ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

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OBSERVE OPTIMIZE OUTSPREAD

ROHINI

COLLEGE OF ENGINEERING & TECHNOLOGY

Unit-III

PROPERTIES OF SURFACES AND SOLIDS

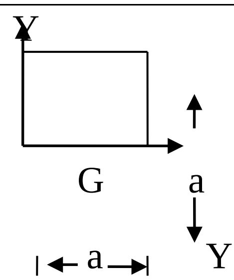
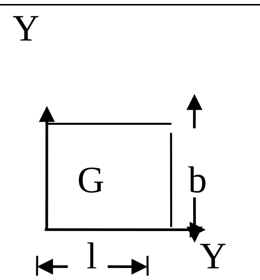
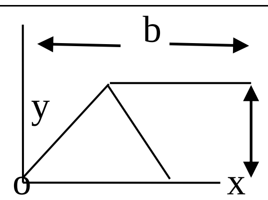
Centroid:

Centroid is defined as a point on a surface the whole area of the surface acts.

Centre of gravity:

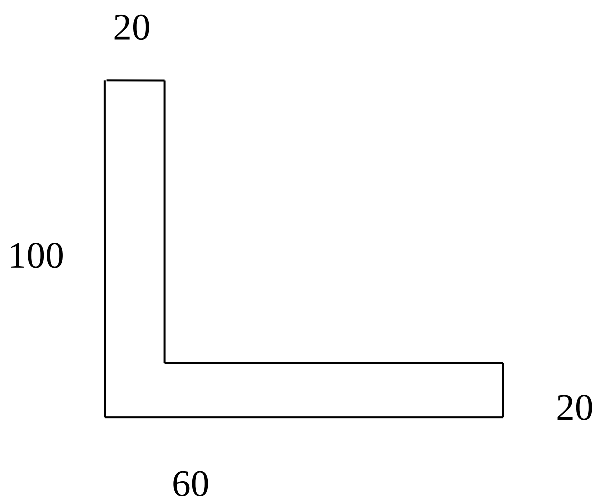
Centre of gravity is defined as the point through which the entire weight of the body acts.

Centroid of simple plane figure:

Sl.No	Name	Shape	X	Y	Area
1.	Square		$\frac{a}{2}$	$\frac{a}{2}$	a^2
2.	Rectangle		$\frac{l}{2}$	$\frac{l}{2}$	lb
3.	Triangle (Isosceles)		$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2}$bh

Problem 1:

Determine the Centroid of L section

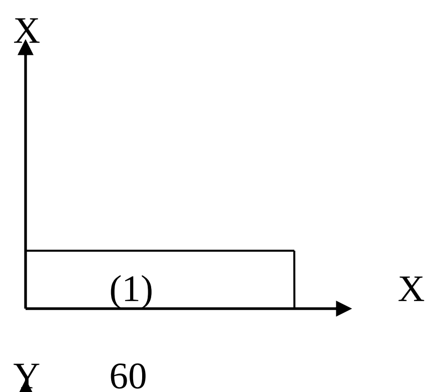


Centroid

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

Section (1)

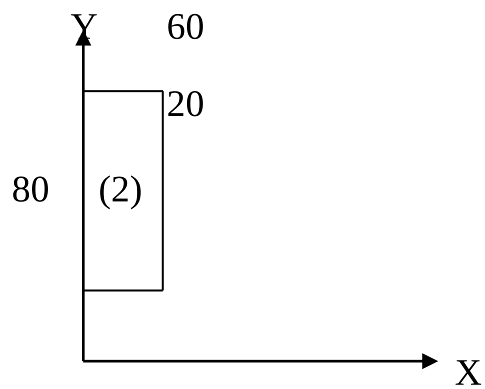


$$a_1 = 60 \times 20 = 1200mm^2$$

$$x_1 = \frac{60}{2} = 30mm$$

$$y_1 = \frac{20}{2} = 10mm$$

Section (2)



$$a_2 = 20 \times 80 = 160mm^2$$

$$x_2 = \frac{20}{2} = 10mm$$

$$y_2 = 20 + \frac{80}{2} = 60mm$$

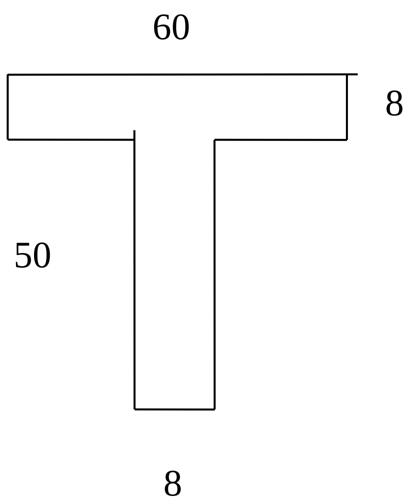
$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{1200\times30+1600\times10}{1200+1600}$$

$$\bar{X} = 18.57mm$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{1200\times10+1600\times60}{1200+1600}$$

$$\bar{Y} = 38.57mm$$

2. Find the Centroid of T section



Section (1)

$$a_1 = 8 \times 50 = 400mm^2$$

$$x_1 = 26 + \frac{8}{2} = 30mm$$

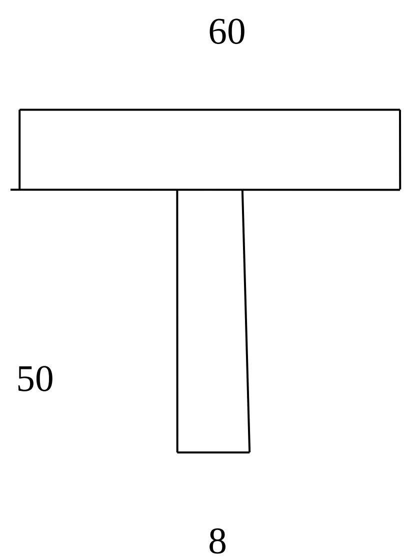
$$y_1 = \frac{50}{2} = 25mm$$

Section(2)

$$a_2 = 60 \times 8 = 480mm^2$$

$$x_2 = \frac{60}{2} = 30mm$$

$$y_2 = 50 + \frac{8}{2} = 54mm$$



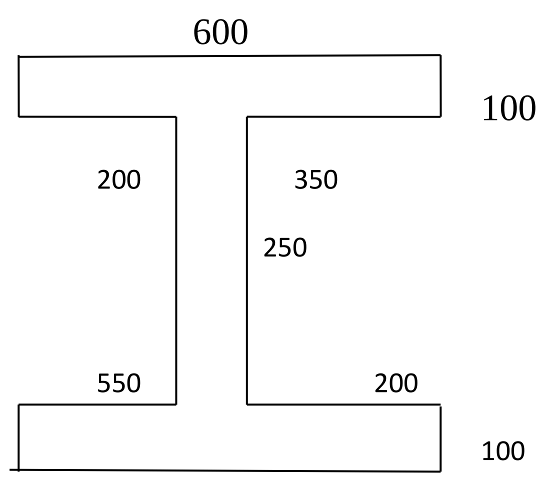
$\bar{X} = \frac{a_1x_1+a_2x_2}{a_1+a_2} = \frac{400\times30+480\times300}{1200+480}$

$\bar{X} = 30mm$

$\bar{Y} = \frac{a_1y_1+a_2y_2}{a_1+a_2} = \frac{400\times25+480\times54}{400+480}$

$\bar{Y} = 40.81mm$

3. Locate the Centroid of the I section shown.



Soln:

$$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$$

$$\bar{Y} = \frac{a_1y_1+a_2y_2+a_3y_3}{a_1+a_2+a_3}$$

Section (1)

