

1.4 INPUT AND TRANSFER IMPEDANCE:

INPUT IMPEDANCE:

The input impedance of a transmission line is given by,

$$Z_{in} = Z_S = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

$$Z_S = Z_0 \frac{Z_R \cosh \gamma l}{Z_R \cosh \gamma l} \left[\frac{1 + \frac{Z_0 \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_0 + \frac{\sinh \gamma l}{\cosh \gamma l}}{Z_R}} \right]$$

$$Z_S = Z_0 \left[\frac{1 + \frac{Z_0 \sinh \gamma l}{Z_R \cosh \gamma l}}{\frac{Z_0}{Z_R} + \tanh \gamma l} \right] \dots\dots(1)$$

TRANSFER IMPEDANCE:

Let,

$$Z_T = \frac{E_S}{I_R} = \frac{\text{Voltage at the receiving end}}{\text{Current at the sending end}}$$

Transfer impedance of the transmission line, Now, the terminating current (I_R) can be written as,

$$I_R = I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l$$

Divide the above equ by I_R ,

$$1 = \frac{I_S}{I_R} \cosh \gamma l - \frac{E_S}{Z_0 I_R} \sinh \gamma l$$

$$1 = \frac{I_S}{I_R} \cosh \gamma l - \frac{Z_T}{Z_0} \sinh \gamma l \dots\dots(2)$$

We know that,

$$E_R = E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$I_S Z_0 \sinh \gamma l = E_S \cosh \gamma l - E_R$$

$$I_S = \frac{E_S \cosh \gamma l - E_R}{Z_0 \sinh \gamma l} \dots\dots(3)$$

Sub equ (3) in equ (2),

$$1 = \frac{\left(\frac{E_S \cosh \gamma l - E_R}{Z_0 \sinh \gamma l} \right)}{I_R} \cosh \gamma l - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 = \left(\frac{E_S \cosh \gamma l - E_R}{I_R Z_0} \right) \frac{\cosh \gamma l}{\sinh \gamma l} - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 = \left(\frac{E_S \cosh \gamma l}{I_R Z_0} - \frac{E_R}{I_R Z_0} \right) \frac{\cosh \gamma l}{\sinh \gamma l} - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 = \left(\frac{Z_T \cosh \gamma l}{Z_0} - \frac{Z_R}{Z_0} \right) \frac{\cosh \gamma l}{\sinh \gamma l} - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 = \frac{Z_T \cosh^2 \gamma l}{Z_0 \sinh \gamma l} - \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 + \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} = \frac{Z_T \cosh^2 \gamma l}{Z_0 \sinh \gamma l} - \frac{Z_T}{Z_0} \sinh \gamma l$$

$$1 + \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} = \frac{Z_T}{Z_0} \left(\frac{\cosh^2 \gamma l}{\sinh \gamma l} - \sinh \gamma l \right)$$

$$1 + \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} = \frac{Z_T}{Z_0} \left(\frac{\cosh^2 \gamma l - \sinh^2 \gamma l}{\sinh \gamma l} \right)$$

$$\cosh^2 \theta - \sinh^2 \theta = 1$$

$$1 + \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} = \frac{Z_T}{Z_0} \left(\frac{1}{\sinh \gamma l} \right)$$

$$Z_T = Z_0 \sinh \gamma l \left[1 + \frac{Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} \right]$$

$$Z_T = Z_0 \sinh \gamma l \left[\frac{Z_0 \sinh \gamma l + Z_R \cosh \gamma l}{Z_0 \sinh \gamma l} \right]$$

$$Z_T = Z_0 \sinh \gamma l + Z_R \cosh \gamma l$$

OPEN AND SHORT CIRCUIT IMPEDANCE:

FINITE LINE TERMINATED IN Z_0 :

In Fig 1.4.1 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

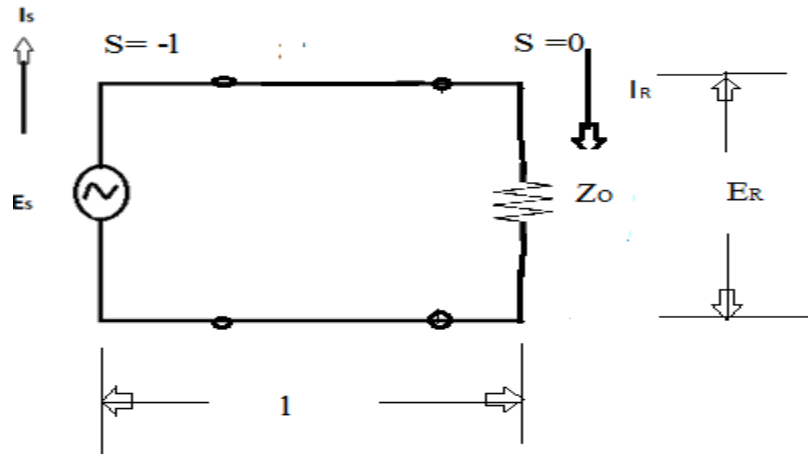


Fig: 1.4.1 Finite line terminated in Z_0

At distance $S=1$, $E = E_R$ and $I = I_R$

$$E_R = E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$I_R = I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l$$

$$Z_R = \frac{E_R}{I_R}$$

$$Z_R = \frac{E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l}{I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l}$$

FINITE LINE OPEN CIRCUITED AT DISTANCE END:

In Fig 1.4.2 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

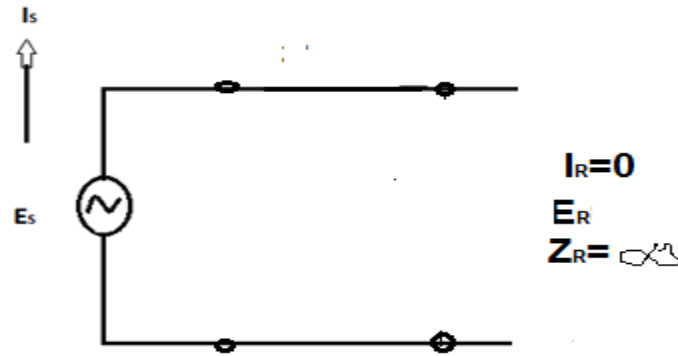


Fig: 1.4.2 Finite line open circuited at distance end

$$E_R = E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$I_R = I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l$$

$$I_R = 0$$

$$0 = I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l$$

$$I_S \cosh \gamma l = \frac{E_S}{Z_0} \sinh \gamma l$$

$$\frac{E_S}{I_S} = Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$

$$Z_{OC} = Z_0 \frac{\cosh \gamma l}{\sinh \gamma l}$$

$$Z_{OC} = Z_0 \coth \gamma l$$

$$\text{If } l = \infty$$

$$Z_{OC} = Z_0 \coth \gamma (\infty)$$

$$Z_{OC} = Z_0$$

FINITE LINE SHORT CIRCUITED AT DISTANCE END:

In Fig 1.4.3 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

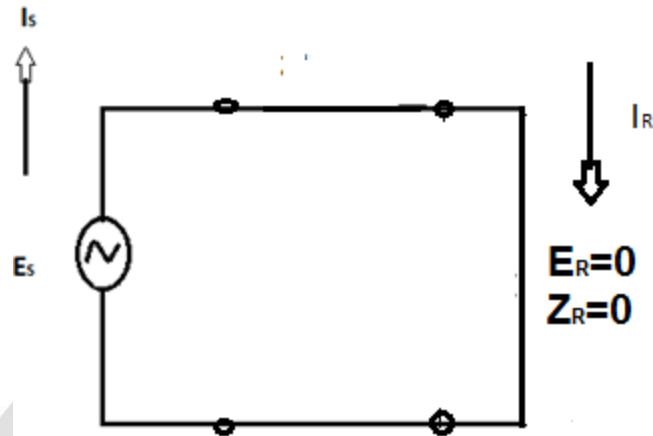


Fig: 1.4.3 Finite line short circuited at distance end

$$E_R = E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$I_R = I_S \cosh \gamma l - \frac{E_S}{Z_0} \sinh \gamma l$$

$$E_R = 0$$

$$0 = E_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$$

$$E_S \cosh \gamma l = I_S Z_0 \sinh \gamma l$$

$$\frac{E_S}{I_S} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}$$

$$Z_{SC} = Z_0 \tanh \gamma l$$

$$\text{If } l = \infty$$

$$Z_{SC} = Z_0 \tanh \gamma(\infty)$$

$$Z_{SC} = Z_0$$

Multiply the Z_{OC} & Z_{SC} expression.

$$Z_{OC} Z_{SC} = Z_0 \coth \gamma l Z_0 \tanh \gamma l$$

$$Z_{OC} Z_{SC} = Z_0^2 \frac{1}{\coth \gamma l} \tanh \gamma l$$

$$Z_{OC} Z_{SC} = Z_0^2$$

$$Z_0 = \sqrt{Z_{OC} Z_{SC}}$$

INPUT IMPEDANCE INTERMS OF Z_0 AND REFLECTION**COEFFICIENT:**

We know that,

Input impedance of the transmission line is,

$$Z_S = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

$$Z_S = Z_0 \left[\frac{Z_R \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right)}{Z_0 \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right) + Z_R \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right)} \right]$$

$$Z_S = \frac{2Z_0}{2} \left[\frac{Z_R(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_0(e^{\gamma l} + e^{-\gamma l}) + Z_R(e^{\gamma l} - e^{-\gamma l})} \right]$$

$$Z_S = Z_0 \left[\frac{Z_R e^{\gamma l} + Z_R e^{-\gamma l} + Z_0 e^{\gamma l} - Z_0 e^{-\gamma l}}{Z_0 e^{\gamma l} + Z_0 e^{-\gamma l} + Z_R e^{\gamma l} - Z_R e^{-\gamma l}} \right]$$

$$Z_S = Z_0 \left[\frac{e^{\gamma l} [Z_R + Z_0] + e^{-\gamma l} [Z_R - Z_0]}{e^{\gamma l} [Z_R + Z_0] - e^{-\gamma l} [Z_R - Z_0]} \right]$$

$$Z_S = Z_0 \frac{[Z_R + Z_0]}{[Z_R + Z_0]} \left[\frac{e^{\gamma l} + e^{-\gamma l} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]}{e^{\gamma l} - e^{-\gamma l} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]} \right]$$

$$Z_S = Z_0 \left[\frac{e^{\gamma l} + e^{-\gamma l} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]}{e^{\gamma l} - e^{-\gamma l} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]} \right]$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$Z_S = Z_0 \left[\frac{e^{\gamma l} + e^{-\gamma l} K}{e^{\gamma l} - e^{-\gamma l} K} \right]$$

The above equation is interms of Z_0 and reflection coefficient.