

UNIT IV

INTERPOLATION, NUMERICAL DIFFERENTIATION AND INTEGRATION

Interpolation and Extrapolation:

Interpolation is the process of finding the values of $f(x)$ for intermediate values of x in the given interval. Extrapolation is the process of finding the values of $f(x)$ for extreme values of x

4.1 Lagrange's Interpolation formula

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 \\ + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

Inverse Lagrange's Interpolation

$$x = f(y) = \frac{(y - y_1)(y - y_2)(y - y_3)}{(y_0 - y_1)(y_0 - y_2)(y_0 - y_3)} x_0 + \frac{(y - y_0)(y - y_2)(y - y_3)}{(y_1 - y_0)(y_1 - y_2)(y_1 - y_3)} x_1 \\ + \frac{(y - y_0)(y - y_1)(y - y_3)}{(y_2 - y_0)(y_2 - y_1)(y_2 - y_3)} x_2 + \frac{(y - y_0)(y - y_1)(y - y_2)}{(y_3 - y_0)(y_3 - y_1)(y_3 - y_2)} x_3$$

1. Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for

x	0	1	2	5
$f(x)$	2	3	12	147

Solution:

Given,

x	x_0	x_1	x_2	x_3
	0	1	2	5
$f(x)$	y_0	y_1	y_2	y_3
	2	3	12	147

Lagrange's Interpolation formula

$$y = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 \\ + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

$$\begin{aligned}
 &= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}(2) + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}(3) + \frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}(12) \\
 &\quad + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}(147) \\
 y = f(3) &= \frac{(3-1)(3-2)(3-5)}{-10}(2) + \frac{(3-0)(3-2)(3-5)}{4}(3) \\
 &\quad + \frac{(3)(3-1)(3-5)}{-6}(12) + \frac{(3)(3-1)(3-2)}{60}(147) \\
 = f(3) &= \frac{(2)(1)(-2)}{-10}(2) + \frac{(3)(1)(-2)}{4}(3) + \frac{(3)(2)(-2)}{-6}(12) + \frac{(3)(2)(1)}{60}(147) \\
 &= \frac{4}{10}(2) - \frac{6}{4}(3) + 2(12) + \frac{1}{10}(147) \\
 &= \frac{8}{10} - \frac{18}{4} + 24 + \frac{147}{10} = 35.
 \end{aligned}$$

2. Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data:

Year	1997	1999	2001	2002
Profit in lakhs of Rs.	43	65	159	248

Solution :

$$\begin{aligned}
 y = f(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 \\
 &\quad + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3 \\
 = f(x) &= \frac{(2000-1999)(2000-2001)(2000-2002)}{(1997-1999)(1997-2001)(1997-2002)}43 \\
 &\quad + \frac{(2000-1997)(2000-2001)(2000-2002)}{(1999-1997)(1999-2001)(1999-2002)}65 \\
 &\quad + \frac{(2000-1997)(2000-1999)(2000-2002)}{(2001-1997)(2001-1999)(2001-2002)}159 \\
 &\quad + \frac{(2000-1997)(2000-1999)(2000-2001)}{(2002-1997)(2002-1999)(2002-2001)}248 \\
 &= \frac{(1)(-1)(-2)}{(-2)(-4)(-5)}43 + \frac{(3)(-1)(-2)}{(2)(-2)(-3)}65 + \frac{(3)(1)(-2)}{(4)(2)(-2)}159 + \frac{(4)(1)(-1)}{(5)(3)(1)}248 \\
 &= \frac{2}{-40}43 + \frac{6}{12}65 + \frac{-6}{-16}159 + \frac{-4}{15}248 \\
 &= -2.15 + 32.5 + 59.625 - 66.13 = 23.845
 \end{aligned}$$

3. Using Lagrange's inverse interpolation formula, find the value of x when $y=20$ from the given data

x	1	2	3	4
y	1	8	27	64

Solution:

$$\begin{aligned}
 x = f(y) &= \frac{(y - y_1)(y - y_2)(y - y_3)}{(y_0 - y_1)(y_0 - y_2)(y_0 - y_3)} x_0 + \frac{(y - y_0)(y - y_2)(y - y_3)}{(y_1 - y_0)(y_1 - y_2)(y_1 - y_3)} x_1 \\
 &+ \frac{(y - y_0)(y - y_1)(y - y_3)}{(y_2 - y_0)(y_2 - y_1)(y_2 - y_3)} x_2 + \frac{(y - y_0)(y - y_1)(y - y_2)}{(y_3 - y_0)(y_3 - y_1)(y_3 - y_2)} x_3 \\
 &= f(y) = \frac{(20 - 8)(20 - 27)(20 - 64)}{(1 - 8)(1 - 27)(1 - 64)} 1 + \frac{(20 - 1)(20 - 27)(20 - 64)}{(8 - 1)(8 - 27)(8 - 64)} 2 \\
 &+ \frac{(20 - 1)(20 - y_1)(20 - 64)}{(27 - 1)(27 - 8)(27 - 64)} 3 + \frac{(20 - 1)(20 - 8)(20 - 27)}{(64 - 1)(64 - 8)(64 - 27)} 4 \\
 &= \frac{12 * -7 * -44}{(-7)(-26)(-63)} 1 + \frac{-19 * -7 * -44}{(7)(-19)(-56)} 2 + \frac{(19)(12)(-44)}{(26)(19)(-37)} 3 + \frac{(19)(12)(-7)}{(63)(56)(37)} 4 \\
 &= 2.8468
 \end{aligned}$$