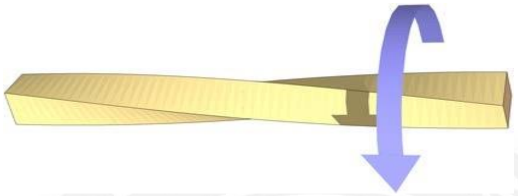
TORSIONAL STRESS AND DEFORMATION

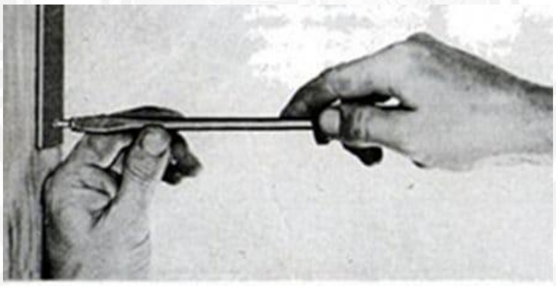
1.4. Torsional Stress and Deformations

Shear stress is produced about a longitudinal axis of a structural member by the application of twisting couple to the end of the structural member is known as



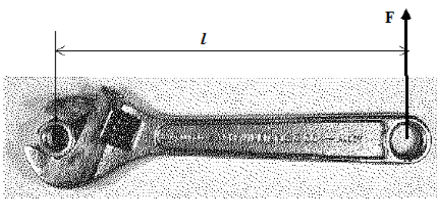
torsional stress. Torsion is the twisting of a straight bar when it is loaded by twisting couple or torque. It tends to produce rotation about the longitudinal axes of the bar.

For instant, when we turn a screw driver to produce torsion our hand applies torque "*T*" to the handle and twist the shank of the screw driver.



1.5. Twisting Couple (Torque)

The twisting of a structural member about its longitudinal axis by two equals and opposite torques is expressed through a certain angle is called twisting couple. The stress is produced in this process is not tensile or compressive, it is said to be



shearing or shear stress. The strain is measured by an angle in unit of radians.

The simple example is that of using a wrench to tighten a nut on a bolt as shown in given figure. If the bolt, wrench, and force are all perpendicular to one another.

1.6. TWISTING COUPLE ON A WIRE

If we have a wire or cylinder, clamped at one end, and twisted at the other through an angle θ, about its axis, it is said to be under tension, due to the elasticity of the material of the wire or the cylinder, a restoring couple is set up in it, equal and opposite to the twisting couple.

Consider a cylindrical wire of length l and radius a. The cylindrical wire is clamped to a fixed support. This wire is made up of a number of cylindrical tubes (co-axial) whose radii vary from zero to ‘r’. Let us consider one such cylinder, as shown in fig (b) with radius ‘x’ and thickness ‘dx’.

Let AB be a line on the elementary tube which is parallel to the axis of the tube. Consider a couple applied at the bottom end of the wire, which results in twisting of the wire through an angle θ. In the twisted state, the position of AB will be taken as AB’ as shown in fig 1.6.1

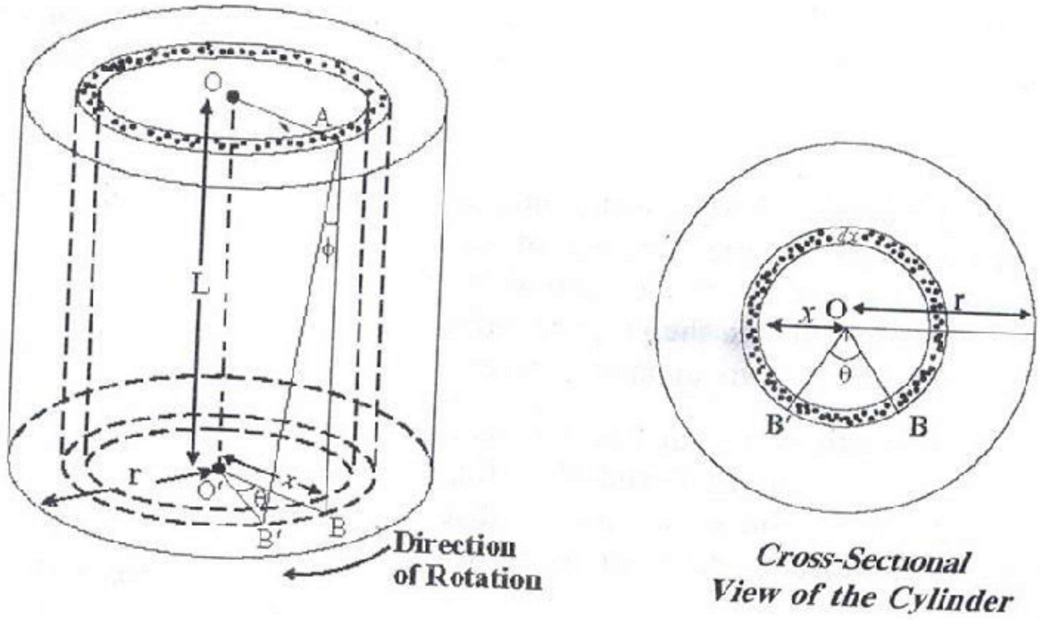


Fig (a) Fig (b)

Fig 1.6.1Twisting couple on a wire.

