1.9 Analysis and design of singly and doubly reinforced rectangular beams by Limit state method

The Concrete beam whose only tension zone of cross-section area is covered with steel rod is known as a **singly reinforced beam**.

ANALYSIS OF SINGLY REINFORCED BEAMS PROBLMS:

TYPE 1 PROBLEM:

GIVEN DATA: Ast in mm2 or number of bars with diameter, size of beam (b, D), type of concrete (fck), type of steel (fy), if load to be calculated then span is given. REQUIRED: Ultimate moment or factored moment or moment or resistance (Mu) or Mu & w.

Note:

1. Ultimate moment or factored moment (Mu) = 1.5 x working moment = 1.5 x M

2. Ultimate load or factored load (wu) = 1.5 x working load = 1.5 x w

DESIGN STEPS:

STEP 1: Note down the value for Xu,max/d by referring IS: 456-2000

F_y in N/mm^2	$X_{u,max}/d$
250	0.53
415	0.48
500	0.46

STEP 2: Determine depth of neutral axis Xu/d Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Where,

Xu = depth of neutral axis

Fy = characteristic tensile strength of steel in N/mm2 Ast = area of steel in tension in mm2

Fck = characteristic compressive strength of concrete in N/mm2 b = breadth or width of member

d = effective depth in mm

Effective depth (d) = overall depth (D) – effective cover (d’) Effective cover (d’) = clear cover + diameter of bar/2

Clear cover for beam = 25mm.

STEP 3: Compare Xu/d and Xu,max/d

If Xu/d < Xu,max/d, then section is under reinforced. The moment of resistance is calculated by

Mu = 0.87.fy.Ast.d. [1- (fy.Ast)/(fck.b.d)]

If Xu/d > Xu,max/d, then section is over reinforced. The moment of resistance is calculated by

Mu,lim = 0.149.fck.b.d2 for Fe250 steel.

Mu,lim = 0.138.fck.b.d2 for Fe415 steel.

Mu,lim = 0.133.fck.b.d2 for Fe500 steel.

If the section is balanced that is Xu/d = Xu,max/d then the limiting moment of resistance (Mu,lim) is calculated.

STEP 4: Working moment = M = Mu/1.5

The maximum bending moment for simply supported beam carrying UDL = wl2/8 Now equating maximum bending moment and working moment

$M = wl^2/8$, $w = 8M/ l^2$

Where w = total load = dead load + live load. DL

self-weight of beam= ρ.b.D =25.b.D kN/m

Live load = w-DL in kN/m.

PROBLEM 1.Find the depth of neutral axis of a singly reinforced R.C beam of 230mm width and 450mm effective depth. It is reinforced with 4 bars of 16mm diameter. Use M20 concrete and Fe415 bars. Also comment on the type of beam.

Given data: b=230mm,

d=450mm,

Ast=4-#16,

fck=20 N/mm2,

fy=415 N/mm2

Required: Xu

Solution:

Step1: As per IS: 456-2000 Xu,max/d = 0.48 for Fe415

Step2: Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Ast= no of bars x π(diameter)2/4 = 4x πx(16)2/4

= 504.24 mm^2

Xu/d = (0.87x415x804.24) / (0.36x20x230x450)

= 0.39

Xu = 0.39xd = 0.39x450 = 175.5mm.

Step3: By comparing Xu/d < Xu,max/d

Section is under-reinforced.

PROBLEM 2.A singly reinforced concrete beam 250mm width is reinforced with 4 bars of 25mm diameter at an effective depth 400mm. If M_{20} grade concrete and Fe_{415} bars are used. Compute moment of resistance of the section.

Given data: b=250mm, d=400mm, Ast=4-#25, fck=20 N/mm2, fy=415 N/mm^2 Required: Mu

Solution:

Step1: As per IS: 456-2000 Xu,max/d = 0.48 for Fe_{415}

Step2: Xu/d = (0.87.fy.Ast)/(0.36.fck.b.d)

Ast= no of bars x π(diameter)2/4

= 4x πx(25)2/4 = 1963.75 mm^2

Xu/d = (0.87x415x1963.75) / (0.36x20x250x400)

= 0.98

Step3: By comparing Xu/d > Xu,max/d

Section is Over-reinforced.