$a_1 = 800 \times 100 = 80000mm^2$

$x_1 = \frac{800}{2} = 400mm$

$y_1 = \frac{100}{2} = 50mm$

Section(2)

$$a_2 = 250 \times 100 = 25 \times 10^3 mm^2$$

$$x_2 = 550 + \frac{100}{2} = 600mm$$

$$y_2 = 100 + \frac{250}{2} = 225mm$$

Section (3)

$$a_3 = 600 \times 100 = 60 \times 10^3 mm^2$$

$$x_3 = 350 + \frac{600}{2} = 650mm$$

$$y_3 = 100 + 250 + \frac{100}{2} = 400mm$$

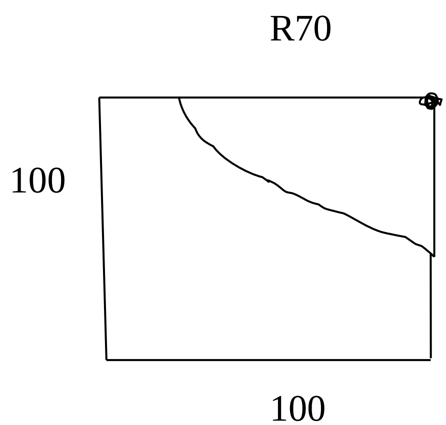
$$\bar{X} = \frac{(80\times10^3\times400)+(25\times10^3\times600)+(60\times10^3\times650)}{80\times10^3+25\times10^3+60\times10^3}$$

$$\bar{X} = 521.21mm$$

$$\bar{Y} = \frac{(80\times10^3\times50)+(25\times10^3\times225)+(60\times10^3\times400)}{80\times10^3+25\times10^3+60\times10^3}$$

$$\bar{Y} = 203.78mm$$

4. Locate the Centroid of the Area.

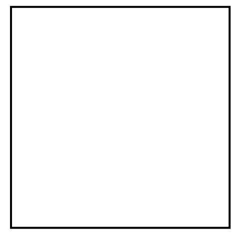


Soln:

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 + a_2}$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 + a_2}$$

Section (1)



$$a_1 = 100 \times 100 = 10 \times 10^3 mm^2$$

$$x_1 = \frac{100}{2} = 50mm$$

$$y_1 = \frac{100}{2} = 50mm$$

Section (2)

$$a_2 = \frac{1}{4} \times \pi r^2 = \frac{1}{4} \times \pi \times 70^2 = 38.48mm^2$$

$$x_2 = \frac{4r}{3\pi} = \frac{4 \times 70}{3\pi} = 70.29mm$$

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 70}{3\pi} = 70.29mm$$

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 + a_2}$$

$$\bar{X} = \frac{(10 \times 10^3 \times 50) - (3848 \times 70.29)}{10 \times 10^3 - 3848}$$

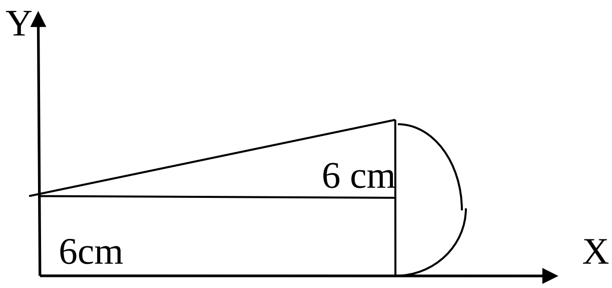
$$\bar{X} = 37.3mm$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

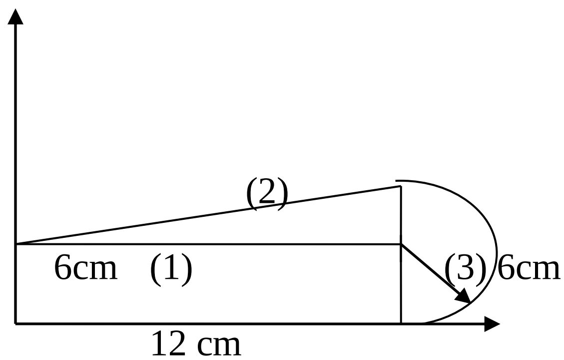
$$\bar{Y} = \frac{(10 \times 10^3 \times 50) - (3848 \times 70.29)}{10 \times 10^3 - 3848}$$

$$\bar{Y} = 373mm$$

5. Locate the Centroid of zero shown in fig.



Soln:



$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Section (1)

$a_1 = 12 \times 6 = 72cm$

$$X_1 = \frac{12}{2} = 6cm$$

$$Y_1 = \frac{6}{2} = 3\ cm$$

Section (2) triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 12 \times 6$$

$a_2 = 36cm$

$$x_2 = \frac{b}{3} = \frac{12}{3} = 4cm$$

$$y_2 = 6 + \frac{h}{3} = 6 + \frac{6}{3} = 8cm$$

Section (3) Semi circle

$$a_3 = \frac{1}{2} \times \frac{\pi d^2}{4^2} = \frac{1}{2} \times \frac{\pi}{4} \times 12^2 = 56.24cm$$

$$x_3 = 12 + \frac{4r}{3\pi} = 12 + \frac{4 \times 6}{3\pi} = 14.5cm$$

$$y_3 = \frac{d}{2} = \frac{12}{2} = 6cm$$

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(72 \times 6) + (36 \times 4) + (56.24 \times 14.5)}{72 + 36 + 56.24}$$

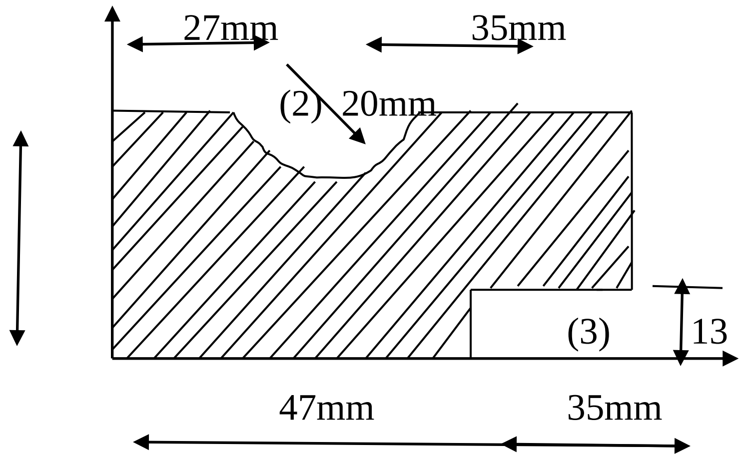
$$\bar{X} = 8.47cm$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(72 \times 3) + (36 \times 8) + (56.24 \times 6)}{72 + 36 + 56.24}$$

$$\bar{Y} = \frac{841.44}{164.24}$$

$$\bar{Y} = 5.12cm$$

5. Find the Centroid of the shaded area shown below



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

Soln:

$$a_1 = 82 \times 40 = 3280mm^2$$

$$x_1 = \frac{82}{2} = 41mm$$

$$y_1 = \frac{40}{2} = 20mm$$

Section (2)

$$a_2 = \frac{1}{2} \times \frac{\pi}{4} \times d^2 = \frac{1}{2} \times \frac{\pi}{4} \times 40^2 = 628.31mm^2$$

$$x_2 = 27mm$$

$$y_2 = 40 - \frac{4r}{3\pi} = 40 - \frac{4 \times 20}{3\pi} = 31.51mm$$

Section (3)

$$a_3 = 35 \times 13 = 455mm^2$$

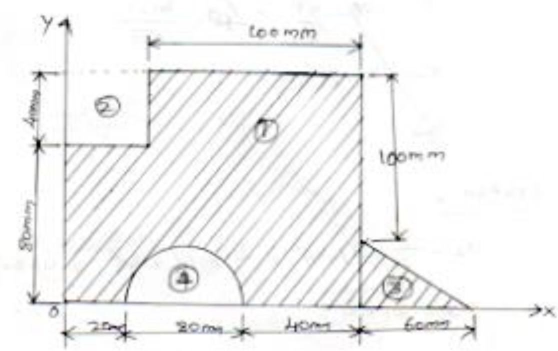
$$x_3 = 82 - \frac{35}{2} = 64.5mm$$

$$y_3 = \frac{13}{2} = 6.5\ mm$$

$$\bar{X} = \frac{(3280 \times 41) - (628.31 \times 27) - (455 \times 64.5)}{3280 - 628.31 - 455} = 40.14mm$$

$$\bar{Y} = \frac{(3280 \times 20) - (628.31 \times 31.51) - (455 \times 6.5)}{3280 - 628.31 - 455} = 19.50mm$$