From fig (a)

The angle $ABB' = \phi$

$$BB' = l\phi \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The displacement decreases, as the move from the rim of the cylinder to the Centre. At the Centre, it will become zero. This means that shearing strain is maximum at the rim and minimum at the Centre.

Consider a hollow cylinder along the plane AB and flattened out. Therefore, we get a rectangle OABO' before twisting and OABB' after twisting. The angle through which this the hollow cylinder is sheared.

From the cross sectional view of the cylinder

$$BB' = x\theta \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From equations (1) and (2), we get,

$$l\phi = x\theta$$

$$\text{Shearing strain } \phi = \frac{x\theta}{l} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

We know that,

$$\text{Rigidity modulus (n)} = \frac{\text{shearing stress}}{\text{shearing strain}}$$

$$\text{Shearing stress} = n \times \text{Shearing strain}$$

$$= n\frac{x\theta}{l} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Let the area of the elementary tube = $2\pi x\, dx$

The shearing force on this area = Shearing stress $\times$ Area

$$= n\frac{x\theta}{l} \times 2\pi\, x\, dx$$

$$= \frac{2\pi n\, \theta x^2 dx}{l} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The moment of force about the axis of the wire $= \frac{2\pi n\theta x^2 dx}{l} \times x$

$$= \frac{2\pi n\theta x^3 dx}{l} \qquad \ldots\ldots\ldots\ldots (6)$$

The twisting couple applied to the whole wire can be obtained by integrating equation 6 between the limit $x = 0$ *and* $x = r$

$$= \int_0^r \frac{2\pi n\theta x^3 dx}{l}$$

$$= \frac{2\pi n\theta}{l}\int_0^r x^3 dx$$

$$= \frac{2\pi n\,\theta}{l}\left.\frac{x^4}{4}\right|_0^r$$

$$C = \frac{1}{2}\frac{n\pi r^4\theta}{l} \qquad \ldots\ldots\ldots\ldots\ldots (7)$$

If θ=1 radian

The twisting couples per unit angular twist of the wire

$$C = \frac{1}{2}\frac{n\pi r^4}{l}$$

1.6.1. TORSIONAL PENDULUM

When a body is fixed at one end and twisted about its axis by means of a torque at the other end, then the body is said to be under torsion. The torsion involves shearing strain and hence modulus involved is the rigidity modulus.

Torsional pendulum consists of a suspension wire with one end is fixed and the other end is fixed to the center of the circular disc. Let l be the length of the suspension wire and r be the radius of the suspension wire. When a heavy circular disc is rotated in a horizontal plane, so that the wire is twisted through an angle θ. The various elements of the wire will undergo shearing strain and restoring couple is produced. Now if the disc is released, the disc will produce torsional oscillations. The couple acting on the disc produces an angular acceleration in it, which is proportional to the angular displacement and is always directed towards its mean position.

Total energy of the torsion pendulum = P.E +K.E

The potential energy confined to the wire is equal to the work done in twisting the disc, thereby creating a restoring couple (C).

The restoring couple through an angle $\theta = \int_0^\theta moment$ of couple $\times$ dθ

$$= \int_0^\theta C\theta d\theta = \frac{C\theta^2}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Let ω be the angular velocity with which the disc oscillates, due to the restoring couple, then

The kinetic energy of the rotating disc (deflecting couple) = $\frac{I\omega^2}{2}$ …………… (9)

I be the moment of inertia of the circular disc

$$\text{Total energy} = \frac{C\theta^2}{2} + \frac{I\omega^2}{2} = \text{constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Differentiating equation (4) with respect to 't' we get,

$$C\theta\frac{d\theta}{dt} + I\omega\frac{d\omega}{dt} = 0 \qquad \ldots\ldots\ldots\ldots (11)$$

Since the angular velocity $\omega = \frac{d\theta}{dt}$ and the angular acceleration $\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

$\because$ Equation (11) becomes

$$C\theta\frac{d\theta}{dt} + I\frac{d\theta}{dt}\frac{d^2\theta}{dt^2} = 0$$

$$\frac{d\theta}{dt}\left[C\theta + I\frac{d^2\theta}{dt^2}\right] = 0$$

Here $\frac{d\theta}{dt} \neq 0$ $\quad \therefore \left[C\theta + I\frac{d^2\theta}{dt^2}\right] = 0$

$\therefore$ Angular acceleration= $\frac{d^2\theta}{dt^2} = -\frac{C\theta}{I}$ …………. (12)

The negative sign indicates that the couple tends to decrease the twist on the wire.

