4.2. FIR DESIGN: WINDOWING TECHNIQUES

The windows are finite duration sequences used to modify the impulse response of the FIR filters in order to reduce the ripples in the pass band and stop band, and also to achieve the desired transition from pass band to stop band.

The FIR filter design starts with the desired frequency response, $H_d(e^{j\omega})$.The desired impulse response, $h_d(n)$ is obtained by taking inverse Fourier transform of $H_d(e^{j\omega})$.

4.2.1.DIFFERENT TYPES OF WINDOWS

1. Rectangular window, $w_R(n)$
2. Bartlet or Triangular window $w_T(n)$
3. Hanning window $w_C(n)$
4. Hamming window $w_H(n)$
5. Blackman window $w_B(n)$
6. Kaiser window $w_K(n)$

1. Rectangular window, $w_R(n)$

The N-point Rectangular window, $w_R(n)$ is defined as

$$w_R(n) = \begin{cases} 1 & ; n = -\frac{N-1}{2} \text{ to } +\frac{N-1}{2} \\ 0 & ; \text{other } n \end{cases}$$

2. Bartlet or Triangular window $w_T(n)$

The triangular window have been chosen such that it has tapered sequences form the middle on either sides.The N point triangular window $w_T(n)$ is defined as

$$w_T(n) = \begin{cases} 1 - \frac{2\lceil n \rceil}{N-1} \; ; \; for -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 \; ; \; other\ n \end{cases}$$

3.Hanning window $w_c(n)$

The Hanning window is one type of raised cosine window.The equation for Hanning window sequence wC (n) is obtained by putting a=0.5

$$w_c(n) = \begin{cases} 0.5 + 0.5\cos\frac{2\pi n}{N-1} \; ; \; for -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0 \; ; \; other\ n \end{cases}$$

4.Hamming window $w_H(n)$:

The equation for Hamming window $w_H(n)$ is obtained by putting a=0.54

$$w_H(n) = \begin{cases} 0.54 + 0.46\cos\frac{2\pi n}{N-1} & ;\ for -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0\ ; & other\ n \end{cases}$$

5.Blackman window w_B (n)

The Blackman window w_B (n) is another type of cosine window defined by the equation,

$$w_B(n) = \begin{cases} 0.42 + 0.5\ \cos\frac{2\pi n}{N-1} + 0.08\ \cos\frac{2\pi n}{N-1} & ;\ for -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0\ ; & other\ n \end{cases}$$

6.Kaiser window w_K (n)

The Kaiser window w_K (n) is defined as

$$w_K(n) = \begin{cases} \frac{I_0\beta_1}{I_0(a)} & ;\ for -\frac{N-1}{2} \le n \le \frac{N-1}{2} \\ 0\ ; & other\ n \end{cases}$$

4.2.2.FIR FILTER DESIGN

Method-1:Symmetry condition h(N-1-n) = h(n)

1. The specifications of digital FIR filter are,

i)The desired frequency response,

$$H_d(e^{j\omega}) = C\ e^{-j\alpha\omega}$$

where C = constant

$$\alpha = \frac{N-1}{2}$$

ii) The cutoff frequency ω_c for low pass and high pass,and ω_{c1} and ω_{c2} for band pass and band stop filters.

iii)The number of samples of impulse response,N.

2. Determine the desired impulse response,h_d(n) by taking Inverse fourier transform of the desired frequency response,$H_d(e^{j\omega})$

$$H_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{d\ (e^{j\omega})(e^{j\omega n})}\ d\omega$$

3.Choose the desired window sequence w(n) defined for n=0 to N-1 from table. Multiply h_d(n) with w(n) to get the impulse response h(n) of the filter. Calculate N samples of the impulse response, for n=0 to N-1.

Impulse response,h(n) = $h_d(n) \times w(n)$; for n= 0 to N-1

The impulse response is symmetric with centre of frequency at (N-1)/2

4.Take Z transform of the impulse response h(n) to get the transfer function H(Z) of the filter.

$$\text{Transfer function },H(z) = Z\{h(n)\} = \sum_{n=0}^{N-1} h(n)z^{-n}$$

5.Draw a suitable structure for realization of FIR filter.

Method-2:Symmetry condition h(-n)=h(n)

1The specifications of digital filter are,

i)The desired frequency response,

$$H_d(e^{j\omega}) = C$$

where C = constant

ii) The cutoff frequency ω_c for low pass and high pass,and ω_{c1} and ω_{c2} for band pass and band stop filters.

Iii)The number of samples of impulse response,N.

2. Determine the desired impulse response,$h_d(n)$ by taking Inverse fourier transform of the desired frequency response, $H_d(e^{j\omega})$

$$H_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})(e^{j\omega n})\, d\omega$$

3. Choose the desired window sequence w(n) defined for n=$-\frac{N-1}{2}$ to $+\frac{N-1}{2}$ from the table 1.Multiply $h_d(n)$ with w(n) to get the impulse response h(n) of the filter calculate N samples of the impulse response, for n =$-\frac{N-1}{2}$ to $+\frac{N-1}{2}$

$$\text{Impulse response, } h(n) = h_d(n) \times w(n)\,;\ \text{for } n = -\frac{N-1}{2} \text{ to } +\frac{N-1}{2}$$

The impulse response is symmetric with centre of symmetry at n=0,and so h(-n) = h(n).Hence it is sufficient if we calculate h(n) for n=0 to (N-1)/2

4. Take Z transform of the impulse response h(n) to get the non causal transfer function of FIR filter, $H_N(z)$

$$H_N(z) = Z\{h(n)\}=\sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n)z^{-n}$$

5. Convert the non causal transfer function, $H_N(z)$ to causal transfer function ,H(z) by multiplying $H_N(z)$ by $z^{-(N-1)/2}$

$$\text{Transfer function, } H(z) = z^{-\frac{N-1}{2}} \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(n)z^{-n}$$

6. Draw a suitable structure for realization of FIR filter.

Design verification:

1. Determine the frequency response, $H(e^{j\omega})$

Method-1: Choose a linear phase magnitude function $|He^{j\omega}|$ from table 1.Using h(n) obtain an equation for $|He^{j\omega}|$

Method-2: The frequency response, $|He^{j\omega}|$ can be obtained by replacing z by $e^{j\omega}$ in the transfer function, H(z).

Frequency response, $H(e^{j\omega}) = H(z)\,|_{z=e^{j\omega}}$

2. Calculate frequency response for various values of ω in the range 0 to π.

3. Calculate the magnitude response , $|He^{j\omega}|$ and sketch the magnitude response to verify the design.