4.3 *Newton's Forward and backward Formula*

Newton's Forward Formula

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \ldots \ldots \ldots \ldots$$

$$u = \frac{x - x_0}{h} \quad and \quad h = x_1 - x_0$$

1. ***Find the value of y when $x = 5$ for the following data.***

x	4	6	8	10
y	1	3	8	10

Given :

$x_0 = 4 y_0 = 1$

$x_1 = 6 \quad y_1 = 3$
$x_2 = 8 \quad y_2 = 8$
$x_3 = 10 \quad y_3 = 10$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		$3 - 1 = 2$		
6	3	(Δy_0)		
			$5 - 2 = 3(\Delta^2 y_0)$	
8	8	$8 - 3 = 5$		$-3 - 3 = -6(\Delta^3 y_0)$
			$2 - 5 = -3$	
10	10	$10 - 8 = 2$		

$h = x_1 - x_0 = 6 - 4 = 2$

$$u = \frac{x - x_0}{h} = \frac{x - 4}{2} = \frac{5 - 4}{2} = \frac{1}{2} = 0.5$$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$y(5) = 1 + \frac{0.5}{1!}(2) + \frac{0.5(0.5-1)}{2!}(3) + \frac{.5(.5-1)(.5-2)}{3!}(-6)$$

$$y(5) = 1 + 1 + \frac{0.5(-0.5)}{2}(3) + \frac{.5(-0.5)(-1.5)}{6}(-6)$$

$$y(5) = 1 + 1 + \frac{-0.25}{2}(3) - (-0.25)(-1.5)$$

$$y(5) = 1 + 1 - 0.375 - 0.375 = 1.25$$

2. ***Find the value of y when $x = 2$ for the following data***

x	0	5	10	15
y	14	379	1444	3584

Given :

$x_0 = 0 y_0 = 14$

$x_1 = 5 \quad y_1 = 379$
$x_2 = 10 \quad y_2 = 1444$
$x_3 = 15 \quad y_3 = 3584$

$$h = x_1 - x_0 = 5 - 0 = 5$$

$$u = \frac{x - x_0}{h} = \frac{x-0}{5} = \frac{2-0}{5} = \frac{2}{5} = 0.4$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$

0	14			
5	379	$379-14=365$ (Δy_0)	$1065-365=700$ $(\Delta^2 y_0)$	$1075-700=375$ $(\Delta^3 y_0)$
10	1444	$1444-379=1065$		
15	3584	$3584-1444=2140$	$2140-1065=1075$	

$$y(x)=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$y(2)=14+\frac{0.4}{1!}(365)+\frac{0.4(0.4-1)}{2!}(700)+\frac{0.4(0.4-1)(0.4-2)}{3!}(375)$$

$$y(2)=14+146+\frac{(-0.24)}{2}(700)+\frac{(-0.384)}{6}(375)$$

$$y(2)=14+146-84+24=100$$

Newton's Backward Formula

$$y(x)=y_n+\frac{v}{1!}\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n+\cdots\ldots\ldots\ldots\ldots$$

$$h=x_1-x_0 v=\frac{x-x_n}{h}$$

1. ***Find the value of*** y ***when and*** $x=43$ $\&x=84$ ***for the following data.***

x	40	50	60	70	80	90
y	184	204	226	250	276	304

Solution:

$$x_0=40 y_0=184$$

$x_1 = 50 \quad y_1 = 204$
$x_2 = 60 \quad y_2 = 226$
$x_3 = 70 \quad y_3 = 250$
$x_4 = 80 \quad y_4 = 276$
$x_5 = 90 \quad y_5 = 304$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
40	184					
		$204 - 184 = 20$				
50	204		2			
		$226 - 204 = 22$		0		
60	226		2		0	
		$250 - 226 = 24$		0		0
70	250		2		0	
		$276 - 250 = 26$		0		
80	276		2			
		$304 - 276 = 28$				
90	304					

(i) when $x = 43$

$$h = x_1 - x_0 = 5 - 40 = 10$$

$$u = \frac{x - x_0}{h} = \frac{x - 40}{10} = \frac{43 - 40}{10} = \frac{3}{10} = 0.3$$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$y(43) = 184 + \frac{0.3}{1!}(20) + \frac{0.3(0.3-1)}{2!}(2) + \frac{0.3(0.3-1)(0.3-2)}{3!}(0)$$

$$y(43) = 184 + 6 + \frac{0.3(-0.7)}{2}(2) + 0$$

$$y(43) = 184 + 6 - 0.21$$

$$y(43) = 189.79$$

(ii) when $x = 84$

$$h = x_1 - x_0 = 50 - 40 = 10$$

$$v = \frac{x - x_n}{h} = \frac{x - 90}{10} \quad v = \frac{84 - 90}{10} = \frac{-6}{10} = -0.6$$

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots \ldots\ldots\ldots\ldots$$

$$y(84) = 304 + \frac{(-0.6)}{1!}(28) + \frac{(-0.6)(-0.6+1)}{2!}(2) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(0) + \cdots$$

$$y(84) = 304 - 16.8 + \frac{(-0.6)(0.4)}{2}(2) + 0$$

$$y(84) = 304 - 16.8 - 0.24$$

$$y(84) = 286.96$$