UNIT-I

TESTING THE HYPOTHESIS

1.4 F-test for Variance Ratio Test

F – TEST

- This test is used to test the significance of two or more sample estimates of Population variance.
- To test whether if there is any significant difference between two estimates of population variance.
- To test if the two sample have come from the same population, we use F – test.
- The test statistic is given by, $F = \frac{S_1^2}{S_2^2}$
- Where $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ (n_1 – first sample size)
- $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$ (n_2 – second sample size)
- And $S_1^2 > S_2^2$
- The degrees of freedom $(v_1, v_2) = (n_1 - 1, n_2 - 1)$
- Note: If $S_2^2 > S_1^2$ then $F = \frac{S_2^2}{S_1^2}$
- : If $S_1^2 > S_2^2$ then $F = \frac{S_2^2}{S_1^2}$ (Always $F > 1$)
- To test whether two independent samples have been drawn from the same population, we have to
- Equality of population means using t – test or z – test, test according to sample size.
- Equality of population variances using F – test.
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Test of significance for equality of population variances

- **Working Rule:**
- Set the null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$
- Set the alternative hypothesis $H_1: \sigma_1^2 \neq \sigma_2^2$
- Level of significance α at 5% (or) 1%
- The test statistic is given by, $F = \frac{S_1^2}{S_2^2}$

Where, $S_1^2 = \frac{1}{n_1 - 1}\sum(x_i - \bar{x})^2$, $S_2^2 = \frac{1}{n_2 - 1}\sum(y_i - \bar{y})^2$

Where $S_1^{\ 2} = \frac{n_1 s_1^{\ 2}}{n_1 - 1}$

$$S_2^{\ 2} = \frac{n_2 s_2^{\ 2}}{n_2 - 1}$$

If $|F| < F_{0.05}$, H_0 is accepted at 5% level of significance.

If $|F| > F_{0.05}$, H_0 is rejected at 5% level of significance.

1. **A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both sample be from populations with the same variance?**
   Solution:

   Given $n_1 = 8, n_2 = 7, S_1^{\ 2} = 3.0, S_2^{\ 2} = 2.5$

   Set the null hypothesis $H_0: \sigma_1^{\ 2} = \sigma_2^{\ 2}$

   Set the alternative hypothesis $H_1: \sigma_1^{\ 2} \neq \sigma_2^{\ 2}$

   Level of significance α at 5%

   The test statistic is given by, $F = \frac{S_1^{\ 2}}{S_2^{\ 2}}$

   $$\Rightarrow F = \frac{3.0}{2.5} = 1.2$$

   Critical value: At 5% level, the tabulated value of F_α is 2.53 for the

   d.f= $(n_1 - 1, n_2 - 1) = (12,\ 14)$

   Conclusion: Since |F| = 1.2 < 2.53

   Hence Null Hypothesis H_0 is accepted at 5% level of significance.

   i.e., The two samples could have come from two normal populations with the same variance.

2. **Two random samples of sizes 8 and 11, drawn from two normal populations are characterized as follows**

	Size	Sum of observations	Sum of square of observations
Sample I	**8**	**9.6**	**61.52**
Sample II	**11**	**16.5**	**73.26**

   You are to decide if the two populations can be taken to have the same variance.

   Solution:

   Given $n_1 = 8, n_2 = 11, \sum(x_i - \bar{x})^2 = 61.52,\ \sum(y_i - \bar{y})^2 = 73.26$

   Set the null hypothesis $H_0: \sigma_1^{\ 2} = \sigma_2^{\ 2}$

Set the alternative hypothesis $H_1: {\sigma_1}^2 \neq {\sigma_2}^2$

Level of significance α at 5%

The test statistic is given by, $F = \frac{{S_1}^2}{{S_2}^2}$

${S_1}^2 = \frac{1}{n_1-1}\sum(x_i - \bar{x})^2 = \frac{1}{8-1}(61.52) = 8.78,$

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{S_2}^2 = \frac{1}{n_2 - 1}\sum(y_i - \overline{y})^2 = \frac{1}{11-1}(73.26) = 7.326
$$

$$
F = \frac{{S_1}^2}{{S_2}^2} = \frac{8.78}{7.326} = 1.1984
$$

Critical value: At 5% level, the tabulated value of F_α is 3.14 for the

d.f= $(n_1 - 1, n_2 - 1) = (7,\ 10)$

Conclusion: Since |F| = 1.1984 < 3.14

Hence Null Hypothesis H_0 is accepted at 5% level of significance.

i.e., The two samples could have come from two normal populations with the same variance.

3. **Two random samples gave the following data**

	Size	Mean	variance
Sample I	**8**	**9.6**	**1.2**
Sample II	**11**	**16.5**	**2.5**

Can we conclude that the two samples have been drawn from the same normal population?

Solution:

The two samples have been drawn from the same population means we have to check

The variance of the population do not differ significantly by F – test.

The sample means (and hence the population means) do not differ significantly by t – test

F – test:

Given $n_1 = 8, n_2 = 11, \overline{x_1} = 9.6, \overline{x_2} = 16.5, {s_1}^2 = 1.2, {s_2}^2 = 2.5$

Set the null hypothesis $H_0: {\sigma_1}^2 = {\sigma_2}^2$

Set the alternative hypothesis $H_1: {\sigma_1}^2 \neq {\sigma_2}^2$

Level of significance α at 5%

$$S_1{}^2 = \frac{n_1 s_1{}^2}{n_1 - 1} = \frac{8 * 9.6}{8 - 1} = 1.37$$

$$S_2{}^2 = \frac{n_2 s_2{}^2}{n_2 - 1} = \frac{11 * 2.5}{11 - 1} = 2.75$$

The test statistic is given by, $F = \frac{S_1{}^2}{S_2{}^2}$

$$\Rightarrow F = \frac{2.75}{1.37} = 2.007$$

Critical value: At 5% level, the tabulated value of F_α is 3.63 for the

d.f= $(n_1 - 1, n_2 - 1) = (10, 7)$

Conclusion: Since |F| = 2.007 < 3.63

Hence Null Hypothesis H_0 is accepted at 5% level of significance.

i.e., The two samples could have come from two normal populations with the same variance.

T – test: (Equality of means)

Given $n_1 = 8, n_2 = 11, \overline{x_1} = 9.6, \overline{x_2} = 16.5, s_1{}^2 = 1.2, s_2{}^2 = 2.5$
Set the null hypothesis H_0: $\mu_1 = \mu_2$

Set the alternative hypothesis H_1: $\mu_1 \neq \mu_2$

Level of significance α= 0.05(5%)

The test Statistic is $t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$S = \sqrt{\frac{n_1 s_1{}^2 + n_2 s_2{}^2}{n_1 + n_2 - 2}} = \sqrt{\frac{8 * 1.2 + 11 * 2.5}{8 + 11 - 2}} = 1.4772$$

$$t = \frac{9.6 - 16.5}{1.4772\sqrt{\frac{1}{8} + \frac{1}{11}}} = -10.0525$$

$$|t| = 10.0525$$

- **Critical value:** At 5% level, the tabulated value of t_α is 2.110 for

$$v = n_1 + n_2 - 2 = 8 + 11 - 2 = 17$$

- **Conclusion:** Since $|t|$= 10.0525 $>$ 2.110
- Hence Null Hypothesis H_0 is rejected at 5% level of significance.

i.e., the two samples could not have been drawn from the same normal population