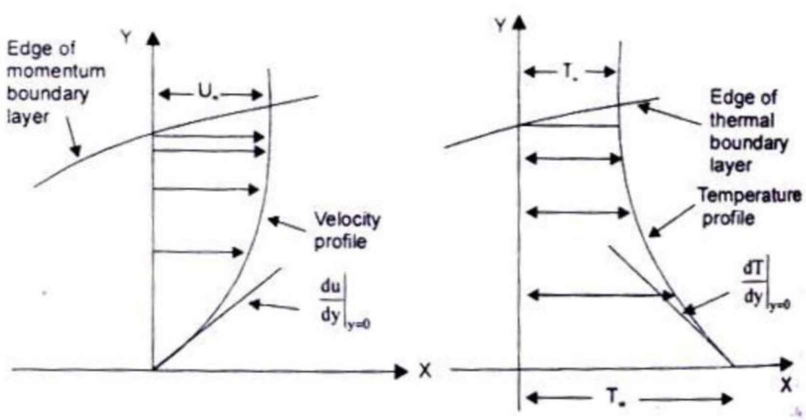


Fig. 2.6 velocity distribution and temperature profile for laminar motion of the fluid over a plane surface

In a two-dimensional flow, the shearing stress is given by $\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$

and the rate of heat transfer per unit area is given by $\frac{\dot{Q}}{A} = \frac{\tau_w k}{\mu}\frac{dT}{du}$

For Pr = $\mu C_p/k = 1$, we have $k/\mu = C_p$ and therefore, we can write after separating the variables,

$$\frac{\dot{Q}}{A\tau_w C_p} du = -dT \tag{5.16}$$

Assuming that Q and τ_w are constant at any station x, we integrate equation (5.16) between the limits: u = 0 when T = T_w, and u = U_∞ when T = T_∞, and we get,

$$\dot{Q}/\left(A\tau_w C_p\right) \times U_\infty = \left(T_w - T_\infty\right)$$

Since by definition, $\dot{Q}/A = h_x\left(T_w - T_\infty\right)$, and $\tau_w = C_{fx} \times \rho U_\infty^2/2$,

Where C_{fx}, is the skin friction coefficient at the station x. We have

$$C_{fx}/2 = h_x/(C_p \rho U_\infty) \tag{5.17}$$

Since $h_x/C_p\rho U_\infty = \left(h_x x/k\right) \times \left(\mu/\rho \times U_\infty\right) \times \left(k/\mu C_p\right) = Nu_x/\left(Re.Pr\right)$,

$$Nu_x/Re.Pr = C_{fx}/2 = \text{Stantonnumer,St.} \tag{5.18}$$

Equation (5.18) is satisfactory for gases in which Pr is approximately equal to unity. Colburn has shown that Eq. (5.18) can also be used for fluids having Prandtl numbers ranging from 0.6 to about 50 if it is modified in accordance with experirnental results.

$$\text{Or, } \frac{Nu_x}{Re_x Pr}.Pr^{2/3} = St_x Pr^{2/3} = C_{fx}/2 \tag{5.19}$$

Eq. (5.19) expresses the relation between fluid friction and heat transfer for laminar flow over a plane wall. The heat transfer coefficient could thus be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

Example 2.4 Glycerine at 35°C flows over a 30cm by 3Ocm flat plate at a velocity of 1.25 m/s. The drag force is measured as 9.8 N (both Side of the plate). Calculate the heat transfer for such a flow system.

Solution: From tables, the properties of glycerine at 35°C are:

ρ = 1256 kg/m^3, C_p = 2.5 kJ/kgK, μ = 0.28 kg/m-s, k = 0.286 W/mK, Pr = 2.4 Re= $\rho VL/\mu$ = 1256 × 1.25 × 0.30/0.28 = 1682.14, a laminar flow.*

Average shear stress on one side of the plate = drag force/area

= 9.8/(2 × 0.3 × 0.3) = 54.4

and shear stress = $C_f \rho U^2/2$

$\therefore$ The average skin friction coefficient, Cr/ 2 = $\dfrac{\tau}{\rho U^2}$

= 54.4/(1256 ×1.25 × 1.25) = 0.0277

From Reynolds analogy, $C_f/2 = St. Pr^{2/3}$

or, h = $\rho C_p U \times C_f/2 \times Pr^{-2/3} = \dfrac{1256 \times 2.5 \times 1.25 \times 0.0277}{(2.45)^{0.667}}$=59.8 kW/m^{2}K.

