Homomorphic Filtering

Homomorphic filters are widely used in image processing for compensating the effect of no uniform illumination in an image. Pixel intensities in an image represent the light reflected from the corresponding points in the objects. As per as image model, image f(z,y) may be characterized by two components: (1) the amount of source light incident on the scene being viewed, and (2) the amount of light reflected by the objects in the scene. These portions of light are called the illumination and reflectance components, and are denoted i (x , y) and r (x , y) respectively.

The functions i (x , y) and r (x , y) combine multiplicatively to give the image function f (x , y):

$$f(x, y) = i(x, y).r(x, y) \qquad (1)$$

where $0 < i(x, y) < a$ and $0 < r(x, y) < 1$.

Homomorphic filters are used in such situations where the image is subjected to the multiplicative interference or noise as depicted in Eq. 1. We cannot easily use the above product to operate separately on the frequency components of illumination and reflection because the Fourier transform of f (x , y) is not separable; that is

F[f(x,y)) not equal to F[i(x, y)].F[r(x, y)]. We can separate the two components by taking the logarithm of the two sides $\ln f(x,y) = \ln i(x, y) + \ln r(x, y)$.

Taking Fourier transforms on both sides we get,

$$F[\ln f(x,y)\} = F[\ln i(x, y)\} + F[\ln r(x, y)].$$

that is, F(x,y) = I(x,y) + R(x,y), where F, I and R are the Fourier transforms ln f(x,y),ln i(x, y) , and ln r(x, y). respectively. The function F represents the Fourier transform of the sum of two images: a low-frequency illumination image and a high-frequency reflectance image. If we now apply a filter with a transfer function that suppresses low- frequency components and enhances high-frequency components, then we can suppress the illumination component and enhance the reflectance component.

Features & Application:

1. Homomorphic filter is used **for image enhancement**.
2. It simultaneously **normalizes the brightness** across an image and **increases contrast.**
3. It is also used to remove **multiplicative noise.**

Images normally consist of light reflected from objects. The basic nature of the image f(*x*,*y*) may be characterized by two components:

(1) The amount of source light incident on the scene being viewed, &

(2) The amount of light reflected by the objects in the scen

These portions of light are called the ***illumination*** and ***reflectance*** components, and are denoted *i*(*x*,*y*) and *r*(*x*,*y*) respectively. The functions *i* and *r* combine multiplicatively to give the image function *F*:

$$f(x,y) = i(x,y)r(x,y),$$

where 0 < i(*x*,*y*) < 1-----indicates perfect black body -----indicates perfect absorption , and 0 < *r*(*x*,*y*) < 1 -----indicates perfect white body ---indicates perfect reflection.

Since i and r combine multiplicatively, they can be added by taking log of the image intensity, so that they can be separated in the frequency domain.

Illumination variations can be thought as a multiplicative noise and can be reduced by filtering in the log domain.To make the illuminations of an image more even, the HF components are increased and the LF

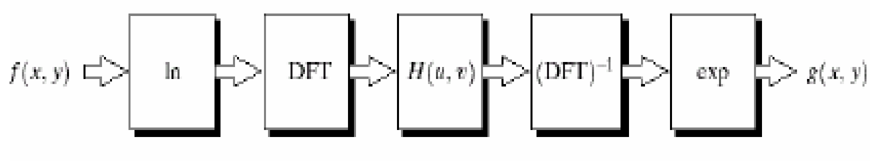

Fig2.4.1: Summary of steps in homomorphic filtering.
(Source: Rafael C. Gonzalez, Richard E. Woods, ‗Digital Image Processing', Pearson, Third Edition, 2010- Page- 292)

Components are filtered, because the HF components are assumed to represent the reflectance in the scene whereas the LF components are assumed to represent the illumination in the scene.i.e., High pass filter is used to suppress LF's and amplify HF's in the log intensity domain. illumination component tends to vary slowly across the image and the reflectance tends to vary rapidly. Therefore, by applying a frequency domain filter the intensity variation across the image can be reduced while highlighting detail.