7. Determine the centroid co-ordinates of the area shown in fig. below with respect to the shown x-y coordinate system.



Soln:

$$\bar{X} = \frac{a_1x_1 - a_2x_2 + a_3x_3 - a_4x_4}{a_1 - a_2 + a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 + a_3y_3 - a_4y_4}{a_1 - a_2 + a_3 - a_4}$$

Section (1)

$$a_1 = 140 \times 120 = 16800mm^2$$

$$x_1 = \frac{140}{2} = 70mm$$

$$y_1 = \frac{120}{2} = 60mm$$

Section (2)

$$a_2 = 40 \times 40 = 1600mm^2$$

$$x_2 = \frac{b}{2} = \frac{40}{2} = 20mm$$

$$y_2 = 80 + \frac{40}{2} = 20mm$$

Section (3)

$$a_3 = \frac{1}{2}bh = \frac{1}{2} \times 60 \times 20 = 600mm^2$$

$$x_3 = 140 + \frac{b}{3} = 140 + \frac{60}{3} = 160mm$$

$$y_3 = \frac{h}{3} = \frac{20}{3} = 6.66mm$$

Section (4)

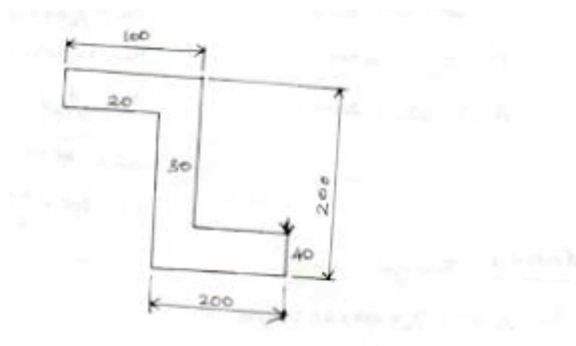
$$a_4 = \frac{1}{2} \times \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (80^2) \times \frac{1}{2} = a_4 = 2513mm^2$$

$$x_4 = 20 + \frac{d}{2} = 20 + \frac{80}{2} = 60mm$$

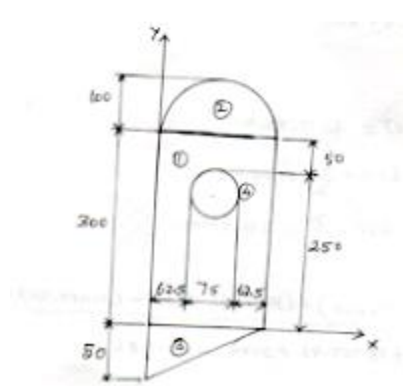
$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 40}{3\pi} = 16.97mm$$

$\bar{X} = 81.97mm \qquad \bar{Y} = 60.97mm$

8. Determine the centroid of Z section



9. Locate the centroid of the plane area shown in fig:



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3 - a_4x_4}{a_1 + a_2 + a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 - a_4y_4}{a_1 + a_2 + a_3 - a_4}$$

Section (1)

Rectangle

$$a_1 = 200 \times 400 = 600mm^2$$

$$x_1 = \frac{200}{2} = 100mm$$

$$y_1 = \frac{400}{2} = 200mm$$

Section (2)

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times 100^2$$

$$a_2 = 15707.96mm^2$$

$$x_2 = \frac{d}{2} = \frac{200}{2} = 100mm$$

$$y_2 = 300 + \frac{4r}{3\pi}$$

$$y_2 = 300 + \frac{4 \times 100}{3\pi} = 342.44mm$$

Section (3)

$$a_2 = \frac{1}{2} bh = \frac{1}{2} \times 200 \times 50 = 5000mm^2$$

$$x_3 = \frac{b}{3} = 66.66mm$$

$$y_3 = -\frac{50}{3} = -16.67mm$$

Section (4)

$$a_4 = \pi r^2 = \pi \times (37.5)^2 = 4417.86mm^2$$

$$x_4 = 62.5 + \frac{d}{2} = 62.5 + \frac{75}{2} = 100m$$

$$y_4 = 250 - \frac{d}{2} = 250 - \frac{75}{2} = 212.5\ mm$$

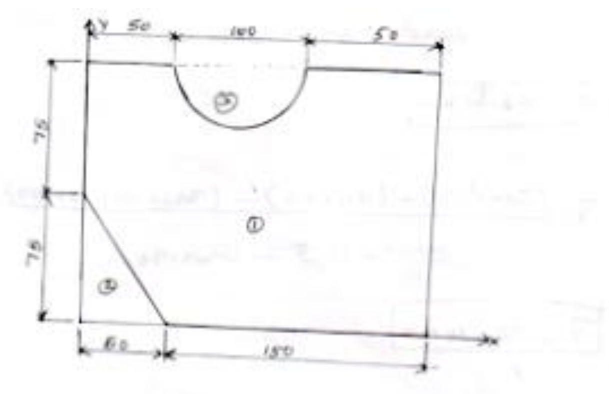
$$\bar{X} = \frac{(6000 \times 100) + (15707.96 \times 100) + (5000 \times 66.66) - (4417.86 \times 100)}{60000 + 15707.96 + 5000 - 4417.86}$$

$\bar{X} = 97.812mm$

$$\bar{Y} = \frac{(6000 \times 200) + (15707.96 \times 342.44) + \left(5000 \times (-16.67)\right) - (4417.86 \times 212.5)}{6000 + 15707.96 + 5000 - 4417.86}$$

$\bar{Y} = 175.07mm.$

10. Locate the centroid of the plane area shown in fig.



Soln:

$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$$

Section (1) Rectangle

$a_1 = 200 \times 150 = 30 \times 10^3 mm^2$

$$x_1 = \frac{200}{2} = 100mm$$

$$y_1 = \frac{150}{2} = 75mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 50 \times 75 = 1875mm^2$$

$$x_2 = \frac{b}{3} = \frac{50}{3} = 16.66mm$$

$$y_3 = \frac{h}{3} = \frac{75}{3} = 25mm$$

Section (3)

$$a_3 = \frac{\pi r^2}{2} = \frac{\pi \times 50^2}{2} = 3926.99mm^2$$

$$y_3 = 150 - \frac{4r}{3\pi} = 150 - \frac{4 \times 50}{3\pi} = 128.77mm$$

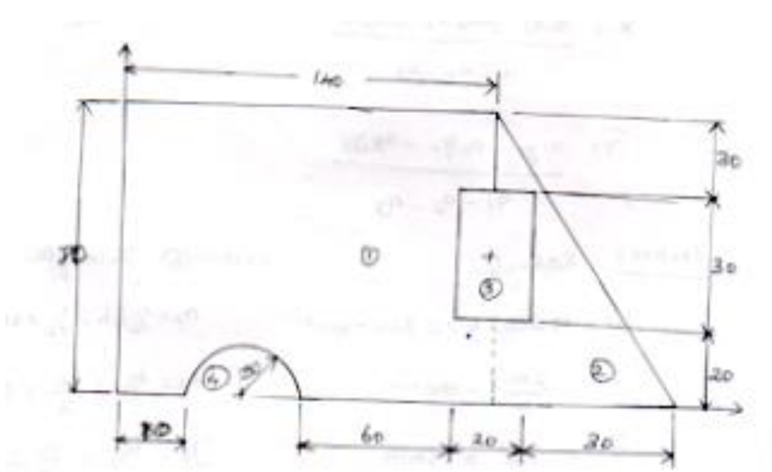
$$\bar{X} = \frac{(30 \times 10^3 \times 100) - (1875 \times 16.66) - (3926.99 \times 100)}{30 \times 10^3 - 1875 - 3926.99}$$

$$X = 105.45mm$$

$$\bar{Y} = \frac{(30 \times 10^3 \times 75) - (1875 \times 25) - (3926.99 \times 128.77)}{30 \times 10^3 - 1875 - 3926.99}$$

$$\bar{Y} = 70.14mm$$

11. Locate the centroid of the sectional area as shown in fig:



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 - a_3x_3 - a_4x_4}{a_1 + a_2 - a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 - a_3y_3 - a_4y_4}{a_1 + a_2 - a_3 - a_4}$$

Section (1) Rectangle

$$a_1 = 140 \times 80 = 11200mm^2$$

$$x_1 = \frac{140}{2} = 70mm$$

$$y_1 = \frac{80}{2} = 40mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 40 \times 80$$

$$a_2 = 1600mm^2$$

$$x_2 = 140 + \frac{40}{3} = 153.33mm$$

$$y_2 = \frac{h}{3} = \frac{80}{3} = 26.66mm$$

Section (3) Rectangle

$$a_3 = 20 \times 30 = 600mm^2$$

$$x_3 = 130 + \frac{20}{2} = 140mm$$

$$y_3 = 20 + \frac{30}{2} = 35mm$$

Section (4) Semicircle

$$a_4 = \frac{\pi r^2}{2} = \frac{\pi \times 30^2}{2} = 1413.71mm^2$$

$$x_4 = 10 + \frac{d}{2} = 10 + \frac{60}{2} = 40mm$$

$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 30}{3\pi} = 12.73mm$$

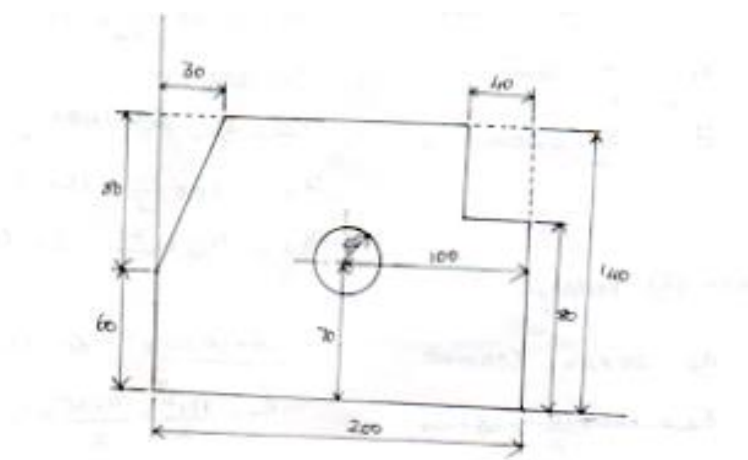
$$\bar{X} = \frac{(11200 \times 70) + (1600 \times 153.33) - (600 - 140) - (1413.71 \times 40)}{11200 + 1600 - 600 - 1413.71}$$

$$\overline{\mathrm{X}} = 82.396\mathrm{mm}$$

$$\bar{Y} = \frac{(11200 \times 40) + (1600 \times 26.68) + (600 \times 35) - (1413.71 \times 12.73)}{11200 + 1600 - 600 - 1413.71}$$

$$\bar{Y} = 41.875cm$$

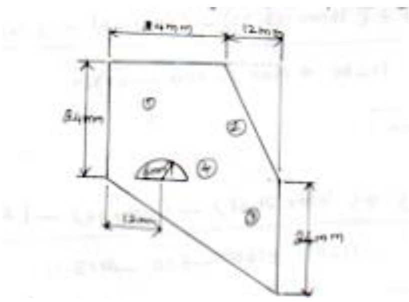
12. Find the centroid of the shaded area shown in fig.



Ans $X = 94.92mm$

$Y = 61.058mm$

13. Locate the centroid for the plane surface shown below.



$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4}{a_1 - a_2 - a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3 - a_4y_4}{a_1 - a_2 - a_3 - a_4}$$

Section (1) rectangle

$$a_1 = 36 \times 48 = 1728mm^2$$

$$x_1 = \frac{36}{2} = 18mm$$

$$y_1 = \frac{48}{2} = 24mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 12 \times 24$$

$$a_2 = 144mm^2$$

$$x_2 = 24 + \frac{12}{3} = 28mm$$

$$y_2 = 24 + \frac{24}{3} = 32mm$$

Section (3)

$$a_3 = \frac{1}{2}bh = \frac{1}{2} \times 36 \times 24$$

$$a_3 = 432mm^2$$

$$x_3 = \frac{b}{3} = \frac{36}{3} = 12mm$$

$$y_3 = \frac{h}{3} = \frac{24}{3} = 8mm$$

Section (4) semicircle

$$a_4 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times 6^2 = 56.54mm^2$$

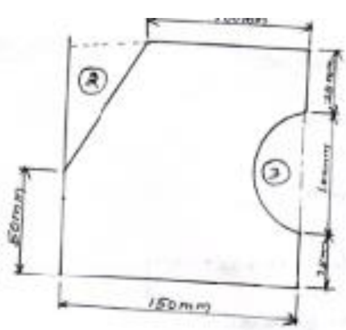
$$x_4 = 12mm$$

$$y_4 = 24 + \frac{4r}{3\pi} = 24 + \frac{4 \times 6}{3\pi} = 26.54mm$$

$$\bar{X} = \frac{(1728 \times 18) - (144 \times 28) - (432 \times 12) - (56.54 \times\times 12)}{1728 - 144 - 432 - 56.54} = 19.35mm$$

$$\bar{Y} = \frac{(1728 \times 24) - (144 \times 32) - (432 \times 8) - (56.54 \times 26.54)}{1728 - 144 - 432 - 56.54} = 29.12mm$$

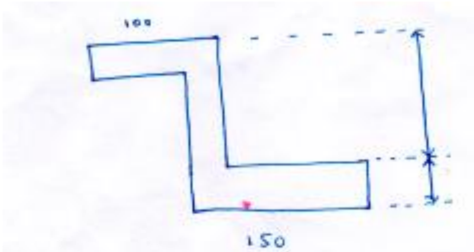
14. Locate the centroid of the plane area shown below.



Ans $X = 70.93mm$

$Y = 68.51mm$

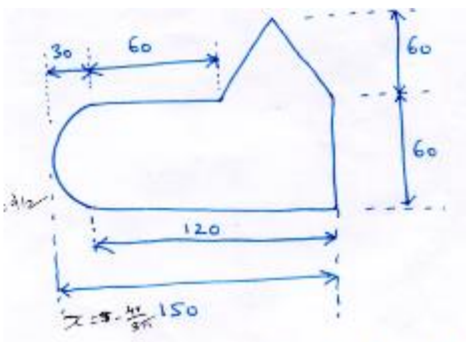
15.Find the centroid for the Z section shown in fig. All dimension are in mm.



