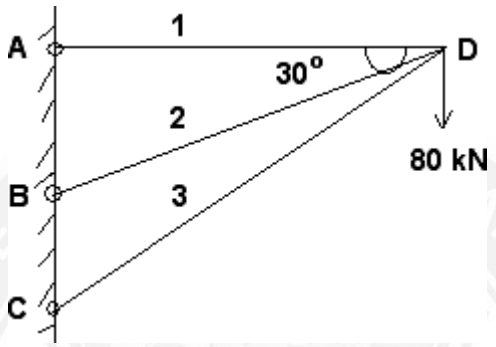
## 5.5. ANALYSIS OF RIGID FRAMES BY STIFFNESS MATRICES METHOD

### 5.5.1. NUMERICAL PROBLEMS ON PIN JOINTED FRAMES;

### PROBLEM NO:01

Using matrix stiffness method, analyze the truss for the member forces in the truss loaded as shown in figure. AE and L are tabulated below for all the three members.

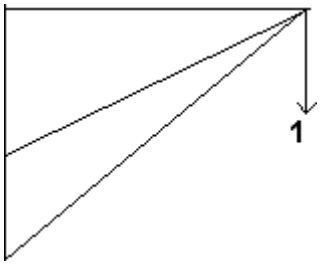



| Member | AE | L |
|---|---|---|
| AD | 400 | 400 |
| BD | 461.9 | 461.9 |
| CD | 800 | 800 |

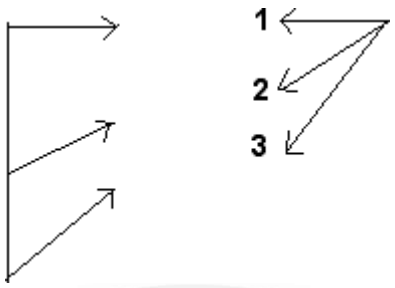
**Solution:**

- **Assigned Co-Ordinates:**

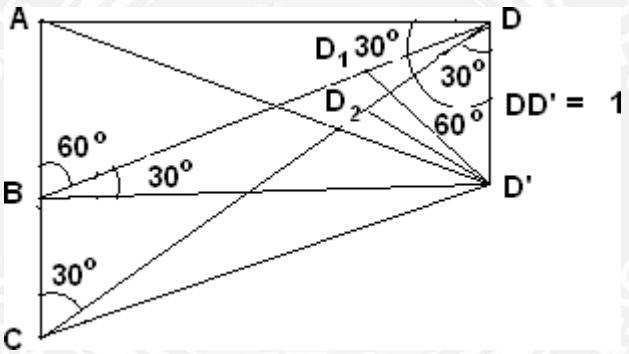
  **Global Co-Ordinates**



**Local Co-Ordinates**




- **Displacement Diagrams:**




- **Formation of [A] Matrix:**

  Apply unit displacement in

  DD'. Displacement along

  1, AD = 0

  Displacement along 2 and 3,

  $DD_1 = \cos 60^o = 0.5$ and $DD_2 = \cos 30^o = 0.866$

$$A = \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix}$$

- **Stiffness Matrix [K]:**

$$K = \frac{AE}{L}\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **System Stiffness Matrix(J):**

$$\mathbf{J = A^T . K . A}$$

$$= \begin{matrix} 0 & \_0.5 & \_0.866 \end{matrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \_0.5 \\ \_0.866 \end{bmatrix}$$

$$= \begin{matrix} 0 & -0.5 & -0.866 \end{matrix} \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix}$$

$$J = 1$$

$$J^{-1} = 1$$

- **Displacement Matrix(Δ):**

$$\mathbf{\Delta = J^{-1} . W}$$

$$\mathbf{= 1 \times 80 = 80\ mm}$$

- **Element Force (P):**

$$\mathbf{P = K . A . \Delta} \qquad = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix} 80$$

$$= \begin{bmatrix} 0 \\ -0.5 \\ -0.866 \end{bmatrix} 80$$

- **Final Force (P):**

$$= \begin{bmatrix} 0 \\ -40 \\ -69.28 \end{bmatrix}$$