Analysis:

WKT , $f(x,y) = i(x,y)r(x,y)$---------------------- (1)

Taking natural log on both sides of the above equation,

we get, $\ln[f(x,y)] = \ln[i(x,y)] + \ln[r(x,y)]$

Let $z = \ln[f(x,y)] = \ln[i(x,y)] + \ln[r(x,y)]$

Taking DFT on both sides of the above equation, we get, $z(x,y) = \ln[f(x,y)] = \ln[i(x,y)] + \ln[r(x,y)]$

$$DFT[z(x,y)] = DFT\{\ln[f(x,y)]\} = DFT\{\ln[i(x,y)] + \ln[r(x,y)]\}$$

$$= DFT\{\ln[i(x,y)]\} + DFT\{\ln[r(x,y)]\} \qquad (2)$$

Since DFT[f(x,y)] = F(u,v), equation (2) becomes,

$Z(u,v) = F_i(u,v) + F_r(u,v)$ ....(3)

The function *Z* represents the Fourier transform of the *sum* of two images: a low frequency illumination image and a high frequency reflectance image.

Figure : Transfer function for homomorphic filtering.

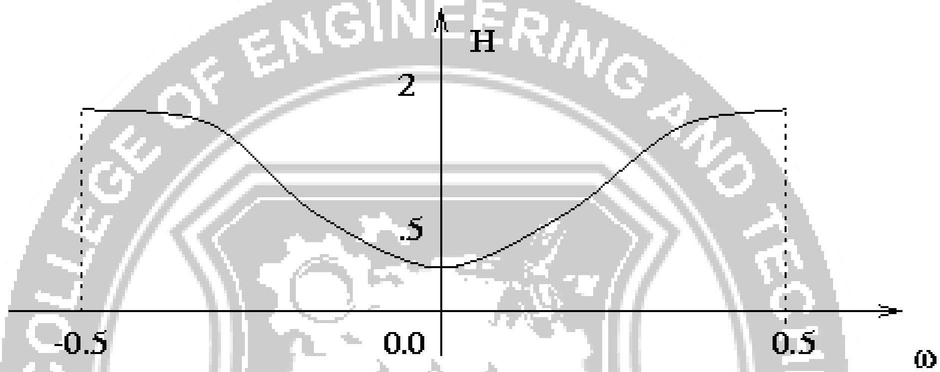

Fig2.4.2:Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and *D* is the distance from the center.
(Source: Rafael C. Gonzalez, Richard E. Woods, ‗Digital Image Processing', Pearson, Third Edition,2010.- Page-292)

If we now apply a filter with a transfer function that suppresses low frequency components and enhances high frequency components, then we can suppress the illumination component and enhance the reflectance component. Thus ,the Fourier transform of the output image is obtained by multiplying the DFT of the input image with the filter function H(u,v).

i.e., $S(u,v) = H(u,v)\ Z(u,v)$ ....(4)

where S(u,v) is the fourier transform of the output image.

Substitute equation (3) in (4),

we ge$S(u,v) = H(u,v)\ [\ F_i(u,v) + F_r(u,v)\ ] = H(u,v)\ F_i(u,v) + H(u,v)\ F_r(u,v)$ --(5) Applying IDFT to equation (6), we get,

$T^{-1}[S(u,v)] = T^{-1}\ [\ H(u,v)\ F_i(u,v) + H(u,v)\ F_r(u,v)]$

$= T^{-1}[\ H(u,v)\ F_i(u,v)] + T^{-1}[H(u,v)\ F_r(u,v)]$

$s(x,y) = i'(x,y) + r'(x,y)$……………………………………………………………(6)

The Enhanced image is obtained by taking exponential of the IDFT

$s(x,y)$, i.e., $g(x,y) = e^{s(x,y)} = e^{i'(x,y)}\, e^{r'(x,y)} = i_o(x,y)\, r_o(x,y)$

where, $i_o(x,y) = e^{i'(x,y)}$, $r_o(x,y) = e^{r'(x,y)}$ are the illumination and reflection components of the enhanced output image.