$\bar{X} = 102.5mm$

$\bar{Y} = 77.5mm$

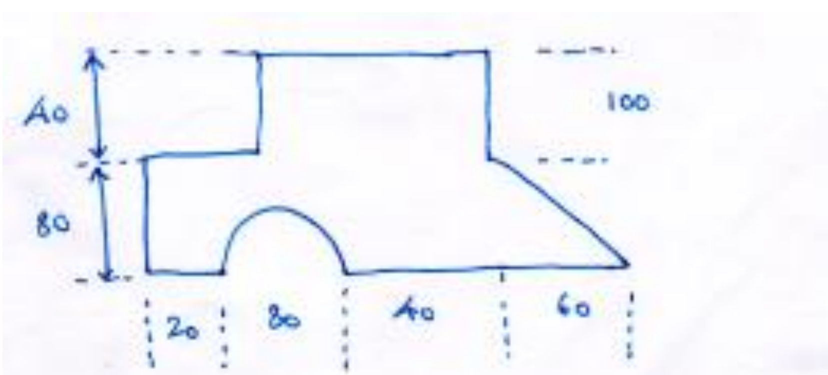
Determine the centroid of the plane uniform lamina as shown in fig. All dimensions are in mm



$\bar{X} = 85.31mm$

$\bar{Y} = 38.64mm$

16.Determine the centroidal coordinate of the given fig. All dimensions are in mm.



Moment of Inertia [polar]

State parallel Axis theorem

It state that, if the moment of inertia of a plane area about an axis through its centroid be denoted by IG the moment of inertia of the area about an axis AB parallel to the first and at a distance ‘h’ from the centroid is given by

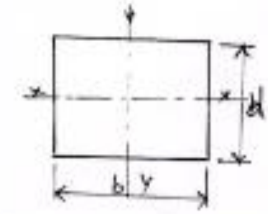
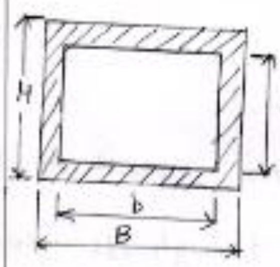
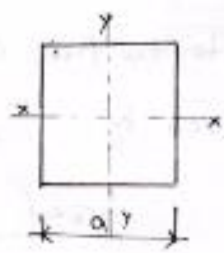
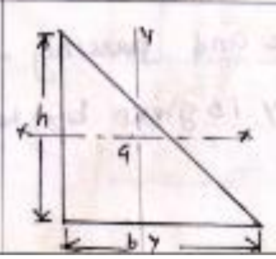
$$I_{AB} = IG + Ah^2$$

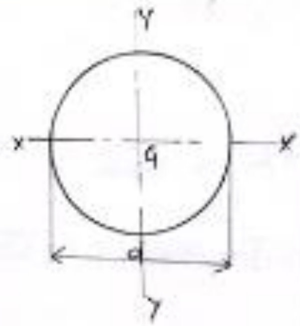
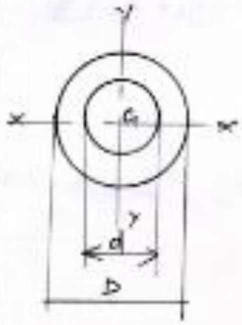
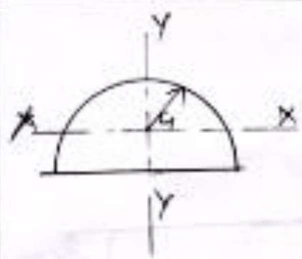
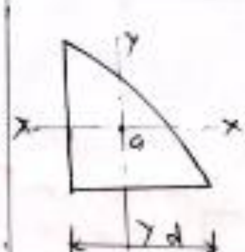
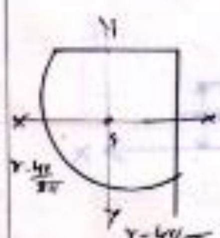
State perpendicular axis Theorem:

It states that ‘If I_{xx} and I_{yy} be the moment of inertia of a plane section about two perpendicular axes meeting at ‘o’ the moment of inertia about I_{zz} about the axis z-z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by the relation

$$I_{zz} = I_{xx} + I_{yy}$$

Moment of inertia.

SL.No	Name	Figure	G ($\bar{x}$ and $\bar{y}$) about Area A $\bar{x}$ from left $\bar{y}$ from base	Ixx, Iyy self centroidal Axis
1	Rectangle		$\bar{x} = b/2$ $\bar{y} = d/2$ $A = b \times d$	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$ $I_{zz} = I_{xx} + I_{yy}$
2	Hollow Rectangle		$\bar{x} = B/2$ $\bar{y} = H/2$ $A = [Bb - Hh]$	$I_{xx} = \frac{1}{12}[BH^3 - bh^3]$ $I_{yy} = \frac{1}{12}[HB^3 - hb^3]$
3	square		$\bar{x} = a/2$ $\bar{y} = a/2$ $A = a^2$	$I_{xx} = \frac{a^4}{12}$ $I_{yy} = \frac{a^4}{12}$
4	Triangle		$\bar{x} = b/2$ $\bar{y} = h/3$ $A = \frac{1}{2} bh.$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{48}$
5	Rightangle Triangle		$\bar{x} = b/3$ $\bar{y} = h/3$ $A = \frac{1}{2} bh.$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{36}$ $I_{xx} + I_{yy} = I_{zz}$

6	circle		$A = \pi r^2$ or $\frac{\pi d^2}{4}$ $\bar{x} = d/2$ $\bar{y} = d/2$	$I_{xx} = \frac{\pi d^4}{64}$ $I_{yy} = \frac{\pi d^4}{64}$ In r $I_{xx} = \frac{\pi r^4}{4}$ $I_{yy} = \frac{\pi r^4}{4}$ $I_{zz} = \pi/2\, r^4$
7	hollow circle		$\bar{x} = D/2$ $\bar{y} = D/2$ $A = \pi/4 [D^2 - d^2]$	$I_{xx} = \pi/4 [R^2 - r^2]$ $I_{yy} = \pi/4 [R^4 - r^2]$ $I_{zz} = \pi/2 [R^2 - r^2]$
8	semi circle 		$\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{2}$	$I_{xx} = 0.11 r^4$ $I_{yy} = \frac{\pi r^4}{8} = \frac{\pi D^4}{128}$
9	Quadrant		$\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{4}$	$I_{xx} = 0.055 r^4$ $I_{yy} = 0.055 r^4$
10	Quadrant		$\bar{x} = r - \frac{4r}{3\pi}$ $\bar{y} = r - \frac{4r}{3\pi}$	

Formula:

Moment of Inertia about the X axis

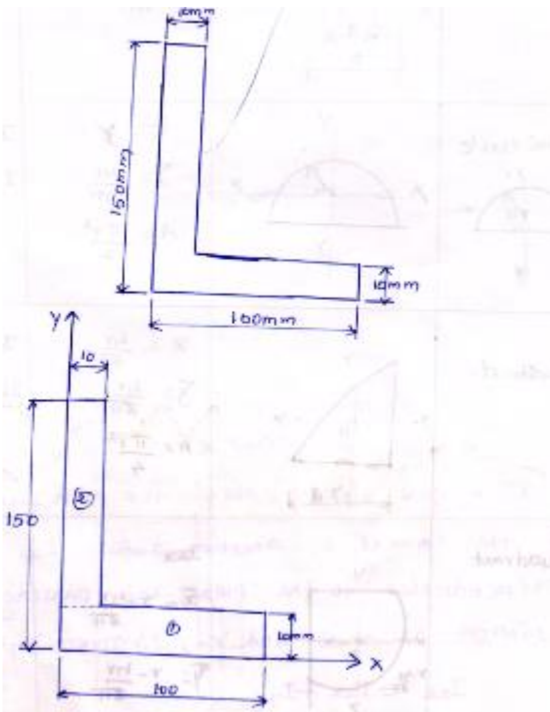
$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2 + \ldots\ldots\ldots\ldots\ldots$

Moment of Inertia about the Y axis

$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$

Problem 1.

