3.3 ANALYSIS OF THREE HINGED PARABOLIC ARCHES

Three hinged arch

The three hinged arches are statically determinate structures in which the horizontal movement at the support is prevented to a certain extent or it is wholly prevented. This type of arch has two hinges at the end supports and also an supplementary intermediate hinge at the crown.

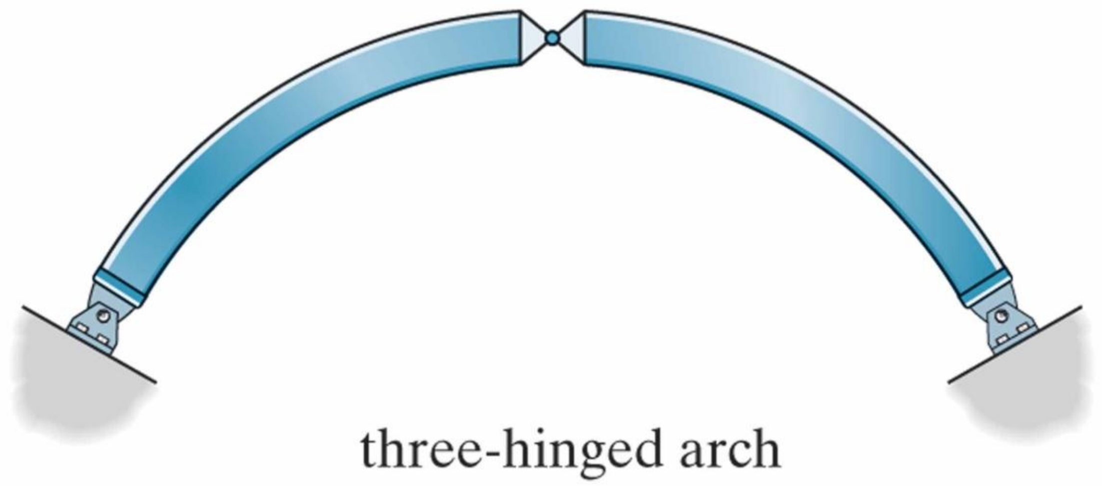

Fig. 3.3.1

Three hinged parabolic arches

An arch which is hinged at three points and whose axis represents a parabolic shape is known as a three hinged parabolic arch.

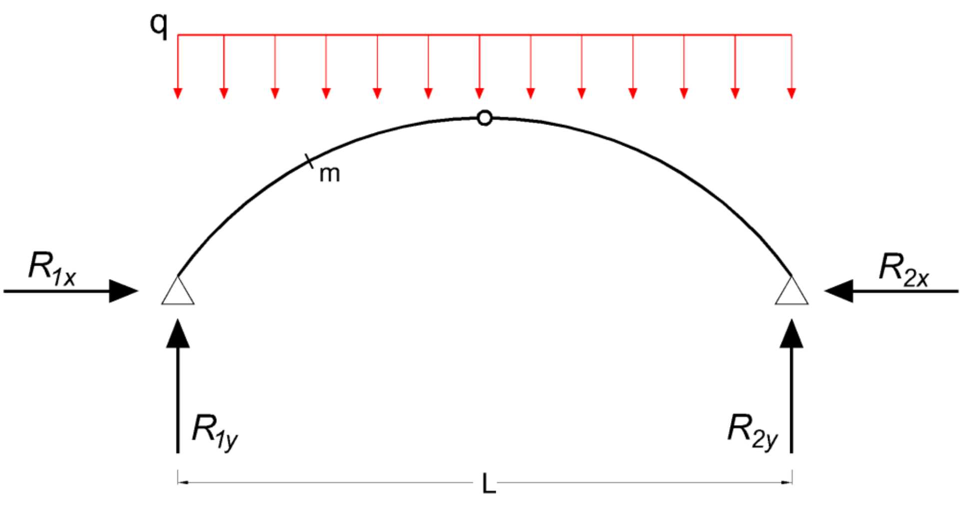

Fig. 3.3.2 Three hinged parabolic arches

Analysis of 3-hinged arches

It is the process of determining external reactions at the support and internal quantities such as normal thrust, shear and bending moment at any section in the arch.