TIME PERIOD OF TORSIONAL OSCILLATION

The time period of torsional oscillation T= $2\pi\frac{angular\ displacement}{angular\ acceleration}$

$$T = 2\pi\sqrt{\frac{\theta}{C\theta/I}} = 2\pi\sqrt{\frac{I}{C}} \quad \text{………. (13)}$$

$$\boxed{\text{Frequency of oscillation } f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{C}{I}}} \quad \text{………. (14)}$$

RIGIDITY MODULUS OF THE WIRE

Let l be the length of the suspension wire and r be the radius of the suspension wire. We know

The twisting couple per unit angular twist of the wire $C = \frac{1}{2}\frac{n\pi r^4}{l}$ ………… (15)

Substituting equation (9) in equation (7) we get

$$T = 2\pi\sqrt{\frac{I}{n\pi r^4/2l}} = 2\pi\sqrt{\frac{2Il}{n\pi r^4}}$$

$$T^2 = 4\pi^2\frac{2Il}{n\pi r^4} = \frac{8\pi Il}{nr^4}$$

$$\boxed{\text{Rigidity modulus of the wire } n = \frac{8\pi Il}{T^2 r^4}} \quad \text{……….. (16)}$$

Thus torsional pendulum is used to find the rigidity modulus of the various materials.

Experimental verification of torsional pendulum

A torsion pendulum is constructed as shown in Figure.

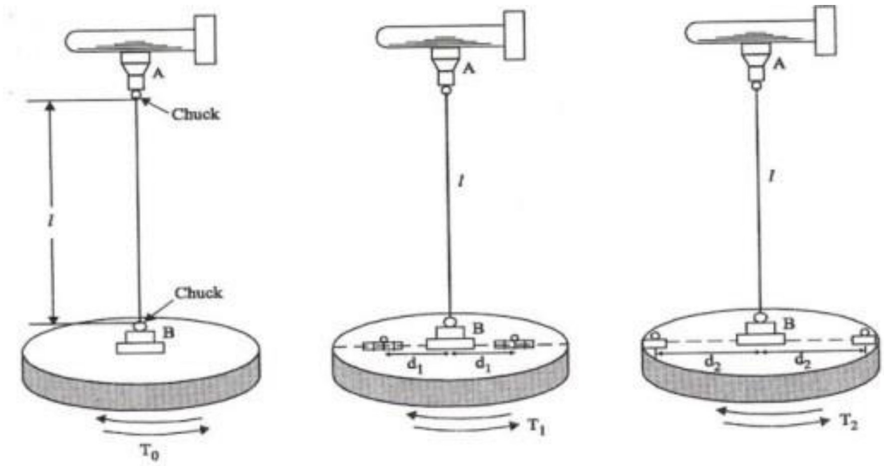


Fig.1.6.1.2 Torsion pendulum

Measure carefully the length of the suspension wire between the two chucks. Standing in front of the pendulum, gently set it in torsional oscillation without any lateral movement.

Note the time for 10 oscillations. T_0, the period of oscillation of the pendulum without any masses in it calculated. Take two readings. Find the mean.

Two equal symmetrical masses (*m*) are placed on the disc on either side, close to the suspension wire. The closest distance 'd_1' from the center of the symmetrical mass and the center of the suspension wire is found. Set the pendulum to oscillate and note the time for 10 oscillations. From that the period of oscillation $T_1$is calculated. Take two readings find the mean.

Two equal masses are now moved to the extreme ends so that the edges of masses coincide with the edge of the disc and the centers are equaled distant. The distance 'd2' from the center of the symmetrical mass and the center of the suspension wire is noted. Set the pendulum to oscillate and note the time for 10 oscillations. Take two readings. Calculate the mean period of oscillation T_2. All the measured parameters are tabulated in the given table,

Position of the Symmetrical masses	**Time taken for 10 oscillations**			**Time period**	**Square of the time period**
	Trail 1	**Trail 2**	**Mean**		
Unit	*s*	*s*	*s*	*s*	*s*
Without any Masses				T_0 =	T_0^2=
With masses at closest distance d_1=……..×10^{-2} m				T_1 =	T_1^2=
With masses at farther distance d_1=… … … .×10^{-2} *m*				T_2 =	T_2^2=

Measure carefully, the diameter (2*r*) of the wire at various places, with a screw gauge. Find the mean of the diameter and calculate the radius. Note the mass (*m*) of the one symmetrical mass. The moment of inertia of the disc and rigidity modulus of the wire is calculated using the formula

$$n = \frac{8\pi I l}{T^2 r^4}$$