1.3 GRADIENT, DIVERGENCE, CURL

Del Operator:

The del operator(∇), is the vector differential operator. In Cartesian co-ordinates,

$$\nabla = \frac{\partial}{\partial_x}\overrightarrow{a_x} + \frac{\partial}{\partial_y}\overrightarrow{a_y} + \frac{\partial}{\partial_z}\overrightarrow{a_z}$$

This vector differential operator, otherwise known as the **gradient operator.**

- The gradient of a scalar V, written as ∇V
- The divergence of a vector A, written as $\nabla . \mathbf{A}$
- The curl of a vector A, written as $\nabla \times \mathbf{A}$
- The Laplacian of a scalar V, written as $\nabla^2 V$

GRADIENT OF A SCALAR

The gradient of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

The gradient of any scalar function is the maximum space rate of change of that function. If the s scalar V represents electric potential, ∇V represents potential gradient.

$$\nabla = \frac{\partial}{\partial_x}\overrightarrow{a_x} + \frac{\partial}{\partial_y}\overrightarrow{a_y} + \frac{\partial}{\partial_z}\overrightarrow{a_z}$$

$$\nabla \mathrm{V} = \frac{\partial V}{\partial_x}\overrightarrow{a_x} + \frac{\partial V}{\partial_y}\overrightarrow{a_y} + \frac{\partial V}{\partial_z}\overrightarrow{a_z}$$

This operation is called the gradient

$$\nabla \mathrm{V} = grad\, V$$

DIVERGENCE OF A VECTOR

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

The divergence of a vector A at any point is defined as the limit of its surface integrated per unit volume as the volume enclosed by the surface shrinks to zero

$$\nabla . A = \lim_{v \to 0} \frac{1}{v} \oint\!\!\oint_s A.\vec{n}\, ds$$

It can be expressed as

$$\nabla = \frac{\partial}{\partial_x}\overrightarrow{a_x} + \frac{\partial}{\partial_y}\overrightarrow{a_y} + \frac{\partial}{\partial_z}\overrightarrow{a_z}$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla . A = \left(\frac{\partial}{\partial_x}\overrightarrow{a_x} + \frac{\partial}{\partial_y}\overrightarrow{a_y} + \frac{\partial}{\partial_z}\overrightarrow{a_z}\right) + \left(A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right)$$

$$\nabla . \mathrm{A} = \frac{\partial A_x}{\partial_x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}$$

This operation is called divergence

$$\nabla . A = div\ A$$

Divergence of a vector is a scalar quantity.

CURL OF A VECTOR

The curl of **A** is an axial (or) rotational vector whose magnitude is the maximum circulation of **A** per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

The curl of vector **A** at any point is defined as the limit of its surface integral of its cross product with normal over a closed surface per unit volume as the volume shrinks to zero.

$$|\mathrm{Curl}\ A| = \lim_{v \to 0} \frac{1}{v} \oint\!\!\oint_s \vec{n} \times A\ ds$$

It can expressed as

$$\nabla = \frac{\partial}{\partial_x}\overrightarrow{a_x} + \frac{\partial}{\partial_y}\overrightarrow{a_y} + \frac{\partial}{\partial_z}\overrightarrow{a_z}$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\overrightarrow{a_x} - \left(\frac{\partial A_z}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\overrightarrow{a_y} + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\overrightarrow{a_z}$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\overrightarrow{a_x} + \left(\frac{\partial A_x}{\partial_z} - \frac{\partial A_z}{\partial_x}\right)\overrightarrow{a_y} + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\overrightarrow{a_z}$$

This operation is called curl.

$$\nabla \times A = Curl\ A$$

SOLENOIDAL AND IRROTATIONAL VECTORS

- A vector $\vec{A}$ is said to be solenoidal if its divergence is zero.

  i.e $\nabla.\vec{A} = 0$, then $\vec{A}$ is said to be solenoidal.

- A vector $\vec{A}$ is said to be irrotational if its curl is zero.

  i.e $\nabla \times \vec{A} = 0$, then $\vec{A}$ is said to be irrotational.