Procedure to find reactions at the supports

Step 1. Sketch the arch with the loads and reactions at the support.

Apply equilibrium conditions namely □ F_X □ 0,□ F_y □ 0and □ M □ 0

Apply the condition that BM about the hinge at the crown is zero (Moment of all the forces either to the left or to the right of the crown).

Example:

A 3hinged arch of span 40m and rise 8m carries concentrated load of 200KN and 150KN at distance of 8m and 16m from the left end and an UDL of 50K/N on the right half of the span.

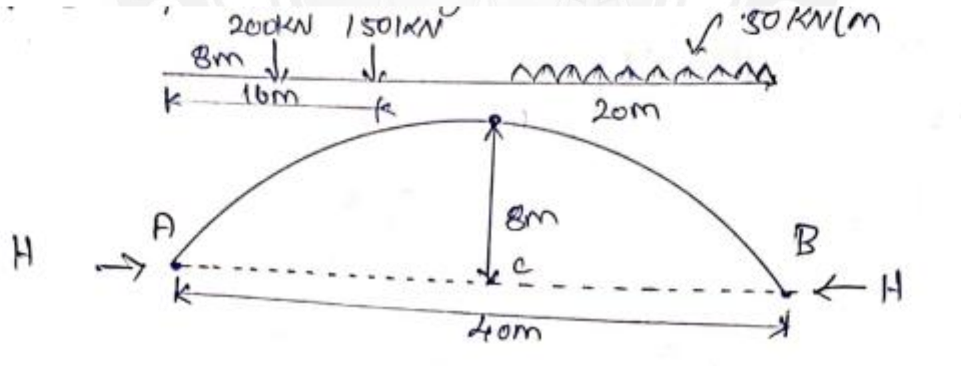

Fig. 3.3.3

SOLUTION

a) vertical reactions VA and VB

Taking moment about A

(200X8)+(150X16)+(50X20X(20+20/2)]-VB X40 = 0

1600+2400+30000-VBX40 =0

VB =850KN

Total load =VA+VB

200+150+(50X20) =850+VA

VA =500KN

b)Horizontal thrust (H)

Taking moment about C

HX8-VA(20)+(200X12)+(150X4) =0

8H-4000+2400+600 =0

H =325KN

Example:

A parabolic 3 hinged arch carries loads as shown in fig. Determine the resultant reactions at supports. Find the bending moment normal thrust and radial shear at D,5m from A. what is the max bending.

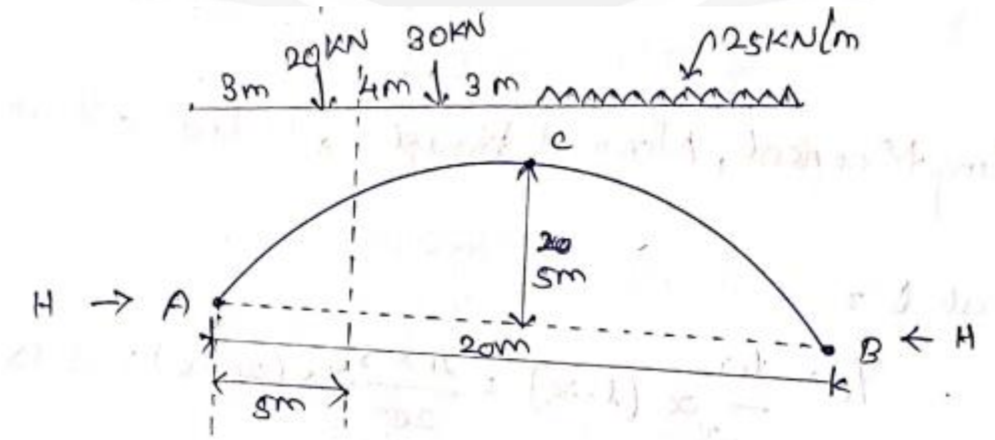

Fig. 3.3.4

SOLUTION

Taking moment about A

$(20x3)+(30x7)+[25x10x(10+10/2)]-VBx20 = 0$

$VB = 201$ KN

$VA = 99$ KN

Horizontal pull

$(H\times5)+(20\times7)+(30\times3)-VA\times10 = 0$

$5H-140+90-990 = 0$

$H = 152$ KN

Resultant Reaction (RA and RB)