2.5. Analytical Evaluation of 'h' for Laminar Flow over a Flat Plat – Assumptions

As pointed out earlier, when the motion of the fluid is caused by the imposition of external forces, such as pressure differences, and the fluid flows over a solid surface, at a temperature different from the temperature of the fluid, the mechanism of heat transfer is called 'forced convection'. Therefore, any analytical approach to determine the convective heat transfer coefficient would require the temperature distribution in the flow field surrounding the body. That is, the theoretical analysis would require the use of the equation of motion of the viscous fluid flowing over the body along with the application of the principles of conservation of mass and energy in order to relate the heat energy that is convected away by the fluid from the solid surface.

For the sake of simplicity, we will consider the motion of the fluid in 2 space dimension, and a steady flow. Further, the fluid properties like viscosity, density, specific heat, etc are constant in the flow field, the viscous shear forces m the Y–direction is negligible and there are no variations in pressure also in the Y –direction.

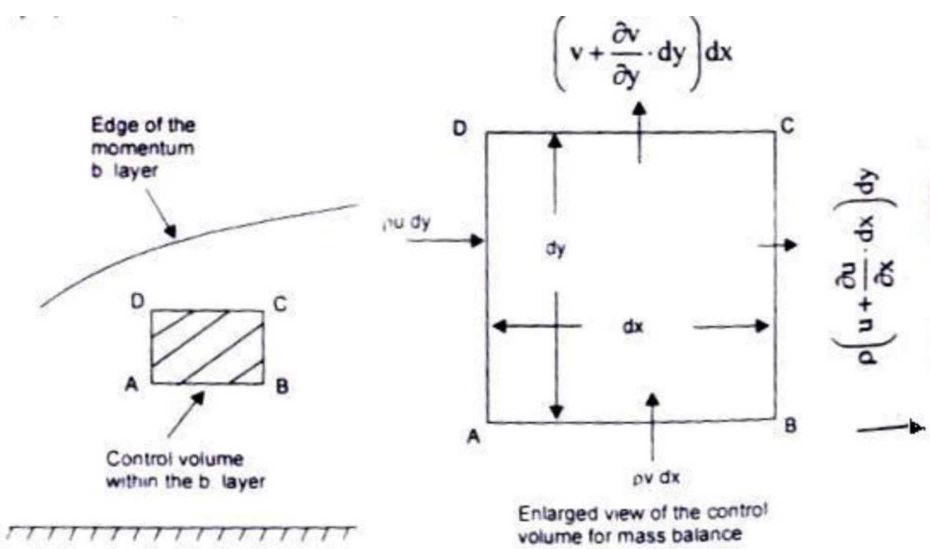
2.6. Derivation of the Equation of Continuity–Conservation of Mass

We choose a control volume within the laminar boundary layer as shown in Fig. 6.2. The mass will enter the control volume from the left and bottom face and will leave the control volume from the right and top face. As such, for unit depth in the Z-direction,

$$\dot{m}_{AD} = \rho u dy \; ; \quad \dot{m}_{BC} = \rho\left(u + \frac{\partial u}{dx}.dx \right)dy;$$

$$\dot{m}_{AB} = \rho v dx \; ; \quad \dot{m}_{CD} = \rho\left(v + \frac{\partial u}{dy}.dy \right)dx;$$

For steady flow conditions, the net efflux of mass from the control volume is zero, therefore,



