Maxwell's Equation

Equation (4.1) and (4.2) gives the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad (4.20a)$$

$$\nabla\times\vec{H} = \vec{J} \quad (4.20b)$$

$$\nabla.\vec{D} = \rho \quad (4.20c)$$

$$\nabla.\vec{B} = 0 \quad (4.20d)$$

In addition, from the principle of conservation of charges we get the equation of continuity

$$\nabla.\vec{J} = -\frac{\partial\rho}{\partial t} \quad (4.21)$$

The equation 4.20 (a) - (d) must be consistent with equation

(4.21). We observe that

$$\nabla.\nabla\times\vec{H} = 0 = \nabla.\vec{J} \quad (4.22)$$

Since $\nabla.\nabla\times\vec{A}$ is zero for any vector $\vec{A}$.

Thus $\nabla\times\vec{H} = \vec{J}$ applies only for the static case i.e., for the scenario when . A classic example for this is given below .

$$\frac{\partial\rho}{\partial t} = 0$$

Suppose we are in the process of charging up a capacitor as shown in fig 4.3.

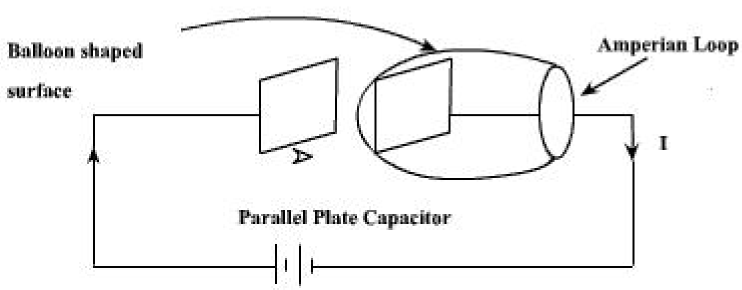

Fig 3.1 process of charging up a capacitor
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

Let us apply the Ampere's Law for the Amperian loop shown in fig 4.3. *Ienc* = *I* is the total current passing through the loop. But if we draw a baloon shaped surface as in fig 4.3, no current passes through this surface and hence *Ienc* = 0. But for non steady currents such as this one, the concept of current enclosed by a loop is ill-defined since it depends on what surface you use. In fact Ampere's Law should also hold true for time varying case as well, then comes the idea of displacement current which will be introduced in the next few slides.

We can write for time varying case, $= \nabla.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$

$$\therefore \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (4.23)$$

$$\nabla.\left(\nabla \times \vec{H}\right) = 0 = \nabla.\vec{J} + \frac{\partial \rho}{\partial t}$$

$$= \nabla.\vec{J} + \frac{\partial}{\partial t}\nabla.\vec{D}$$

(4.24)

The equation (4.24) is valid for static as well as for time varying case.

Equation (4.24) indicates that a time varying electric field will give rise to a magnetic $\frac{\partial \vec{D}}{\partial t}$ (A/m^2) $\vec{J}$

field even in the absence of . The term has a dimension of densities and is called the displacement current density.

The main $\frac{\partial \vec{D}}{\partial t}$ $\nabla \times \vec{H}$ equation is one of the major contributions of Jame's Clerk Maxwell. The modified set of equations

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (4.24b)$$

$$\nabla.\vec{D} = \rho \quad (4.24c)$$

$$\nabla.\vec{B} = 0 \quad (4.24d)$$

is known as the Maxwell's equation and this set of equations apply in the time varying scenario, static fields are being a particular. $\left(\frac{\partial}{\partial t} = 0\right)$

In the integral form

$$\oint_c \vec{E}.d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}.d\vec{S} \quad (4.26a)$$

$$\oint_c \vec{H}.d\vec{l} = \int_s \left(J + \frac{\partial D}{\partial t}\right).d\vec{S} = I + \int_s \frac{\partial \vec{D}}{\partial t}.d\vec{S} \quad (4.26b)$$

$$\int_v \nabla.\vec{D}\,dv = \oint_s \vec{D}.d\vec{S} = \int_v \rho\,dv \quad (4.26c)$$

$$\oint \vec{B}.d\vec{S} = 0 \quad (4.26d)$$

The modification of Ampere's law by Maxwell has led to the development of a unified electromagnetic field theory. By introducing the displacement current term, Maxwell could predict the propagation of EM waves. Existence of EM waves was later demonstrated by Hertz experimentally which led to the new era of radio communication.