$RA = \sqrt{H^2+VA^2}$

$= \sqrt{152^2+99^2}$

$=181.39$KN

$RB = \sqrt{H^2+VB^2}$

$= \sqrt{152^2+201^2}$

$=252$ KN

$\emptyset = \tan^{-1} VA/H$

$=\tan^{-1} 99/152$

$=33°4'36''.6$

$\emptyset = \tan^{-1} VB/H$

$=\tan^{-1} 201/152$

$=52°54'9''.86$

Bending moment, Normal thrust, radial SF at D

BM at D

Y_D $=4r/l^2\ x(l-x)$

$=4\times5/20^2\ X\ 5(20-5)$

$=3.75$ m

BMD $=+VA\times5\overset{+}{-}H_{YD}\overset{+}{-}20\times2$

$=495\overset{+}{-}570\overset{+}{-}40$

$=-11.5$ KNm

Slope of the arch at D

Slope $=\tan^{-1}[4r/l^2\ (l-2x)]$

$=\tan^{-1}[4\times5/20^2(20-2\times5)]$

$=26°33'55''.18$

Normal thrust

P =Vxsin + H cos

Vx =net beam shear force

Vx = VA -20

=99-20

= 79 KN

P =79 sin26°33'55''.18+152 cos26°33'55''.18

= 171.28KN

Radial shear force

F =vx cos - H sin

= 79 cos - 152 sin

=2.683 KN

Max BM in CB

BMx =VBx - Wx^2/2-Hyx

Yx =4r/l^2 x(l- x)

=4×5/20^2 X x(20-x)

=0.05x(20-x)

Mx = 201x-25×x^2/2 -152(0.05x(20-x))

=201x-12.5x^2 7.6x(20-x)

=201x -12.5x^2 -152x +7.6x^2

Mx =49x - 4.9x^2

Diff W.r to x

dM/dX =49-9.8 x

BM to be max

9.8x-49 =0

x =5m

Mx =49(x5)-4.9(5^2)

=122.5 KN

Example:

A symmetrical three hinged parabolic arch of span 40m and rise 8m carries a UDL of 30KN/m, over the left half of the span. The hinges are provide at the support and at the centre of the arch. Calculation the reaction at the support.als calculate the bending moment, radial shear and normal thrust a distance of 10m from the left support.