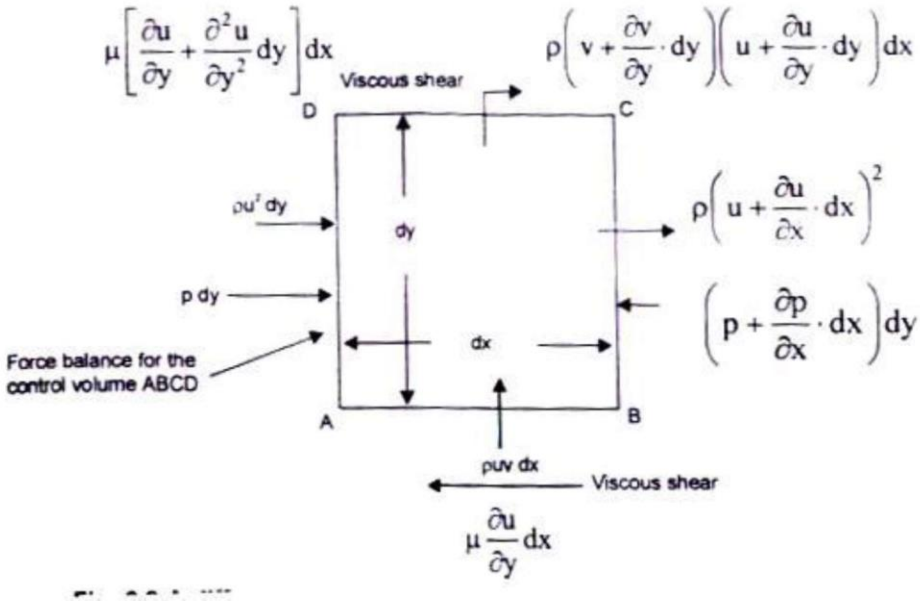


Fig. 2.7 a differential control volume within the boundary layer for laminar flow over a plane wall

$$\rho^{udy} + \rho^{xdx} = \rho^{udy} + \rho\frac{\partial^{u}}{\partial x}dxdy + \rho^{vdx} + \rho\frac{\partial^{v}}{\partial x}.dxdy$$

or, $\partial u/\partial x + \partial v + \partial y = 0,$ the equation of continuity. (2.20)

Concept of Critical Thickness of Insulation

The addition of insulation at the outside surface of small pipes may not reduce the rate of heat transfer. When an insulation is added on the outer surface of a bare pipe, its outer radius, r_0 increases and this increases the thermal resistance due to conduction logarithmically whereas t he thermal resistance to heat flow due to fluid film on the outer surface decreases linearly with increasing radius, r_0. Since the total thermal resistance is proportional to the sum of these two resistances, the rate of heat flow may not decrease as insulation is added to the bare pipe.

Fig. 2.7 shows a plot of heat loss against the insulation radius for two different cases. For small pipes or wires, the radius r_l may be less than re and in that case, addition of insulation to the bare pipe will increase the heat loss until the critical radius is reached. Further addition of insulation will decrease the heat loss rate from this peak value. The insulation thickness (r^{*} r_l) must be added to reduce the heat loss below the uninsulated rate. If the outer pipe radius r_l is greater than the critical radius re any insulation added will decrease the heat loss.

2.7 Expression for Critical Thickness of Insulation for a Cylindrical Pipe

Let us consider a pipe, outer radius r_1 as shown in Fig. 2.18. An insulation is added such that the outermost radius is r a variable and the insulation thickness is $(r - r_1)$. We assume that the thermal conductivity, k, for the insulating material is very small in comparison with the thermal conductivity of the pipe material and as such the temperature T_1, at the inside surface of the insulation is constant. It is further assumed that both h and k are constant. The rate of heat flow, per unit length of pipe, through the insulation is then,

$\dot{Q}/L = 2\pi(T_1 - T_\infty)/(\ln(r/r_1)/k + 1/hr)$, where T_∞ is the ambient temperature.