An area in the form of L section is shown in fig.



$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2$

$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2$

Section (1) Rectangle

$a_1 = 100 \times 10 = 1000mm^2$

$$x_1 = \frac{100}{2} = 50mm$$

$$y_1 = \frac{10}{2} = 5mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{100 \times 10^3}{12} = 8333.33mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{10 \times 100^3}{12} = 833.33 \times 10^3 mm^4$$

Section (2) rectangle

$$a_2 = 10 \times 140 = 1400mm^2$$

$$x_2 = \frac{10}{2} = 5mm$$

$$y_2 = 10 + \frac{140}{2} = 80mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{10 \times 140^3}{12} = 2.286 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{140 \times 10^3}{12} = 11.66 \times 10^3 mm^4$$

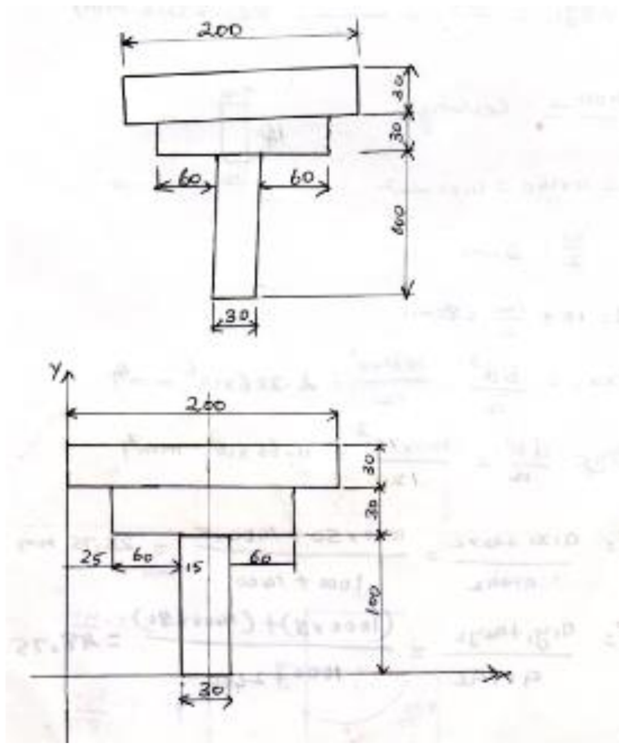
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{1000 \times 50 + 1400 \times 5}{1000 + 1400} = 23.75mm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{1000 \times 5 + 1400 \times 80}{1000 + 2400} = 48.75mm$$

$$I_{xx} = 8333.33 + 1000[48.75 - 5]^2 + 2.286 \times 10^6 [1400(48.75 - 80)^2]$$
$$= 575 \times 10^6 mm^4$$

$$I_{yy} = 833.33 \times 10^3 + 1000[23.75 - 50]^2 + 11.66 \times 10^3 [1400(23.75 - 5)^2]$$
$$= 2.02 \times 10^6 mm^4$$

2. Find the moment of inertia of the built up section shown in fig. about the axes passing through the centre of gravity parallel to the flange plate.



$$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2$$

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$$

Section (1) Rectangle

$$a_1 = 30 \times 100 = 3000mm^2$$

$$x_1 = 25 + 60 + \frac{30}{2} = 100mm$$

$$y_1 = \frac{100}{2} = 50mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{30 \times 100^3}{12} = 2.5 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{100 \times 30^3}{12} = 2.25 \times 10^5 mm^4$$

Section (2) Rectangle

$$a_2 = 150 \times 30 = 4500mm^2$$

$$x_2 = 25 + \frac{150}{2} = 100mm$$

$$y_2 = 100 + \frac{30}{2} = 115mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{150 \times 30^3}{12} = 3.37 \times 10^5 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 150^3}{12} = 8.43 \times 10^5 mm^4$$

Section (3)

$$a_3 = 300 \times 30 = 9000mm^2$$

$$x_2 = \frac{200}{2} = 100mm$$

$$y_2 = 100 + 30 + \frac{30}{2} = 145mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{300 \times 30^3}{12} = 6.75 \times 10^5 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 300^3}{12} = 67.5 \times 10^5 mm^4$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 100) + (4500 \times 100) + (9000 \times 100)}{3000 + 4500 + 9000}$$

$$\bar{X} = 100mm$$

$$\bar{Y} = \frac{a_1 x_1 + a_2 x_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 50) + (4500 \times 115) + (9000 \times 145)}{3000 + 4500 + 9000}$$

$$\bar{Y} = 119.54mm$$

$$I_{xx} = 2.5 \times 10^6 + 3000[119.54 - 50]^2 + 3.37 \times 10^5 + 4500[119.54 - 115]^2 + 6.75 \times 10^5 + 9000[119.54 - 145]^2$$

$$I_{xx} = 23.94 \times 10^6 mm^4$$

$$I_{yy} = 2.25 \times 10^5 + 3000[100 - 100]^2 + 8.43 \times 10^6 + 4500[100 - 100]^2 + 6.75 \times 10^5 + 9000[100 - 100]^2$$

$$I_{yy} = 76.15 \times 10^6 mm^4$$

Product of Inertia:

The moment of inertia of a plane fig. above a set of perpendicular axis is called product of inertia.

$$I_{xy} = \int_A xy\ da$$

$$= \sum axy$$

Problem: 1

Find the product of inertia and principal moment of inertia of the section about the centroidal axis

Product of inertia

$$I_{xy} = I_{x_1y_1} + I_{x2y2} + I_{x3y3}$$

$$I_{xy} = I_{xy} + a_1(\overline{x_1} - \bar{X})(\overline{y_1} - \bar{Y})$$

$I_{x_1y_1} = a_1x_1^1y_1^1$ $\qquad X_1^1 = x_1 - \bar{X}$ $\qquad Y_1^1 = y_1 - \bar{Y}$

$I_{x_2y_2} = a_2x_2^1y_2^1$ $\qquad X_2^1 = x_2 - \bar{X}$ $\qquad Y_2^1 = y_2 - \bar{Y}$

$I_{x_3y_3} = a_3x_3^1y_3^1$ $\qquad X_3^1 = x_3 - \bar{X}$ $\qquad Y_3^1 = y_3 - \bar{Y}$