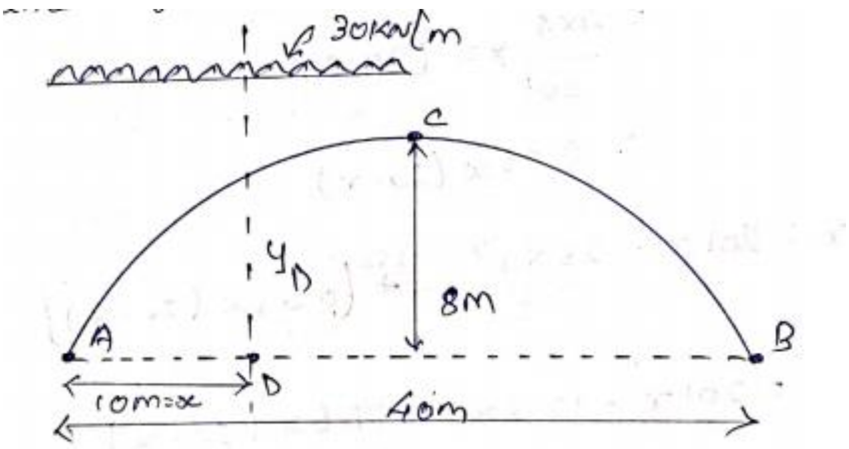

Fig. 3.3.5

Solution

Vertical Components VA and VB

Taking moment about A

$$VB\times40-30\times20^2/2 = 0$$

$$VB = 150KN$$

$$VA = \text{total load} - VB$$

$$= 30\times20-150$$

$$= 450KN$$

Horizontal components

$$VA\times20-H\times8-30\times20\times20/2 = 0$$

H =375KN

Resultant Reaction RA,RB

RA $=\sqrt{H^2+VA^2}$

$=\sqrt{450^2+375^2}$

=585.71KN

RB $=\sqrt{H^2+VB^2}$

$=\sqrt{150^2+375^2}$

=403.89KN

Bending moment at 10m from A

Y $=4r/l^2\ x(l-x)$

$=4\times 8/40^2\times 10(40-10)$

=6m

Bending moment at 10m

$=VA(10)-HA(Y)-30\times 10\times 10/2$

=450(10)-(375y)-30(50)

=3000-375Y

=3000-375(6)

=750KNm

Radial shear force at x=10m

R $=V x \cos\theta - H \sin\theta$

Vx =VA-30×10

=450-300

=150KN

slope at D

θ $=\tan^{-1}[4r/l^2 (l-2x)]$

θ $=\tan^{-1}[4\times8/40^2(40-2\times10)]$

=21°48'

R =150 cos 21°48'-375 sin 21°48'

= 0

Normal thrust

P = Vx sinθ+H cosθ

=150sin21°48'+375 cos 21°48'

=403.89KN

Example:

A three hinged parabolic arch of 40m span has abutments at unequal levels. The highest point of the arch is 4m above the left support and 9m above the right support abutments. The arch is subjected to an UDL of 15KNm over its entire horizontal span. Find the horizontal thrust and bending moment at the point 8m from the left support.