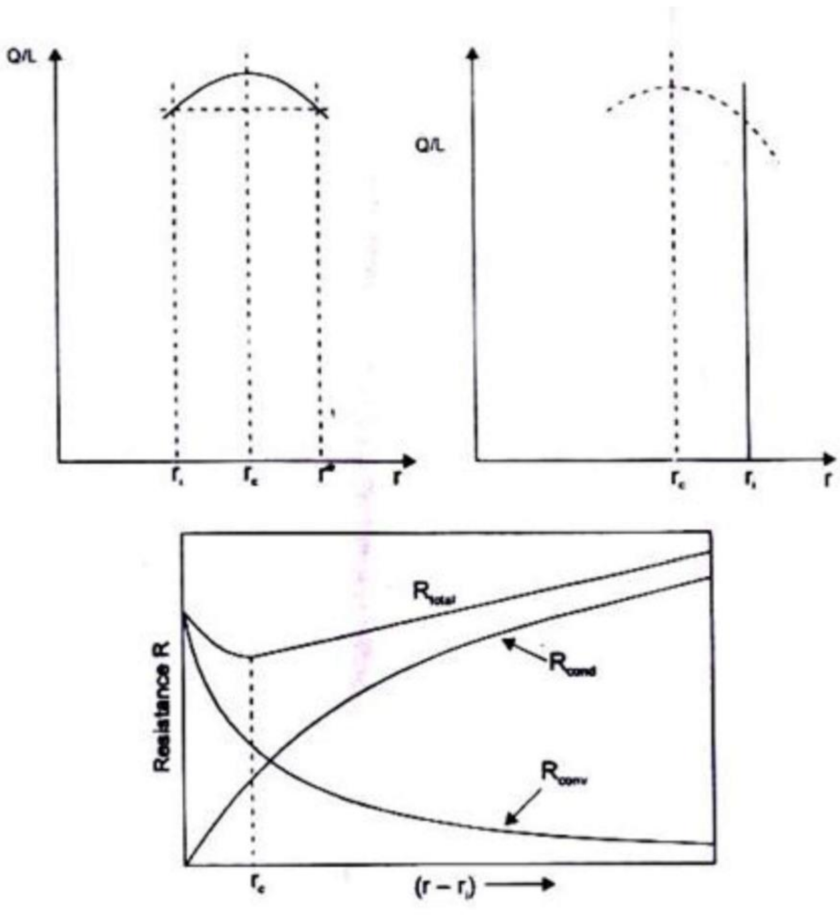


Fig 2.8 Critical thickness for pipe insulation

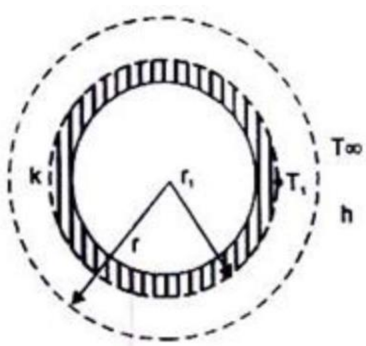


Fig 2.9 critical thickness of insulation for a pipe

An optimum value of the heat loss is found by setting $\dfrac{d(\dot{Q}/L)}{dr} = 0$.

$$\text{or,}\quad \frac{d(\dot{Q}/L)}{dr} = 0 = -\frac{2\pi(T_1 - T_\infty)(1/kr - 1/hr^2)}{(\ln(r/r_1)/k + 1/hr^2)}$$

or, $(1/kr) - (1/hr^2) = 0$ and $r = r_c = k/h$ (2.21)

where r_c denote the ‘critical radius’ and depends only on thermal quantities k and h.

If we evaluate the second derivative of (Q/L) at $r = r_c$, we get

$$\left.\frac{d^2(Q/L)}{dr^2}\right|_{r=r_c} = -2\pi(T_1 - T_\infty)\left[\frac{\frac{k}{hr} + \mathrm{In}\left(\frac{r}{r_1}\right)\left(\frac{2k}{hr} - 1\right) - 2\left(1 - \frac{k}{hr}\right)^2}{\frac{1}{kr}\left(\frac{k}{h} + r\,\mathrm{In}\left(\frac{r}{r_1}\right)\right)}\right]_{r=r_c}$$

$$= -\left[2\pi(T_1 - T_\infty)h^2/k\right]/\left[1 + \mathrm{1n}\ r_c/r_1\right]^2$$

Which is always a negative quantity. Thus, the optimum radius, $r_c = k/h$ will always give a maximum heal loss and not a minimum.

2.8. An Expression for the Critical Thickness of Insulation for a Spherical Shell

Let us consider a spherical shell having an outer radius r_1 and the temperature at that surface T_1 . Insulation is added such that the outermost radius of the shell is r, a variable. The thermal conductivity of the insulating material, k, and the convective heat transfer coefficient at the outer surface, h, and the ambient temperature T_∞ is constant. The rate of heat transfer through the insulation on the spherical shell is given by

$$\dot{Q} = \frac{(T_1 - T_\infty)}{(r - r_1)/4\pi k r r_1 + 1/h\,4\pi r^2}$$

$$\frac{d\dot{Q}}{dr} = 0 = \frac{4\pi(T_1 - T_\infty)(1/kr^2 - 2/hr^3)}{\left[(r - r_1)/k r r_1 + 1/hr^2\right]^2}$$

which gives, $1/Kr^2 - 2/hr^3 = 0$;

or $r = r_c = 2\ k/h$ (2.22)

2.9 Heat and Mass Transfer

Example 2.5 Hot gases at 175°C flow through a metal pipe (outer diameter 8 cm). The convective heat transfer coefficient at the outside surface of the insulation (k = 0.18 W /mK) IS 2.6 W m1K and the ambient temperature IS 25°C. Calculate the insulation thickness such that the heat loss is less than the uninsulated case.