Section (1)

$$a_1 = 40 \times 8 = 320mm^2$$

$$x_1 = 25 + \frac{30}{2} = 52mm$$

$$y_1 = \frac{8}{2} = 4mm$$

Section (2)

$$a_3 = 40 \times 8 = 320mm^2$$

$$x_3 = \frac{40}{2} = 20mm$$

$$y_2 = 8 + \frac{44}{2} = 30mm$$

Section (3)

$$a_1 = 40 \times 8 = 320mm^2$$

$$x_3 = \frac{40}{2} = 20mm$$

$$y_3 = 8 + 44 + \frac{8}{2} = 56mm$$

$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3} = \frac{(320 \times 52) + (352 \times 36) + (320 \times 20)}{320 + 352 + 320} = 36mm$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3} = \frac{(320 \times 4) + (352 \times 30) + (320 \times 56)}{320 + 352 + 320} = 30mm$$

$$I_{x_{1}y_{1}} = a_1 x_1^1 y_1^1 \qquad X_1^1 = x_1 - \bar{X} = 52 - 36 = 16$$

$$Y_1^1 = y_1 - \bar{Y} = 4 - 30 = -26$$

$$I_{x_{1}y_{1}} = 320 \times 16 \times (-26)$$

$I_{x_{1y1}} = -133120mm^4$

$I_{x_{2y2}} = a_2 x_2^1 y_2^1$ $\qquad X_2^1 = x_2 - \bar{X} = 36 - 36 = 0$

$Y_2^1 = y_2 - \bar{Y} = 30 - 30 = 0$

$I_{x_{2y2}} = 0$

$I_{x_{3y3}} = a_3 x_3^1 y_3^1$ $\qquad X_3^1 = x_3 - \bar{X} = 20 - 36 = -16$

$Y_3^1 = y_3 - \bar{Y} = 56 - 30 = 26mm$

$I_{x_{3y3}} = -133120mm^4$

Product of Inertia:

$I_{xy} = I_{X1Y1} + I_{X2Y2} + I_{X3Y3}$

$= -133120 + 0 + (-133120)$

$I_{xy} = -266240mm^4$

Principal Moment of Inertia:

Maximum Principal moment of inertia:

$$I_{Max} = \frac{I_{XX} + I_{YY}}{2} \pm \sqrt{\left(\frac{I_{XX} + I_{YY}}{2}\right)^2 + I^2XY}$$

$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$

$I_{YY} = I_{YY1} + I_{YY2} + I_{YY3}$

$$I_{XX1} = \frac{bd^3}{12} + a_1(Y_1 - \bar{Y}_1)^2$$

$$I_{XX2} = \frac{bd^3}{12} + a_2(Y_1 - \bar{Y}_1)^2$$

$$I_{XX3} = \frac{bd^3}{12} + a_3(Y_1 - \bar{Y}_1)^2$$

$$I_{XX} = I_{XX1} + a_1(Y_1 - \bar{Y}_1)^2 + I_{XX2} + a_2(\bar{Y} - Y_2)^2 + I_{XX3} + a_3(\bar{Y} - Y_3)^2$$

$$= 1706.66 + 320(30-4)^2 + 28394.66 + 352(30-30)^2 + 1706.66 + 320(30-56)^2$$

$$I_{XX} = 464.44 \times 10^3 mm^4$$

$$I_{yy} = I_{yy1} + a_1(X - X_1)^2 + I_{yy2} + a_2(X - X_2)^2 + I_{yy3} + a_3(X - X_3)^2$$

$$= 42666.66 + 320(36-52)^2 + 1877.33 + 352(36-36)^2 + 42666.66 + 320(36-20)^2$$

$$I_{yy} = 251.05 \times 10^3 mm^4$$

I_{Max}

$$I_{Max} = \frac{I_{xx} + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I_{yy}}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44 \times 10^3 + 251.05 \times 10^3}{2} + \sqrt{\left(\frac{464.44 \times 10^3 - 251.05 \times 10^3}{2}\right)^2 + (-266240)^2}$$

$$I_{Max} = 357745 + \sqrt{1.138 \times 10^{10} + 7.083 \times 10^{10}}$$

$$I_{Max} = 357745 + 286.81 \times 10^3$$

$$I_{Max} = 644.55 \times 10^3 mm^4$$

$$I_{Min} = \frac{I_{xx} + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I_{yy}}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44 \times 10^3 + 251.05 \times 10^3}{2} - \sqrt{\left(\frac{464.44 \times 10^3 + 251.05 \times 10^3}{2}\right) + (-266240)^2}$$

$$I_{Min} = 357745 - 268.81 \times 10^3$$

$I_{Min} = 88.935 \times 10^3 mm^4$

The position of Principal Axes is given by

$$tan2\theta = \left(\frac{I_{xy}}{\frac{I_{xx}-I_{yy}}{2}}\right) = \left(\frac{266240}{\frac{464.44\times10^3-251.05\times10^3}{2}}\right)$$

$$tan2\theta = \left(\frac{-2I_{xy}}{I_{xx}-I_{yy}}\right)$$

$$tan2\theta = 2.49$$

$$2\theta = tan^{-1}(2.49)$$

$$2\theta = 68.16 \;\; \theta = \frac{68.16}{2}$$

$$\theta = 34°4'$$

Principal Moment of inertia:

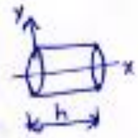
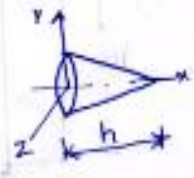
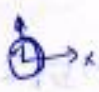
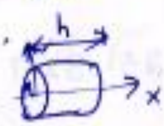
The perpendicular axis above which product of inertia is zero called principal axis and the moment of inertia with respect to these axis are called principal moment of inertia.

$$I_{max} \& I_{min} = \left(\frac{I_{xx}+I_{yy}}{2}\right) \pm \left(\frac{I_{xx}+I_{yy}}{2}\right)^2 + I_{xy}{}^2$$

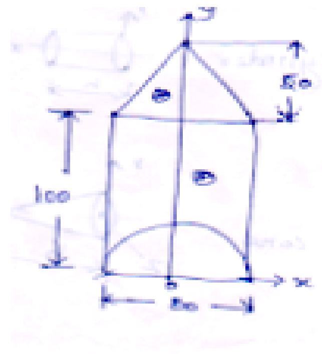
Location of principal Axes

$$tan2\theta = \frac{-2Ixy}{I_{xx}-I_{yy}}$$

Centre of gravity of common volume

Shape	Fig	volume	centroid
cylinder		$V = \pi r^2 h$	$\bar{x} = h/2$, $\bar{y}_1 = $ half
cone		$V = 1/3\, \pi r^2 h$	$\bar{x} = h/4$
Pyramid		$V = \frac{1}{3}\, abh$	$\bar{x} = h/4$
Hemisphere		$V = 2/3\, \pi r^2$	$\bar{x} = \frac{3r}{8}$
parabola		$V = 1/2\, \pi a^2 h$	$\bar{x} = h/3$
semi ellipse		$V = 2/3\, \pi a^2 h$	$\bar{x} = \frac{3h}{8}$

1. Find the position of the centroid of the solid combine shown in fig. consist of a solid cone of height 50 mm and base diameter 80 mm and a cylinder of height 100 mm and diameter 80mm with a semicircular cut as shown.



$$\bar{X} = \frac{v_1x_1 + v_2x_2 - v_3x_3}{v_1 + v_2 - v_3}$$

$$\bar{Y} = \frac{v_1y_1 + v_2y_2 - v_3y_3}{v_1 + v_2 - v_3}$$

$$v_1 = {}^1\!/_3\,\pi r^2 h = {}^1\!/_3 \times \pi \times 40^2 \times 50 = 83775.80mm^3$$

$$v_2 = \pi r^2 h = \pi \times 40^2 \times 100 = 502654.82mm^3$$

$$v_3 = \frac{\pi r^2 h}{2} = \frac{\pi \times 40^2 \times 100}{2} = 251327.41mm^3$$

$$X_1 = {}^b\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_1 = 100 + {}^{50}\!/_3 = 116.66mm$$

$$X_2 = {}^b\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_2 = {}^{100}\!/_2 = 50mm$$

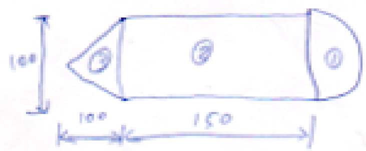
$$X_3 = {}^d\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_3 = \frac{4r}{3\pi} + \frac{4\times40}{3\pi} = 16.97mm$$

$$\bar{X} = \frac{(83775.80 \times 40) + (502654.82 \times 40) - (251327.4 \times 40)}{83775.5 + 502654.82 - 251327.41}$$

$$\bar{X} = 40mm$$

$$\bar{Y} = \frac{(83775.80 \times 416.66) + (502654.82 \times 50) - (251327.41 \times 16.97)}{83775.80 + 502654.82 - 251325.41}$$

2. A hemisphere of diameter 100 mm is fixed to cylinder is OA hemisphere diameter 100mm and cone is fixed another and of the cylinder its length is 100mm as shown fig. Locate the centroid of combine fig.