Step4:

Mu,lim=0.138.fck.bd^2

=0.138x20x250x400 ^{2}

= 110400000 N-mm = 110.4 kN-m

PROBLEM 3.A simply supported singly reinforced beam having 250mm wide and 500mm effective depth provided with Fe_{415} steel and M_{20} grade of concrete. Determine the ultimate moment of resistance of beam.

Given data: b=250mm,

d=500mm,

fck=20 N/mm2,

fy=415 N/mm^2

Required: Mu,lim

Solution:

$Mu,lim=0.138.fck.bd^2$

$=0.138x20x250x500^2$

= 172500000 N-mm

= 172.5 kN-m

PROBLEM 4.A rectangular section of 230x500mm is used as a simply supported beam for effective span of 6m. The beam consists of tensile reinforcement of 4000 mm2 and center of reinforcement is placed at 35mm from the bottom edge. What maximum total UDL can be allowed on the beam? Given M20 concrete and Fe415 steel.

Given data:

b=230mm,

D=500mm,

simply supported beam, l=6m,

Ast=4000mm2, d’=35mm,

fck=20 N/mm2,

f_y=415 N/mm^2

Required: w

Solution:

Step1: As per IS: 456-2000

Xu,max/d = 0.48 for Fe415

Step2:

Xu/d = (0.87.fy.Ast)/ (0.36.fck.b.d)

d=D-d'=500-35=465mm

Xu/d = (0.87x415x4000) / (0.36x20x230x465)

= 1.875

Step3: by comparing Xu/d > Xu,max/d

Section is Over-reinforced.

Step4: Mu,lim=0.138.fck.bd^2 =0.138x20x230x465^2

= 137259630 N-mm

= 137.26 kN-m

Step5: M=Mu/1.5

=137.26/1.5=91.506kN-m

Maximum bending moment for simply supported beam with UDL

M=wl2/8,

w=8M/l2

=8.91.506/62=20.33kN-m

ANALYSIS OF DOUBLY REINFORCED BEAM

Definition: the RCC beam section in which steel reinforcement is provided to resist both compression and tension is called doubly reinforced beam.

The circumstances under which doubly reinforced sections are provided:

1. When there are architectural restrictions on the depth of otherwise singly reinforced section.

2. Restriction in the depth at the location of beam at plinth level, along with the provision of ventilator between the ground level and the bottom of plinth beam.

3. In a continuous beam floor system, where the beam acts as a T-beam in the midspan and acts as a rectangular beam at the supports where the B.M may be much greater than at the mid span.

4. Where it is required to increase the stiffness of the beam.

5. It is found that the compression steel increases the rotation capacity and ductility

TYPE I PROBLEMS (Mu):

Given data: Ast, Asc, size of beam, effective cover for compression steel (d'), type of concrete (fck) and steel (fy). If load to be calculated then span is given.

Required: ultimate moment or factored moment or moment of resistance (Mu) and super imposed load (w).

Solution:

Step1: calculate Asc= no of bars x π (ϕc)2/4 mm2, Ast= no of bars x π (ϕt)2/4 mm2 Where, ϕc= diameter of compression steel, ϕt= diameter of tension steel

Step2: Xu,max= 0.46d for Fe500

0.48d for Fe415

0.53d for Fe250

Step3: stress in compression (fsc): from the table F of SP16 by linear interpolation Calculate d'/d

For Fe250 fsc=0.87.fy

Step4: Ast_2= Asc.fsc / (0.87. fy)

Step5: Ast= Ast_1+ Ast_2

Ast1= Ast- Ast_2

Step 6: depth of neutral axis

Xu/d= (0.87.fy.Ast1) / (0.36.fck.b.d) or Xu= 0.87.fy.Ast-fsc.Asc / (0.36.fck.b)

If, Xu < Xu,max section is under reinforced.

Therefore, calculate the moment of resistance by the following expression Mu-Mu,lim=fsc.Asc.(d-d')

Mu= 0.36. fck.b.xu.(d-0.42.xu) + fsc.Asc(d-d') If, Xu > Xu,max section is over reinforced.