Solution: (a) Pipe without Insulation

Neglecting the thermal resistance of the pipe wall and due to the inside convective heat transfer coefficient, the temperature of the pipe surface would be 175°C.

$\dot{Q}/L = h \times 2\pi r\ (T_1 - T_\infty) = 2.6 \times 2 \times 3.14 \times .04\{175-25) = 98$ W/m (b) Pipe Insulated. Outermost Radius, r*

$$\dot{Q}/L = 98 = (T_1 - T_\infty) / \left(\frac{1n(r^*/4)}{2\pi \times 0.18} + \frac{100}{2.6 \times 2\pi \times r^*}\right)$$

or $\frac{150}{98}$ = 08841n (r*/4)+6.12/r*; which gives r* = 13.5 cm.

Therefore, the insulation thickness must be more than 9.5 cm.

(Since the critical thickness of insulation is $r_c = k/h = 0.18/2.6 = 6.92$ cm, and is greater than the radius of the bare pipe, the required insulation thickness must give a radius greater than the critical radius.)

If the outer radius of the pipe was more than the critical radius, any addition of insulating material will reduce the rate of heat transfer. Let us assume that the outer radius of the pipe is 7 cm ($r > r_c$)

$\dot{Q}$/L, without insulation = hA (ΔT) = 2.6 × 2 ×3.142 × 0.07× (175-25)

= 171.55 W/m

By adding 4 cm thick insulation, outermost radius = 7.0 + 4.0 = 11.0 cm.

and $\dot{Q}/L = (175 - 25) / \left[\frac{1n(11/7)}{2\pi \times 0.18} + \frac{1}{2.6\pi \times 2 \times 0.11}\right]$ =133.58W/m.

Reduction in heat loss = $\frac{171.55 - 133.58}{171.55}$ = 0.22 or 22%.

Example 2.6 An electric conductor 1.5 mm in diameter at a surface temperature of 80°C is being cooled in air at 25°C. The convective heat transfer coefficient from the conductor surface is 16W/m^{2}K. Calculate the surface temperature of the conductor when it is covered with a layer of rubber insulation (2 mm thick, k = 0.15 W /mK) assuming that the conductor carries the same current and the convective heat transfer coefficient is also the same. Also calculate the increase in the current carrying capacity of the conductor when the surface temperature of the conductor remains at 80°C.

Solution: When there is no insulation,

$\dot{Q}$/ L = hA (ΔT) = 16 × 2 × 3.142 × 0.75 × 10^{-3} = 4.147 W/m

When the insulation is provided, the outermost radius = 0.75 + 2 = 2.75 mm

$\dot{Q}/L = 4.147 = (T_1-25) / \left(\frac{1n\,2.75/0.75}{2\pi \times 0.15} + \frac{1000}{16 \times 2\pi \times 2.75}\right)$

or $T_1 = 45.71^oC$

i.e., the temperature at the outer surface of the wire decreases because the insulation adds a resistance.

The critical radius of insulation, r c = k/h = 0.15/16 = 9.375 mm

i.e., when an insulation of thickness (9.375 - 0.75) = 8.625 mm is added, the heat transfer rate would be the maximum and the conductor can carry more current. The heat transfer rate with outermost radius equal to r_c = 9.375 mm

$$\dot{Q}/L = (80 - 25) \;/\; \left(\frac{1n\,9.375/0.75}{2\pi\times0.15}+\frac{1000}{16\times2\pi\times9.375}\right) = 14.7\ \text{W/m}$$

The rate of heat transfer is proportional to (current)^2, the new current I_2 would be:

$I_2/I_1 = (14.7 / 4.147)^{1/2} = 1.883$

or, the current carrying capacity can be increased 1.883 times. But the maximum current capacity of wire would be limited by the permissible temperature at the centre of the wire.

The surface temperature of the conductor when the outermost radius with insulation is equal to the critical radius, is given by

$$\dot{Q}/L = 4.147 = (T-25/ \left(\frac{1n\,9.375/0.75}{2\times3.142\times0.15}+\frac{1000}{16\times2\times3.142\times9.375}\right)$$

or $\quad T = 40.83°C.$