UNIT – III

NUMERICAL DIFFERENTIATION AND INTEGRATION

PROBLEMS BASED ON DOUBLE INTEGRAL

TRAPEZOIDAL RULE AND SIMPSON'S RULE

Trapezoidal rule for Double Integral

$$I = \frac{hk}{4}[(Sum\ of\ four\ corners) + 2(Sum\ of\ nodes\ on\ boundary) + 4(Sum\ of\ interior\ nodes)]$$

1. ***Evaluate*** $\int_1^2 \int_3^4 \frac{1}{(x+y)^2}\,dxdy$ ***with*** $h = k = 0.5$

Solution :

Let $f(x, y) = \frac{1}{(x+y)^2}$

(i)***Range for*** x : 3 ***to*** 4 ***and*** $h = 0.5$

(ii)***Range for*** y : 1 ***to*** 2 ***and*** $k = 0.5$

$y \backslash x$	3	3.5	4
1	0.0625	0.0494	0.04
1.5	0.0494	0.04	0.0331

2	0.04	0.0331	0.0278

$$f(x,y)=\frac{1}{(x+y)^2}$$

$$f(3,1)=\frac{1}{(3+1)^2}=\frac{1}{16}=0.0625$$

$$f(3.5,1)=\frac{1}{(3.5+1)^2}=\frac{1}{(4.5)^2}=0.0494$$

$$f(4,1)=\frac{1}{(4+1)^2}=\frac{1}{25}=0.04$$

$$I=\frac{hk}{4}[(Sum\ of\ four\ corners)+2(Sum\ of\ nodes\ on\ boundary)+4(Sum\ of\ interior\ nodes)]$$

$$I=\frac{(0.5)(0.5)}{4}[(0.0625+0.04+0.04+0.0278)+2(0.0494+0.0494+0.0331+0.0331)+4(0.04)]$$

$$I=\frac{0.25}{4}[(0.1703)+0.330+0.16]$$

$$I=0.0413$$

$Simpson's\ \frac{1}{3}\ rule\ for$ ***Double Integral***

Simpson's 1/3 rule $= \frac{hk}{9}[(\textit{Sum of the corner of the boundary})$

$+2(\textit{sum of the odd nodes of the boundary})$
$+4(\textit{sum of the even nodes of the boundary})$
$+4(\textit{sum of the odd nodes of the odd rows})$
$+8(\textit{sum of the even nodes of the odd rows})$
$+8(\textit{sum of the odd nodes of the even rows})$
$+16(\textit{sum of the even nodes of the even rows})]$

$$I = \frac{(0.5)(0.5)}{9}[(0.0625 + 0.04 + 0.04 + 0.0278) + 4(0.0494 + 0.0494 + 0.0331 + 0.0331) + 16(0.04)]$$

$$I = \frac{0.25}{9}[(0.1703) + 0.660 + 0.64]$$

$$I = 0.0408$$

Evaluate the integral $\int_1^{1.4}\int_2^{2.4} \frac{dx\,dy}{xy}$ using Trapezoidal rule. Verify your results by actual integration.

Solution: $f(x,y) = \frac{1}{xy}$, x varies from $(2, 2.4)$

y varies from $(1, 1.4)$

Divide the range of x and y into 4 equal parts.

$$h = \frac{2.4-2}{4} = 0.1,\ k = \frac{1.4-1}{4} = 0.1$$

The values of $f(x,y)$ at the nodal points are given in the table :

y \ x	2	2.1	2.2	2.3	2.4
1	0.5	0.4762	0.4545	0.4348	0.4167
1.1	0.4545	0.4329	0.4132	0.3953	0.3788
1.2	0.4167	0.3698	0.3788	0.3623	0.3472
1.3	0.3846	0.3663	0.3497	0.3344	0.3205
1.4	0.3571	0.3401	0.3247	0.3106	0.2976

By Trapezoidal rule for double integration

$$I = \frac{hk}{4}\left[\begin{array}{l}\text{Sum of values of } f \text{ at the four corners} \\ +2\left(\begin{array}{l}\text{Sum of values of } f \text{ at the nodes} \\ \text{on the boundary except the corners}\end{array}\right) \\ +4\,(\text{Sum of the values at the interior nodes})\end{array}\right]$$

$$= \frac{(0.1)(0.1)}{4}\left[\begin{array}{l}(0.5 + 0.4167 + 0.2976 + 0.3571) \\ +2\left(\begin{array}{l}0.4762 + 0.4545 + 0.4348 + 0.3788 + 0.3472 + 0.3205 \\ +0.3106 + 0.3247 + 0.3401 + 0.3846 + 0.4167 + 0.4545\end{array}\right) \\ +4\left(\begin{array}{l}0.4329 + 0.4132 + 0.3953 + 0.3623 + 0.3344 \\ +0.3497 + 0.3663 + 0.3698 + 0.3788\end{array}\right)\end{array}\right]$$