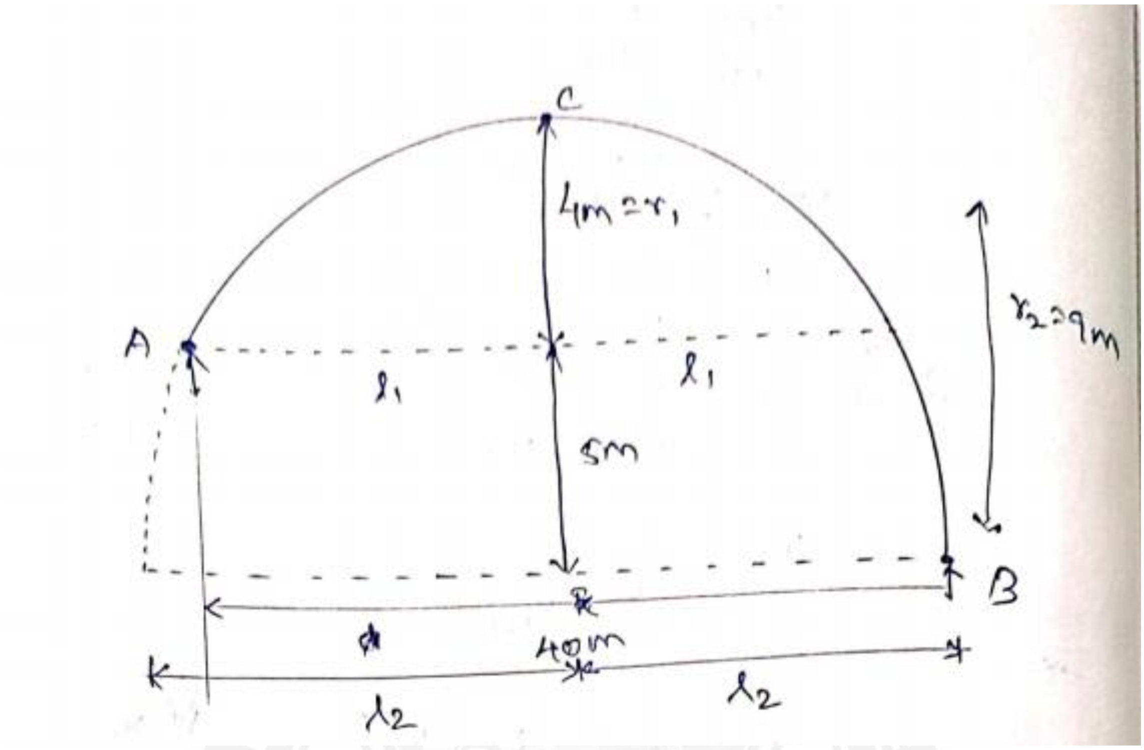

Fig. 3.3.6

Solution

Reaction A,B and H

Find l1, l2

$$l_1/l_2 = \sqrt{r_1/r_2}$$

$$l_1/(40-l_1) = \sqrt{4/9}$$

$$l_1 = (40-l_1)\times 2/3$$

$$l_1 = 16m$$

$$l_2 = 40-l_1$$

$$= 40-16$$

$$= 24m$$

considering left slab of C

VA(16)-4H-15×16×16/2 =0

16VA-4H-1920 =0

4VA-H-480 =0 _____(1)

Considering the right slab of C

_VB(24)+H(9)+15×24×24/2 =0

-24VB+9H+4320 =0

-8VB+3H+1440 =0_____(2)

VA+VB =600

VB =600-VA_____(3)

Sub 3 in 2

-8(600-VA)+3H+1440 =0

8VA-4800+3H+1440 =0

8VA+3H-3360 =0______(4)

1&4 solve

4VA-H-480 =0 **8VA-2H-960 =0**

VA-3H-3360 =0 **8VA+3H-3360 =0**

×4= -4VA+12H+13440 =0 - - +

11H +12960 = 0 -5H =2400

H =480KN

4VA -H -480 =0

4VA -480-480 =0

VA =240KN

VB =360 KN

Bending moment x= 8m

BMx =VA(8)-15×8×8/2-Hy

$Y = 4r/l^2\ x(l - x)$

$Y = 4\times4/(2\times16)^2\ x8(2\times16 - 8)$

Y =3m

BM = 240×8-15×32-480×3

= 0

Radial shear

R =Vxcosθ-H sinθ

Vx =VA-(15×8)

=240-(15×8)

=120KN

$\theta = \tan^{-1}[4r/l^2\ (l - 2x)]$

$\theta = \tan^{-1}[4\times4/(2\times16)^2\ (32 - 2\times8)]$

=14°2'

F = 120 cos 14°2' -480 sin 14°2'

F = 0

Normal thrust

$$P_N = V\times \sin\theta + H \cos\theta$$

$= 120 \sin 14°2' + 480 \cos 14°2'$

$= 494.77$ KN

Example:

A three hinge arch is circular 25m in span with a central rise of 5m.It is loaded with a concentration load of 10KN at 7.5m from the left hand hinge. Find the horizontal thrust, reaction at each end hinge bending moment under the load.