Mass Moment of irertia:

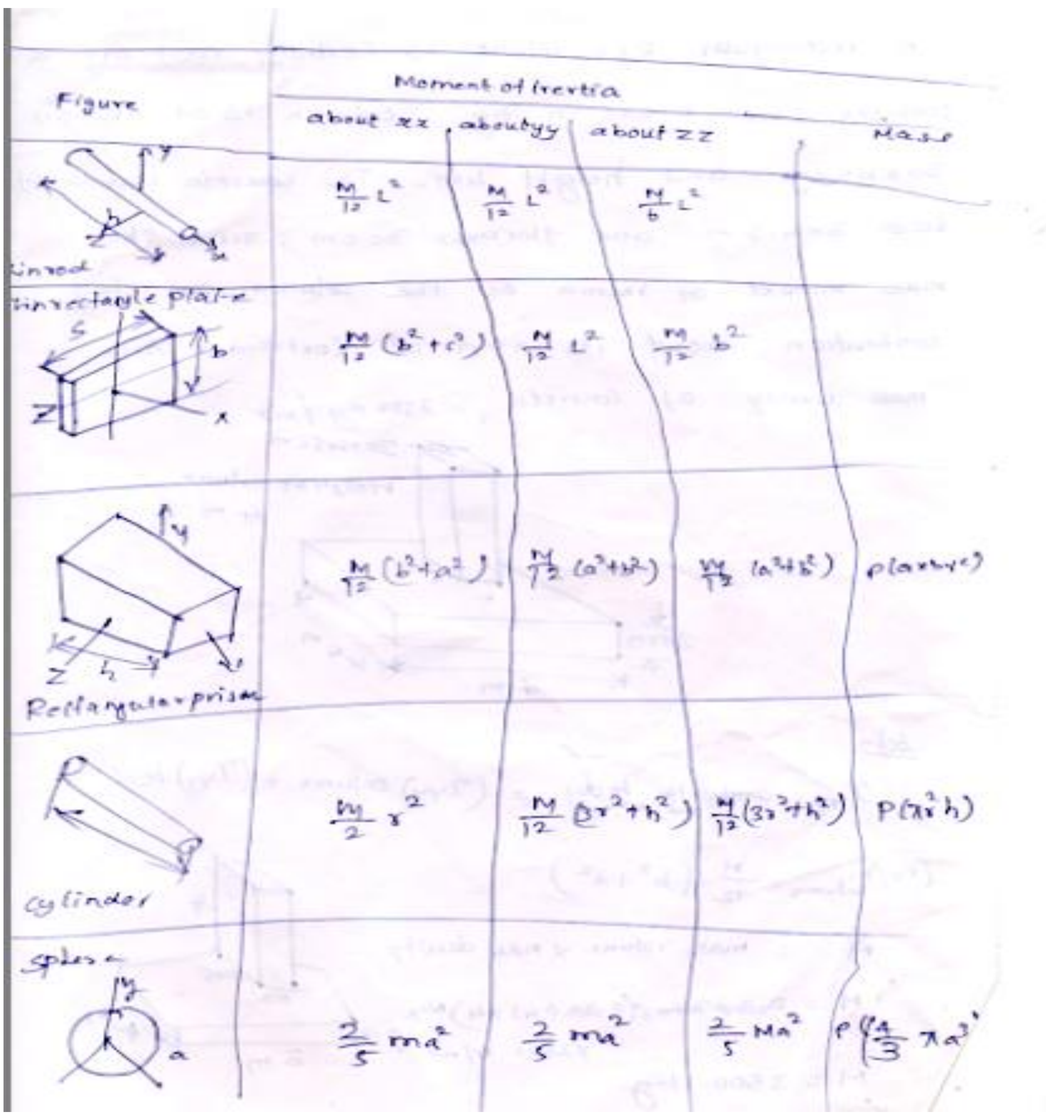
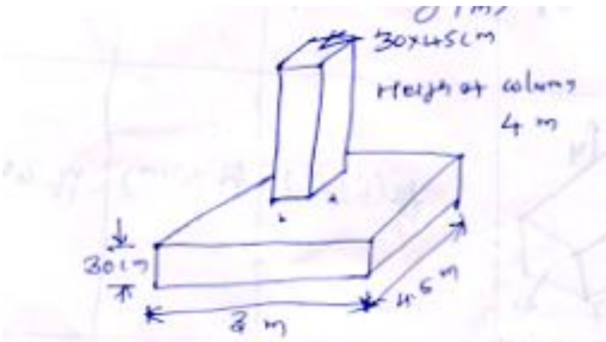


Figure	Moment of Inertia about xx	about yy	about zz	Mass
Thin rod	$\frac{M}{12}L^2$	$\frac{M}{12}L^2$	$\frac{M}{6}L^2$	
Thin rectangle plate	$\frac{M}{12}(b^2+c^2)$	$\frac{M}{12}L^2$	$\frac{M}{12}b^2$	
Rectangular prism	$\frac{M}{12}(b^2+a^2)$	$\frac{M}{12}(a^2+h^2)$	$\frac{M}{12}(a^2+b^2)$	$\rho(a \times b \times c)$
cylinder	$\frac{M}{2}r^2$	$\frac{M}{12}(3r^2+h^2)$	$\frac{M}{12}(3r^2+h^2)$	$\rho(\pi r^2 h)$
sphere	$\frac{2}{5}ma^2$	$\frac{2}{5}ma^2$	$\frac{2}{5}Ma^2$	$\rho(\frac{4}{3}\pi a^3)$

3. A rectangular RCC column is centrally cast over a concrete bed R.C.C. in Fig. column is of section 30$\times$ 45 cm and height 4m.the concrete bed is of size

3$\times$ 4.5m and thickness 30cm. find the mass moment of inertia of the column and bed combination about its vertical centroidal axis.

Mass density of concrete=2500 kg/m^3



Soln:

I_{yy} Combosite body= $(I_{yy})column + (I_{yy})bed$

$$(I_{yy})_{column} = \frac{M}{12}(b^2 + d^2)$$

M=mass volume$\times$ mass density

M=$(0.3 \times 0.45 \times 4)m^3 \times 2500 \,{}^{kg}/_{m^3}$

M=3500 kg

I_{yy} Column $= \frac{M}{12}(b^2 + d^2)$

$= \frac{1350}{12}(0.3^2 + 0.45^2)$

I_{yy} Column=32.91kg.m^2

I_{yy}bed $= \frac{M}{12}(b^2 + d^2)$ M=massvolume$\times$ $density$

$= \frac{10125}{12}(3^2 + 4.5^2)$ $(3 \times 0.3 \times 4.5) \times 2500m^3 \times {}^{kg}/_{m^3}$

M=10125kg

I_{yy} bed = 24679.69kgm^2

I_{yy}Composite body $= Iyy$ column +I_{yy} bed

= 32.91+24679.69

I_{YY}= 24712.6kg.m^2