Put Xu = Xu,max value and calculate moment of resistance by the following expression, Mu= 0.36. fck.b.xu,max.(d-0.42.xu,max) + fsc.Asc(d-d')

To calculate safe udl of live load: follow same steps as in singly reinforced beams.

Poblem 1 A doubly reinforced beam section is 250mm wide and 450mm deep to center of the tensile reinforcement. It is reinforced with 2 bars of 16mm diameter as compressive reinforcement at an effective cover 50mm and 4 bars of 25mm diameter as tensile steel. Using M15 concrete and Fe250 steel. Calculate the ultimate moment of resistance of the beam.

Given data:

b=250mm,

d=450mm,

fck=15 N/mm2,

fy=250 N/mm2,

d'=50mm,

Asc= 2- #16, Ast=4-#25

Required: Mu Solution:

Step1: calculating Asc= no of bars x π (ϕc)2/4

=2 πx(16)2/4=402.12mm2 Ast

= no of bars x π (ϕt)2/4=4x πx(25)2/4=1963.49mm^2

Step2: Xu,max=0.53d for Fe250

=0.53x450=238.5mm

Step3: stress in compression (fsc)

fsc=0.87.fy=0.87x250=217.5 N/mm2

step4: Ast2=(Asc. fsc)/(0.87. fy)

=(402.12x217.5)/(0.87x250)=402.12mm2

step5: Ast=Ast1+Ast2, Ast1

=Ast-Ast2=1963.49-402.12=1561.37mm2

step6: depth of neutral axis (Xu)

Xu= 0.87.fy.Ast1/ (0.36.fck.b)

= 0.87x250x1561.37/ (0.36x15x250)

=251.5mm

Xu> Xu,max section is over reinforced.

Step7: Mu=0.149.fck.b.d2+fsc.Asc.

(d-d')= 0.149x15x250x4502+217.5x402.12x (450- 50) =148.13x106 N-mm

Mu=148.13 kN-m.

Problem 2 A doubly reinforced beam section is 250mm wide and 500mm deep to the center of the tensile reinforcement. It is reinforced with 2 bars of 18mm diameter as compression reinforcement at an effective cover of 40mm and 4 bars of 25mm diameter as tensile reinforcement using M15 concrete and Fe415 steel. Calculate MR of the section.

Given data:

b=250mm,

d=500mm,

fck=15 N/mm^2,

fy=415 N/mm^2,

d'=40mm,

A_{sc}= 2-#18,

A_{st}=4-#25

Required: M_u

Solution:

Step1: calculating,

A_{st} =no's.xπ.$(\phi t)^2/4$

= $4x\pi x(25)^2/4$= 1963.49mm^2 A_{sc}

= no of bars x π $(\phi c)^2/4=2x\ \pi x(18)^2/4=508.93mm^2$

Step2:

$X_{u,max}$=0.48d for Fe415

=0.48x500

=240mm

Step3: stress in compression (f_{sc})d'/d

=40/500

=0.08

By referring table –F of sp16 d'/d fsc

0.05	355	Y1
0.08	?	Y
0.1	353	Y2

Y = Y1 + ((Y2-Y1)/(X2-X1))*(X-X1)

= 355+((353-355)/(0.1-0.05))*(0.08-0.05)=353.8 N/mm^2

F_{sc}= 353.8 N/mm^2

step4:

A_{st2} =(A_{sc}. f_{sc})/(0.87. fy)

=(508.93x353.8)/(0.87x415)=498.71mm^2

step5:

A_{st} =A_{st1}+A_{st2},

A_{st1} =A_{st}-A_{st2}

=1963.49-498.71

=1464.78mm^{2}step6: depth of neutral axis (X_u)

X_u = (0.87.fy.A_{st}-f_{sc}.A_{sc})/ (0.36.f_{ck}.b)

= (0.87x415x1963.49-353.8x508.93)/(0.36x15x250)

=391.74mm

X_u> $X_{u,max}$ section is over reinforced.

Step7: M_u=0.138.f_{ck}.b.d^2+f_{sc}.A_{sc}. (d-d')

= 0.138x15x250x500^2+353.8x508.93x (500-40)

=212.2x10^6 N-mm

M_u=212.2 kN-m.