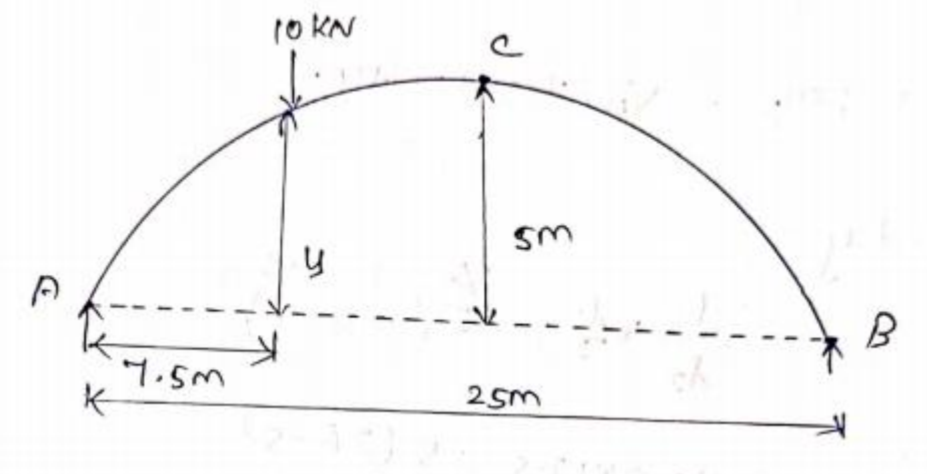

Fig. 3.3.7

solution

Vertical reaction VA and VB

Taking moment about A

$VB\times25-10\times7.5 = 0$

$VB = 3KN$

$VA = 7KN$

Horizontal thrust

$VB \times 12.5 - H \times 5 = 0$

$3 \times 12.5 - H(5) = 0$

$H = 7.5KN$

Reaction RA and RB

$RA = \sqrt{H^2 + VA^2}$

$= \sqrt{7^2 + 7.5^2}$

$= 10.26$ KN

$RB = \sqrt{VB^2 + H^2}$

$= \sqrt{3^2 + 7.5^2}$

$= 8.08$ KN

Bending moment under the load

$BMD = VA(7.50) - Hy$

Find y

$l/l2 \times l/2 = 5\ (2R-5)$

$12.5 \times 12.5 = 5\ (2R-5)$

$R = 18.125m$

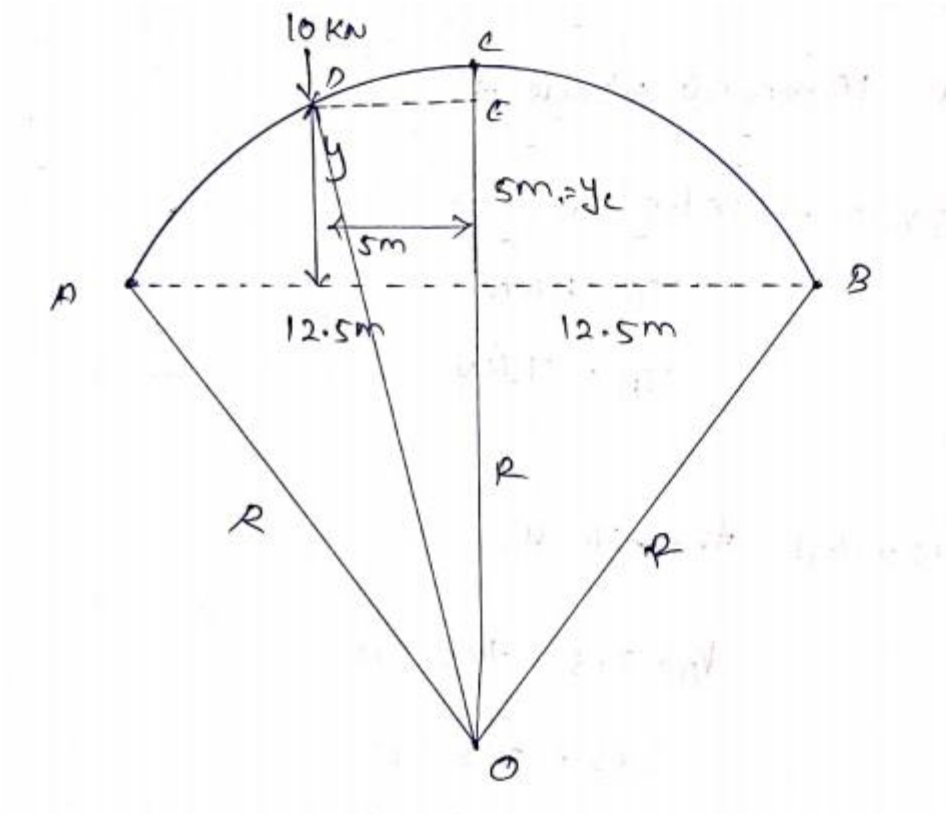

Fig. 3.3.8

$R^2 = (R - Yc + y)^2 + x^2$

$18.125^2 = (18.125 - 5 + y)^2 + 5^2$

$303.515 = (13.125 + Y)^2$

$17.421 = 13.125 + y$

$Y = 4.3m$

BMD =7(7.5)-7.5(4.3)

=20.25KNm