$= 0.0614$

By actual integration

$$\int_1^{1.4}\int_2^{2.4} \frac{1}{xy}dxdy = \int_1^{1.4}\left(\int_2^{2.4} \frac{1}{x}dx\right)\frac{1}{y}dy = \int_1^{1.4} (\log x)_2^{2.4}\,\frac{1}{y}dy$$

$$= (\log 2.4 - \log 2)\,(\log y)_1^{1.4}$$

$$= 0.0613$$

Evaluate the integral $\int_1^{1.4}\int_2^{2.4} \frac{dxdy}{xy}$ using Simpson's rule. Verify your results by actual integration.

Solution: $f(x,y) = \frac{1}{xy}$, x varies from $(1, 1.4)$

y varies from $(2, 2.4)$

Divide the range of x and y into 4 equal parts.

$$h = \frac{2.4-2}{4} = 0.1,\ k = \frac{1.4-1}{4} = 0.1$$

By Extended Simpson's rule

$$I = \frac{hk}{9}\left[\begin{array}{l} (\text{Sum of the values of } f \text{ at the four corners}) \\ +\,2\,(\text{Sum of the values of } f \text{ at the odd positions on the boundary except the corners}) \\ +\,4\,(\text{Sum of the values of } f \text{ at the even positions on the boundary except the corners}) \\ +\,4\left(\begin{array}{l}\text{Sum of the values of } f \text{ at the odd positions} \\ \text{on the odd rows of the matrix except boundary rows}\end{array}\right) \\ +\,8\left(\begin{array}{l}\text{Sum of the values of } f \text{ at the even positions} \\ \text{on the odd rows of the matrix except boundary rows}\end{array}\right) \\ +8\left(\begin{array}{l}\text{Sum of the values of } f \text{ at the odd positions} \\ \text{on the even rows of the matrix except boundary rows}\end{array}\right) \\ +16\left(\begin{array}{l}\text{Sum of the values of } f \text{ at the even positions} \\ \text{on the even rows of the matrix except boundary rows}\end{array}\right) \end{array}\right]$$

$$= \frac{(0.1)(0.1)}{9}\left[\begin{array}{l} (0.5 + 0.4167 + 0.2976 + 0.3571) \\ +2\,(0.4545 + 0.3472 + 0.3247 + 0.4167) \\ +4\left(\begin{array}{c} 0.4762 + 0.4348 + 0.3788 + 0.3205 + 0.3106 \\ +0.3401 + 0.3846 + 0.4545 \end{array}\right) \\ +4\,(0.3788) \\ +8\,(0.3968 + 0.3623) \\ +8\,(0.3497 + 0.4132) \\ +16\,(0.3663 + 0.3344 + 0.4329 + 0.3953) \end{array}\right]$$

The values of $f(x,y)$ at the nodal points are given in the table :

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1	0.5	0.4762	0.4545	0.4348	0.4167
1.1	0.4545	0.4329	0.4132	0.3953	0.3788
1.2	0.4167	0.3698	0.3788	0.3623	0.3472
1.3	0.3846	0.3663	0.3497	0.3344	0.3205
1.4	0.3571	0.3401	0.3247	0.3106	0.2976

Anna University Questions

1. Evaluate $\int_1^5 \left[\int_1^4 \frac{1}{x+y} dx\right] dy$ by Trapezoidal rule in x−direction with $h = 1$ and Simpson's one-third rule in y−direction with $k = 1$. (ND10)

   Solution: [By Trap. : $I = 2.4053$, Simp. : $I = 2.122$]

2. Evaluate $\int_0^2 \int_0^1 4xy\,dx\,dy$ using Simpson's rule by taking $h = \frac{1}{4}$ and $k = \frac{1}{2}$. (ND12)

   Solution: [3.1111]

3. Evaluate $\int_1^{1.4} \int_2^{2.4} \frac{1}{xy} dx\,dy$ using Simpson's one-third rule. (MJ13)

   Solution: [0.0613]

4. Taking $h = k = \frac{1}{4}$, evaluate $\int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} \frac{\sin(xy)}{1+xy} dx\,dy$ using Simpson's rule . (AM14)

   Solution: [0.0141]